SECTION A

Question 1)

(a) The common mode rejection ratio quantifies the ability of a differential amplifier to reject common mode signals. It is defined as the ratio of the differential gain to the common mode gain and is often specified in decibels.

[3]

(b) The voltage at the non-inverting v_+ is calculated from the potential divider formed from R_3 and R_4 with an input v_1 such that

$$v_+ = \frac{v_1 R_4}{R_3 + R_4}$$

Considering conservation of current at the inverting input v_- (with infinite input impedance) gives

$$\frac{v_2 - v_-}{R_1} = \frac{v_- - v_{out}}{R_2}$$

Hence

$$R_2(v_2 - v_-) = R_1(v_- - v_{out})$$

Therefore

$$v_- = \frac{R_2 v_2 + R_1 v_{out}}{R_1 + R_2}$$

Since the op-amp has infinite gain $v_+ = v_-$ giving

$$\frac{v_1 R_4}{R_3 + R_4} = \frac{R_2 v_2 + R_1 v_{out}}{R_1 + R_2}$$

Therefore

$$v_{out} = v_1 \frac{R_4}{R_1} \frac{R_1 + R_2}{R_3 + R_4} - v_2 \frac{R_2}{R_1}$$

[4]

(c)

$$v_{out} = A_{diff}(v_1 - v_2) + A_{cm}\left(\frac{v_1 + v_2}{2}\right)$$

To determine A_{diff} let $v_1 = \frac{v_{in}}{2}$ and $v_2 = -\frac{v_{in}}{2}$ such that $v_{out} = A_{diff} v_{in}$

Hence

$$v_{out} = \frac{v_{in}}{2} \frac{R_4}{R_1} \frac{R_1 + R_2}{R_3 + R_4} + \frac{v_{in}}{2} \frac{R_2}{R_1}$$

Therefore

$$A_{diff} = \frac{1}{2}\left(\frac{R_4}{R_1} \frac{R_1 + R_2}{R_3 + R_4} + \frac{R_2}{R_1}\right)$$

Likewise to determine A_{cm} let $v_1 = v_{in}$ and $v_2 = v_{in}$ such that $v_{out} = A_{cm}v_{in}$

Therefore

$$v_{out} = v_{in}\frac{R_4}{R_1}\frac{R_1 + R_2}{R_3 + R_4} - v_{in}\frac{R_2}{R_1}$$

Thus

$$A_{cm} = \frac{R_4}{R_1}\frac{R_1 + R_2}{R_3 + R_4} - \frac{R_2}{R_1}$$

[6]

(d) The CMRR is defined as

$$CMRR = \frac{A_{diff}}{A_{cm}}$$

Therefore

$$A_{diff} = \frac{1}{2}\left(\frac{R_4}{R_1}\frac{R_1 + R_2}{R_3 + R_4} + \frac{R_2}{R_1}\right)$$

Therefore

$$A_{diff} = \frac{1}{2}\frac{R_4(R_1 + R_2) + R_2(R_3 + R_4)}{R_1(R_3 + R_4)}$$

$$A_{diff} = \frac{1}{2}\frac{R_1R_4 + 2R_2R_4 + R_2R_3}{R_1(R_3 + R_4)}$$

Likewise

$$A_{cm} = \frac{R_4}{R_1}\frac{R_1 + R_2}{R_3 + R_4} - \frac{R_2}{R_1}$$

$$A_{cm} = \frac{R_4(R_1 + R_2) - R_2(R_3 + R_4)}{R_1(R_3 + R_4)}$$

$$A_{cm} = \frac{R_1R_4 - R_2R_3}{R_1(R_3 + R_4)}$$

Therefore

$$CMRR = \frac{A_{diff}}{A_{cm}} = \frac{\frac{1}{2}\frac{R_1R_4 + 2R_2R_4 + R_2R_3}{R_1(R_3 + R_4)}}{\frac{R_1R_4 - R_2R_3}{R_1(R_3 + R_4)}} = \frac{R_1R_4 + 2R_2R_4 + R_2R_3}{2(R_1R_4 - R_2R_3)}$$

[4]

(e) For the worst case CMRR we want the denominator to be maximum, i.e. we want to maximise $R_1R_4 - R_2R_3$ which means R_1 and R_4 should be maximum and R_2 and R_3 should be minimised, hence if resistors all resistors have the same nominal value R then the worst case $CMRR$ occurs when $R_1 = R(1 + t)$, $R_2 = R(1 - t)$, $R_3 = R(1 - t)$ and $R_4 = R(1 + t)$

