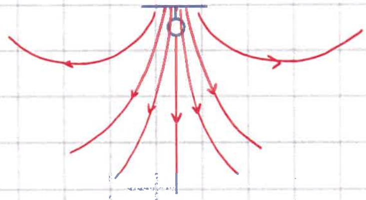
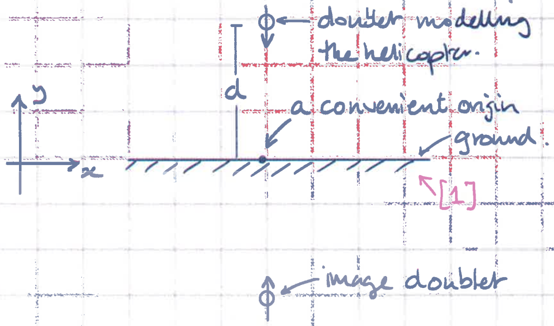
1. The modelled flow is a doublet aligned with the vertical axis.



a) The helicopter is hovering distance d above the ground. An image doublet is required so that the vertical velocity at the ground is zero. For an x-wise doublet with strength $2\pi\mu$ at the origin, $F(z) = -2\pi\mu/2\pi z = -\mu/z$. This has positive x-velocity along the x-axis. Substitute $z \mapsto z e^{-i\alpha}$ to rotate the flow by angle α. For the helicopter doublet, $\alpha = -\pi/2$ so $F(z) = -\mu/z e^{i\pi/2} = i\mu/z$.
For the image doublet, $F(z) = -i\mu/z$. Their positions are shifted with $z \mapsto z - id$ and $z \mapsto z + id$.



$F(z) = \dfrac{i\mu}{z - id}$

[1]

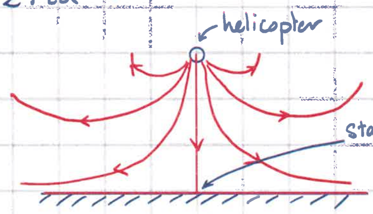
$F(z) = \dfrac{-i\mu}{z + id}$

$$F(z) = i\mu\left(\frac{1}{z-id} - \frac{1}{z+id}\right) \quad \text{[2]}$$
$$= i\mu\,\frac{z+id-(z-id)}{z^2+d^2}$$
$$= -\frac{2d\mu}{z^2+d^2}$$

Most students answered this well.

Few students sketched the streamlines well. You should start with the stagnation points and then the stagnation streamlines.

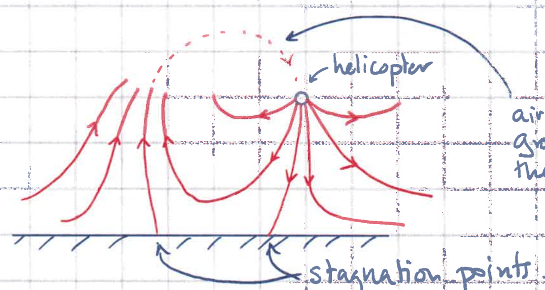
b) i) When $U = 0$, the flow is left/right symmetric around the y axis with a stagnation point at the origin.



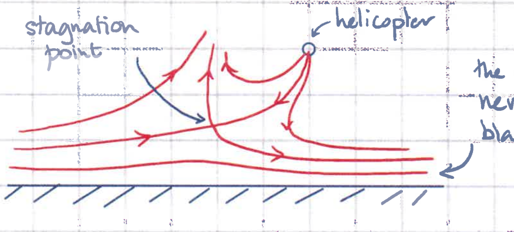
z/d = non-dimensional position

We now add $F(z) = Uz$ onto this complex potential: $F(z) = Uz - \dfrac{2d\mu}{z^2+d^2} = d\left\{U\dfrac{z}{d} - \dfrac{\mu}{d^2}\,\dfrac{2}{(z/d)^2+1}\right\}$

ii) When $U \ll \mu/d^2$ the pair of doublets dominate the flow between the helicopter and the ground. The stagnation point remains close to the origin. There must also be another stagnation point upstream.



iii) When $U \gg \mu/d^2$ the uniform flow dominates the flow between the helicopter and the ground. The stagnation points move above and below the ground.



c) Substitute $z/d \mapsto z$ and rescale F to obtain the simple form $F(z) = \dfrac{Ud^2}{\mu}z - \dfrac{2}{z^2+1} = Vz - \dfrac{2}{z^2+1}$

The minimum value of V at which the helicopter will avoid sucking dust from the ground occurs where the stagnation points on the ground coincide. No students spotted this, so nobody answered this well.