[3]

(f) Putting $R_1 = R(1+t)$, $R_2 = R(1-t)$, $R_3 = R(1-t)$ and $R_4 = R(1+t)$ into the expression for the CMRR gives

$$CMRR = \frac{R_1R_4 + 2R_2R_4 + R_2R_3}{2(R_1R_4 - R_2R_3)} = \frac{R^2(1+t)^2 \ + 2R^2(1-t^2) + R^2(1-t)^2}{2[R^2(1+t)^2 - R^2(1-t)^2]}$$

$$CMRR = \frac{(1+t)^2 \ + 2(1-t^2) + (1-t)^2}{2[(1+t)^2 - (1-t)^2]}$$

$$CMRR = \frac{1 + 2t + t^2 \ + 2 - 2t^2 + 1 - 2t + t^2}{2[(1 + 2t + t^2) - (1 - 2t + t^2)]}$$

$$CMRR = \frac{4}{2[4t]} = \frac{1}{2t}$$

Hence if $t = 0.1\% = 0.001$

$$CMRR = \frac{1}{0.002} = 500 = 54 \text{ dB}$$

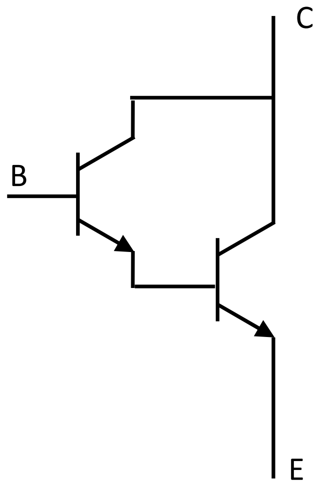
[5]

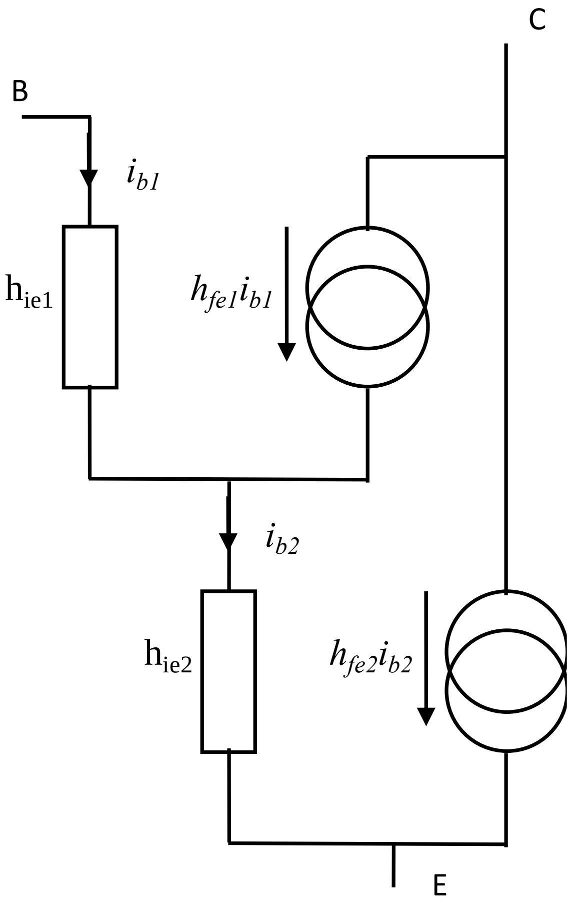
Question 2)

(a)
For Q2, $V_{CE} = 5$ V therefore $V_{E2} = 10 - 5 = 5$ V. Given $I_{E2} = 10$ mA, then R_E =500 Ω. Given $V_{E2} = 5$ V, $V_{B2} = V_{E1} = 5.7$ V therefore $V_{B1} = 6.4$ V, therefore voltage across R_1 is 3.6 V. Given current through R_1 is I_{B1} =1 μA then $R_1 = 3.6$ MΩ.

[4]

(b)
(i) Small signal model for the Darlington Pair is





(ii) For equivalent transistor $v_{be} = i_{b1}h_{ie1} + i_{b2}h_{ie2}$ but $i_{b2} = i_{b1} + h_{fe1}i_{b1} = i_{b1}(1 + h_{fe1})$

Therefore the equivalent base resistance is $h'_{ie} = v_{be}/i_{b1} = h_{ie1} + h_{ie2}\left(1 + h_{fe1}\right)$

(iii) Total collector current is $i_c = h_{fe1}i_{b1} + h_{fe2}i_{b2} = h_{fe1}i_{b1} + h_{fe2}i_{b1}(1 + h_{fe1})$

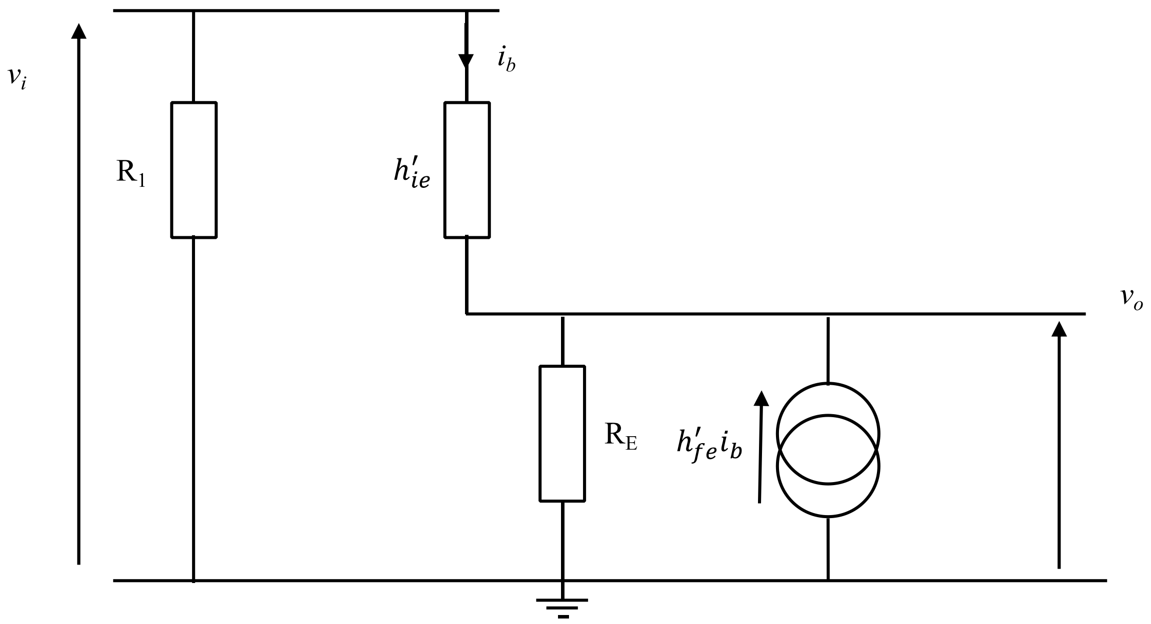
Therefore current gain is $h'_{fe} = i_c/i_{b1} = h_{fe1} + h_{fe2}(1 + h_{fe1}) = h_{fe1} + h_{fe2} + h_{fe1}h_{fe2}$

[6]

(c)

For the small signal model we will use the equivalent model of the Darlington Pair from (b), with small signal parameters $h'_{ie} = h_{ie1} + h_{ie2}\left(1 + h_{fe1}\right)$ and $h'_{fe} = h_{fe1} + h_{fe2} + h_{fe1}h_{fe2}$

Therefore $h'_{ie} = 50 \times 10^3 + 500 \times 101 = 100.5$ kΩ and $h'_{fe} = 100 + 100 + 100 \times 100 = 10200$



[4]

(d)
(i) Summing currents at emitter

$$i_b + h'_{fe}i_b = \frac{v_o}{R_E}$$

But $i_b = (v_i - v_o)/h'_{ie}$
Therefore

$$\frac{v_o}{R_E} = \frac{(1 + h'_{fe})(v_i - v_o)}{h'_{ie}}$$

Therefore

$$v_o\left(\frac{1}{R_E} + \frac{(1 + h'_{fe})}{h'_{ie}}\right) = \frac{v_i(1 + h'_{fe})}{h'_{ie}}$$