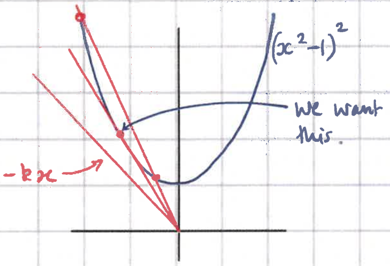
$$\frac{dF}{dz} = V + 2(z^2+1)^{-2}(2z) = V + 4z(z^2+1)^{-2} = 0 \quad \text{at a stagnation point.}$$

We need the solution on $z = x$

Find the value of V at which two real solutions to $V + 4x(x^2+1)^{-2} = 0$ coincide.

$\Rightarrow \quad (x^2+1)^2 = -kx \qquad$ where $k \equiv 4/V$

Sketch both sides:



This sketch shows lines with no real solutions, two real solutions, and one real solution, which is the one we want to find.

At the desired solution, the lines touch and their gradients are equal.

$$\left.\begin{aligned} &\Rightarrow (x^2+1)^2 = -kx \\ &\text{and } 4x(x^2+1) = -k \end{aligned}\right\} \text{ solve simultaneously}$$

Eliminate k first: $(x^2+1)^2 = 4x^2(x^2+1)$

$\Rightarrow \quad x^2+1 = 4x^2 \qquad$ or $x^2 = -1$, which are not real solutions

$\Rightarrow \quad 3x^2 = 1$

$\Rightarrow \quad x = \pm(1/3)^{1/2} \qquad$ and we want the solution at $x = -(1/3)^{1/2}$

Substitute back in to find k: $\quad k = 4(1/3)^{1/2}(4/3) = \dfrac{16}{3\sqrt{3}} \quad \Rightarrow \quad V \equiv 4/k = \dfrac{3\sqrt{3}}{4}$

The minimum value of V at which the helicopter will avoid sucking dust from the ground is $V = 3\sqrt{3}/4$. [9]

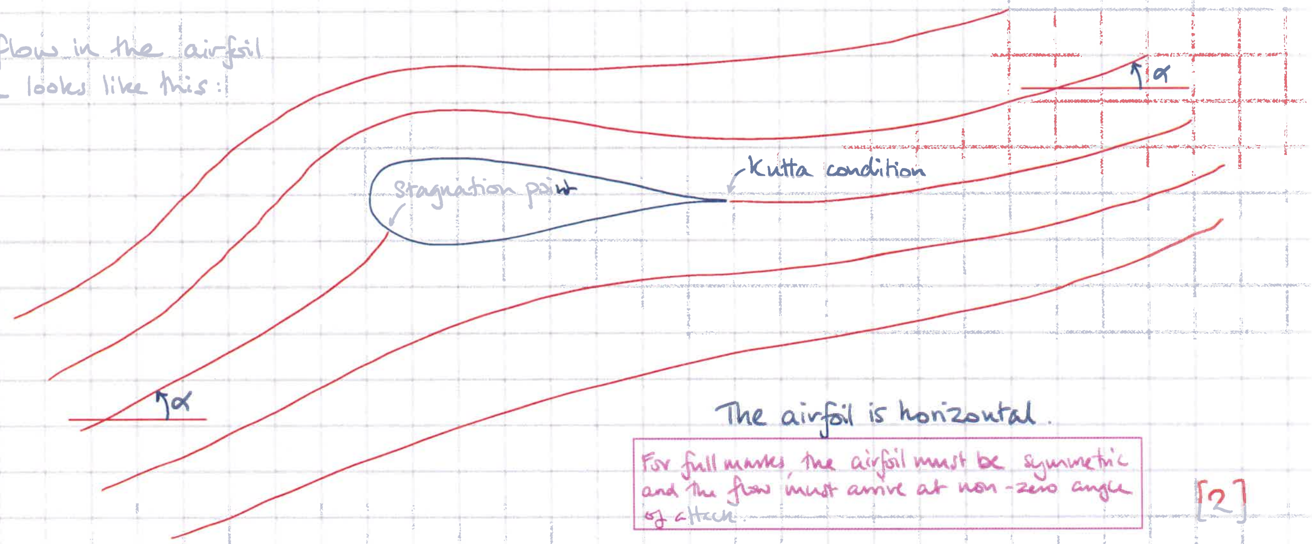
d) The doublet crudely models the kinematics (ie motion) of the flow through the helicopter blades but it does not lead to realistic dynamics (ie forces). Firstly, the pressure at the blades is $-\infty$. Secondly the momentum change and stagnation pressure change across the blades is zero, meaning that there is no force from the flow on the helicopter and no K.E. input into the flow by the helicopter. [2]

A more realistic model would incorporate a stagnation pressure increase and a momentum change at the rotor blades. These models are called "actuator disk" models.