Therefore

$$\frac{v_o}{v_i} = \frac{1}{\frac{h'_{ie}}{R_E(1 + h'_{fe})} + 1} = \frac{1}{\frac{100.5 \times 10^3}{500 \times (1 + 10200)} + 1} = 0.981$$

(ii)

$$i_{in} = \frac{v_i}{R_1} + i_b = \frac{v_i}{R_1} + \frac{v_i - v_o}{h'_{ie}} = v_i$$

Therefore

$$r_{in} = \frac{v_i}{i_{in}} = \frac{v_i}{\frac{v_i}{R_1} + \frac{v_i - v_o}{h'_{ie}}} = \frac{1}{\frac{1}{R_1} + \frac{1 - v_o/v_i}{h'_{ie}}} = \frac{1}{\left(\frac{1}{3.6 \times 10^6} + \frac{1 - 0.981}{100.5 \times 10^3}\right)} = 2.1\ \text{M}\Omega$$

(iii)
Consider current i_x inputted at the output such that summing the currents at the emitter resistor gives

$$i_x + i_b h'_{fe} + i_b = \frac{v_o}{R_E}$$

setting $v_i = 0$ and considering current through h'_{ie} gives

$$\frac{0 - v_0}{h'_{ie}} = i_b$$

Therefore combining gives

$$i_x = \frac{v_0\left(1 + h'_{fe}\right)}{h'_{ie}} + \frac{v_o}{R_E}$$

hence the small-signal output resistance r_o is

$$r_o = \frac{v_o}{i_x} = \frac{1}{\frac{1 + h'_{fe}}{h'_{ie}} + \frac{1}{R_E}} = \frac{1}{\frac{10201}{100.5 \times 10^3} + \frac{1}{500}} = 9.7\ \Omega$$

[6]

C_i forms a high pass filter since

$$v_i = \frac{r_{in}}{r_{source} + r_{in} + \frac{1}{j\omega C_i}} v_{source}$$

Therefore $2(r_{source} + r_{in})\omega C_i = 1$
Hence

$$C_i = \frac{1}{(r_{source} + r_{in})\omega} = \frac{1}{(10^6 + 2.1 \times 10^6) \times 2\pi \times 300} = 171\ \text{pF}$$

Likewise

$$C_o = \frac{1}{(r_{load} + r_o)\omega} = \frac{1}{(20 + 9.7) \times 2\pi \times 300} = 18\ \mu\text{F}$$

In practice might want larger capacitor values e.g. C_0 =100 µF and C_i=1 nF to avoid attenuating signals at 300 Hz (with 3 dB attenuation reduced to 0.1 dB).

[5]

Question 3)

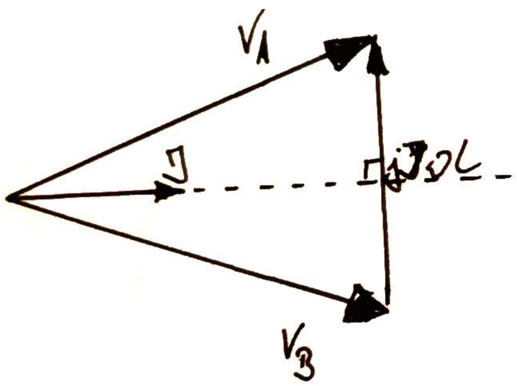
(a)
Three ideally sinusoidal phase voltages of the same magnitude shifted in time by ± 120° and likewise three ideally sinusoidal phase currents also shifted by ± 120° to each other (which we get in case we have balanced linear loads) do not need a return wire, but the currents compensate each other (Kirchhoff) already. So three wires only are needed if properly balanced. A small fourth wire at around zero volts can often be seen and is often elevated against the others for lightning protection (Faraday).
In the low-voltage system, where we as normal customers and most normal businesses are connected, at least four wires with a dedicated neutral line are used (3 phases, 1 neutral, protective earth typically only generated at your home). That allows us to plug in loads that only need one phase (most of our daily needs). However, therefore, the load is locally not balanced anymore and the neutral current (sometimes called zero sequence) is not zero. [5]

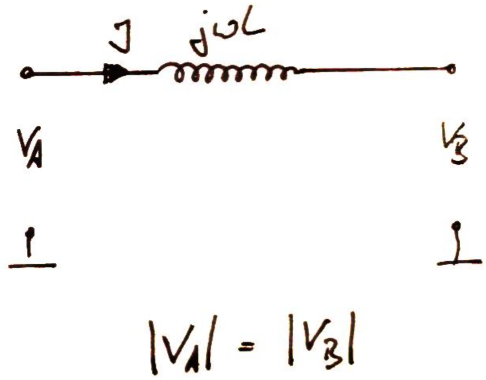
(b)
(i) Symmetrical three-phase fault to earth are faults that are modelled as a connection of all phases to ground at the location where the fault occurs. Properties required from the protection system:
- Discrimination: Only isolates the faulty part of the system.

- Stability: Breaker stays closed when the fault is outside the protected zone.
- Sensitivity: Ability to set the current level at which the beaker opens. [3]

(ii) A breaker operates by moving metal contacts apart. The inductivity of loads and the lines themselves cause an arc. The arc current is also ac and reaches zero eventually. The breaker removes the ionisation products by a blast of gas or fluid (air, oil, SF6) to prevent arc re-striking.
An isolator in series with circuit breaker opens when current has been interrupted and provides a barrier, typically with a large gap (in contrast to the hidden short gap in the breaker) also visible for maintenance crews. [3]

(c)





[5]

(d)

$$P_{\text{max}} = \sqrt{3}\,130\text{ A}\cdot\ 33\text{ kV} = 7.43\text{ MW}$$

Power lost and reactive power of the line:

$$P_{\text{line}} = 3\cdot(130\text{ A})^2\cdot 24\ \Omega = 1.22\text{ MW}$$

$$Q_{\text{line}} = 3\cdot(130\text{ A})^2\cdot 88\ \Omega = 4.46\text{ MVA}$$

Power at the PV plant:

$$S^2 = P^2 + Q^2 = [(7.43+1.22)^2 + 4.46^2]\ (\text{MVA})^2$$

$$= 94.71\ (\text{MVA})^2$$

[4]

(e)

$$S = 9.73\text{ MVA}$$

Required compensation:

$$Q_{\text{line}} = 4.46\text{ MVar}$$

$$V = \frac{S}{\sqrt{3}\cdot 130\text{ A}} = 43.2\text{ kV}$$

$$= 94.71\ (\text{MVA})^2$$

Star connection of capacitors:

$$Q = \frac{3V_\mathrm{c}^2}{X_\mathrm{c}}$$

$$X_c = \frac{V_c^2}{Q} = \frac{(43.2\ \mathrm{kV})^2}{4.46\ \mathrm{MVar}} = 418\ j\Omega$$

$$X_c = \frac{3V_c^2}{Q} = \frac{3(43.2\ \mathrm{kV})^2}{4.46\ \mathrm{MVar}} = \frac{1}{j\omega C}$$

$$C = 7.6\ \mu\mathrm{F}$$

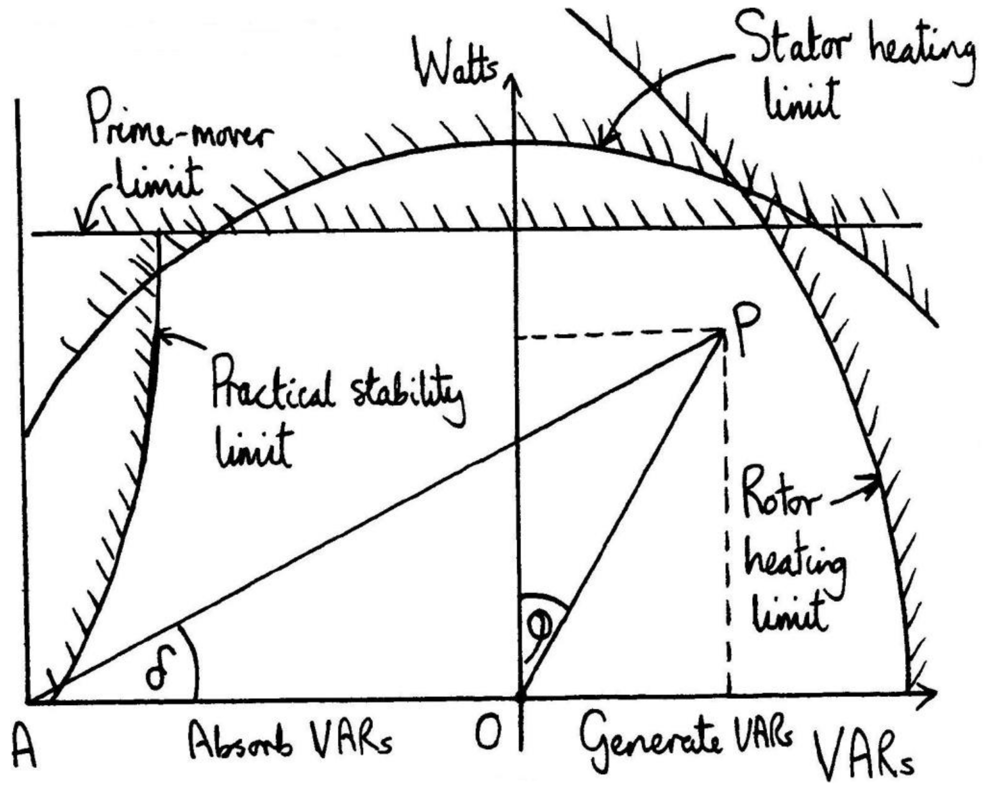
[5]

Question 4)

(a)

The vertical axis in the diagram represents real power, and the horizontal axis reactive power.

- For a machine operating from the infinite bus, V is fixed. The synchronous reactance, Xs, is also fixed, and so the length AO may be determined.
- The real power is limited by the maximum mechanical power which the prime-mover can provide, i.e., a horizontal line at Pmax (prime-mover limit).
- The generator has a volt-amp rating given by 3VI. Since 3VI is the length OP, i.e., a circle about O with radius 3VImax. Because this limitation is due to the heating of the stator windings owing to RI^2 losses, it is known as the stator heating limit. The length AP is limited by the maximum excitation voltage, E_{max} (V and Xs are fixed). In turn, since E is proportional to the rotor field current, E_{max} is physically limited by the maximum rotor field current, which is the maximum current the rotor winding can tolerate without overheating (circle of radius $3VE_{max}/Xs$ is drawn, centre A).



[5]

(b)
Steam power (i.e., all large thermal plants with a furnace for combustion gas and steam production, e.g., coal, gas, nuclear, etc.): High speed machines with cylindrical rotors as steam turbines most efficient at high speed. Therefore, the generator has only few poles, e.g., two poles (3000 rpm) or four pole (1500 rpm). Rotors must be thin to reduce centrifugal mechanical stresses and long to still provide enough power. The rotor experiences constant field (rotating field is perfectly in sync with mechanical rotation so that a point on the rotor does not experience any temporal changes of the magnetic conditions). Therefore, there is no need to laminate steel sheets. Instead, the rotor can be made out of massive solid steel pieces that are (hot) formed.
Water power (and similarly slow prime movers): salient-pole rotor for low speed machines. Hydro-turbines are most efficient at low speed. Therefore, the generators have many poles, e.g., 30 poles (200 rpm) or more. The diameter is large (large torque at low speed to still achieve a high power level mechanically). The design of the rotor (and sometimes also the stator) is highly modular. Separately wound poles are assembled onto a steel wheel (with spokes to reduce material need). [4]

(c)

The per-unit system is a representation in which physical quantities are expressed in dimensionless form (p.u.) by normalizing them with corresponding base values. Its significance is that it simplifies the analysis and avoids changes in voltages due to transformers in a large power system with many levels and branches. [3]

(d)

$$P = 3V_{\mathrm{ph}} I_{\mathrm{ph}} \cos(\varphi)$$

$$I_{\mathrm{ph}} = \frac{P}{3V_{ph} cos\ (\varphi)} = \frac{250\ \mathrm{MW}}{3 \cdot 11\ kV/\sqrt{3} \cdot 0.75} = 17.50\ \mathrm{kA}$$

$$E = \sqrt{\left(I_{\mathrm{ph}} X_{\mathrm{S}} \cos\varphi\right)^2 + \left(V_{\mathrm{ph}} + I_{\mathrm{ph}} X_{\mathrm{S}} \sin\varphi\right)^2}$$

$$= \sqrt{(17.5\ \mathrm{kA} \cdot 0.3\ \Omega \cdot 0.75)^2 + \left(\frac{11\ \mathrm{kV}}{\sqrt{3}} + 17.5 kA \cdot 0.3\ \Omega \cdot 0.66\right)^2}$$

$$= 10.58\ \mathrm{kV}$$

$$\sin\delta = \frac{I_{\mathrm{ph}} X_{\mathrm{S}} \cos\varphi}{E} = 0.372$$

$$\delta = 21.86°$$

[6]

(e)

$$I_{\mathrm{ph}} = \frac{P}{3V_{\mathrm{ph}}} = 13.12\ \mathrm{kA}$$

$$E = \sqrt{V_{\mathrm{ph}}^2 + \left(I_{\mathrm{ph}} X_{\mathrm{S}}\right)^2} = 7.47\ \mathrm{kV}$$

[4]

(f)
Reduction from 17.5 kA (Part d) to 13.1 kA (e) for same power at the load.
=> VA rating of generator reduced to 13.1/17.5 = 75%
=> I^2R stator losses: $(13.1/17.5)^2$ = 56%
=> Excitation voltage reduced to 7.47/10.58 = 71%
=> field power losses reduced by 50% (squared voltage due to V^2/R)

[3]

Question 5)

(a)

From

$$\nabla \times \underline{E} = -\dot{\underline{B}}$$

$=-j\omega\mu_0\underline{H}$

$\nabla \times \underline{E}$ has only one component in the x direction= -j dE_Y/dz

Thus

$$H_x = -\frac{E_0}{Z_0}e^{j(\omega t-\beta z)}$$

With

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Average power:

$$\frac{1}{2}Re\left\{\underline{\overline{E}} \times \underline{\overline{H}}^*\right\} = \frac{E_O^2}{2Z_0} \qquad [7]$$

(b) loop is perpendicular to the direction of H. Therefore the flux through the loop is:

$$\frac{\pi}{4}d^2\mu_0\frac{E_0}{Z_0}e^{j(\omega t-\beta z)}$$

emf= $-\frac{\partial\phi}{\partial t} = \frac{\pi}{4}d^2\mu_0\frac{E_0}{Z_0}e^{j(\omega t-\beta z)}$

$emf_{rms} = \frac{\pi}{4\sqrt{2}}d^2\mu_0\frac{E_0}{Z_0} = \frac{\pi^2}{2\sqrt{2}\lambda}d^2E_0$ [6]

(c) Power density at a distance r:

$P/2\pi r^2 = E_O^2/2Z_0$

Hence, given that $Z_0=377\Omega$

$E_O^2=120P/r^2$

In absence of a receiver, the induced emf V_0, for an input resistance of 50Ω, and power=5 x 10^{-9} W:

$V_0^2/2x50=5x10^{-9}$

Therefore $V_0^2=5x10^{-7}$ V^2

From (b):

emf$_{rms}$= $\frac{\pi^2}{2\sqrt{2}\lambda} d^2 E_0$

hence:

$5x10^{-7}$ V^2= $\left(\frac{\pi^2}{2\sqrt{2}\lambda} d^2\right)^2 E_0^2 = \left(\frac{\pi^2}{2\sqrt{2}\lambda} d^2\right)^2 \frac{120P}{r^2}$

For a 800MHz frequency

800MHz=c/λ

Hence: λ=0.375m

d=0.1m

P=2kW

r=$\frac{\pi^2}{2\sqrt{2}\lambda} d^2 \times \frac{120P}{5x10^{-7}}$=45.6km [7]

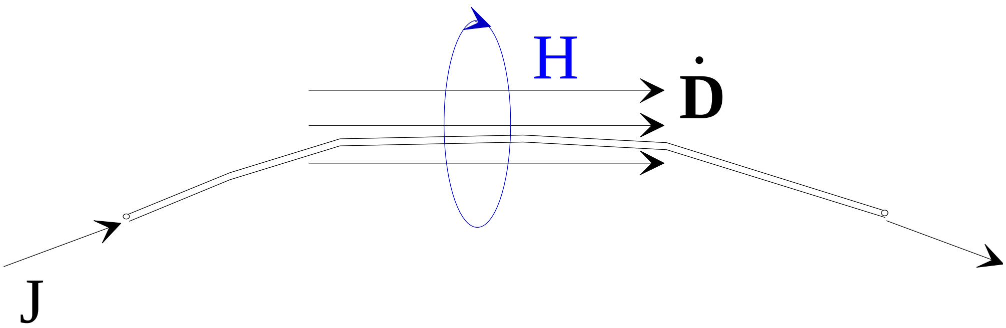
(d) If the power requirement is 50% lower, The induced emf can be reduced by a factor of $\sqrt{2}$

Hence angle = $\cos^{-1}(1/\sqrt{2})=45^0$

[5]

Question 6)

(a) We can work out the inductance per unit length L of the cable from Ampere's law:



$$\oint_C \underline{H}\cdot\underline{dl} = \int_S \left(\underline{J}+\underline{\dot{D}}\right)\cdot\underline{dS}$$

Where S is any surface bounded by a closed curve C.

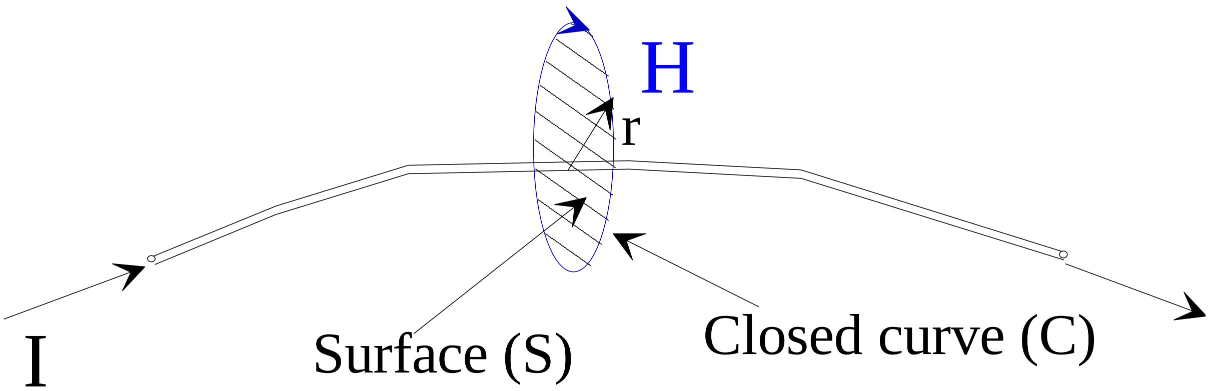
In the absence of a changing electric field we can simplify the equation to:

$$\oint_C \underline{H} \cdot \underline{dl} = \int_S \left(\underline{J} \right) \cdot \underline{dS}$$

Since J is the current density then:

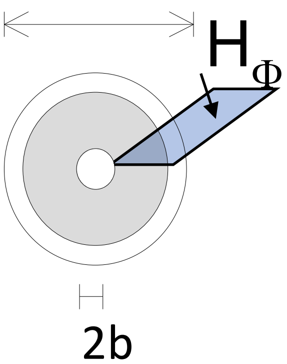
$$\int_S \left(\underline{J} \right) \cdot \underline{dS} = I$$

for the surface shown below



The inner conductor carries current flowing in one direction, the outer in the opposite direction. Thus, summing the currents for a surface which includes both inner and outer conductors gives a total current of 0, hence a field strength of 0

The field strength H in the cable is then:



$$H_\phi = \frac{I}{2\pi r}$$

The total magnetic flux per unit length is derived by integrating the field strength between inner and outer conductors.

dS=drdl

$$\psi = \int_a^b \mu H_\phi dr = \frac{\mu I}{2\pi} ln\left(\frac{a}{b}\right)$$

Since

$$L = \frac{\psi}{I}$$

the inductance per unit length L is:

$$L = \frac{\mu}{2\pi} ln\left(\frac{a}{b}\right)$$

[9]

(b) The characteristic impedance at any point is:

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{\mu_0}{2\pi} ln(\,a/b)}{\frac{2\pi\varepsilon_0\varepsilon_r}{ln(\,a/b)}}}$$

$$Z_0 = \frac{ln(\,a/b)}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}}$$

For a=3mm, b=1.5mm and ε_r=2:

Z_0=29.4Ω

[9]

(c) The power reflection coefficient is:

$$|\rho_L|^2 = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|^2 = \left|\frac{50 - 29.4}{50 + 29.4}\right|^2 \approx 0.07$$

To reduce the reflected power to zero the transmission line impedance must be increased to 50Ω.

By decreasing the diameter of the inner conductor, the new diameter x should be:

ln(3/x)/ln(2)=50/29.4=

x=3/$e^{1.18}$=0.92mm [7]