There were some creative answers to this question, which received full marks.

Matthew Juniper 2026

2. a) The flow in the airfoil plane looks like this:

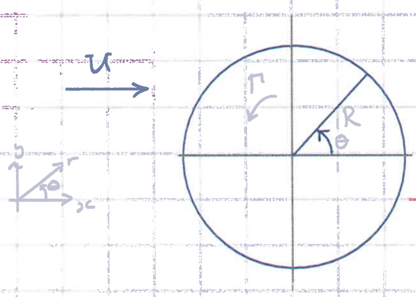


The airfoil is horizontal.

For full marks, the airfoil must be symmetric and the flow must arrive at non-zero angle of attack.

[2]

b)



The flow around a cylinder without circulation is a uniform flow plus an x-wise doublet whose strength, μ, creates a circular streamline at $r=R$. The strength, μ, can be derived in many ways, e.g.:

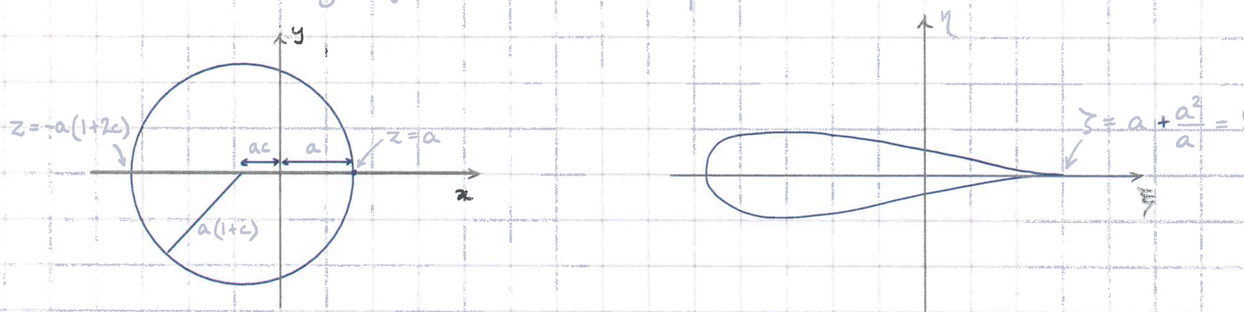
$$u_r = U\cos\theta + \frac{\mu\cos\theta}{2\pi r^2} = 0 \text{ at } r=R \quad \Rightarrow \quad -\frac{\mu}{2\pi} = R^2 U$$ [2]

Adding a vortex at the origin and substituting $z \mapsto z e^{-i\alpha}$ to create a flow at incidence α: [2]

$$F(z) = U z e^{-i\alpha} + \frac{UR^2}{z} e^{+i\alpha} - \frac{i\Gamma}{2\pi}\ln z + \text{constant}$$

c) The Kutta condition states that the physical value of Γ is that which causes the flow to leave the trailing edge smoothly. This models the influence of viscosity through the generation of boundary layers that separate from the surface unless and until the Kutta condition is satisfied. [2]

d) There are many ways to calculate the circulation from the Kutta condition. First we must locate the trailing edge in the z and ζ-planes.



The trailing edge is at $z = a$, which maps to $\zeta = 2a$. For the Kutta condition to be satisfied, the rear stagnation point needs to sit here.

Now we consider the complex potential around the above cylinder.

$$F(z) = Uze^{-i\alpha} + \frac{UR^2}{z}e^{+i\alpha} - \frac{i\Gamma}{2\pi}\ln z + \text{constant}.$$ for a cylinder with radius R at the origin

$$\Rightarrow F(z) = U(z+ac)e^{-i\alpha} + \frac{Ua^2(1+c)^2}{(z+ac)}e^{+i\alpha} - \frac{i\Gamma}{2\pi}\ln(z+ac) + \text{constant}$$ for our cylinder.

$$\Rightarrow \frac{dF}{dz} = Ue^{-i\alpha} - \frac{Ua^2(1+c)^2}{(z+ac)^2}e^{+i\alpha} - \frac{i\Gamma}{2\pi(z+ac)} = 0 \quad \text{at} \quad z=a$$ if the rear stagnation point is at $z=a$

$$\Rightarrow \quad Ue^{-i\alpha} - \frac{\cancel{Ua^2(1+c)^2}}{\cancel{(a+ac)^2}}e^{+i\alpha} - \frac{i\Gamma}{2\pi(a+ac)} = 0$$

$$\Rightarrow \quad -2Ui\sin\alpha - \frac{i\Gamma}{2\pi(a+ac)} = 0$$

$$\Rightarrow \quad \Gamma = -4\pi Ua(1+c)\sin\alpha$$ [8]

e) $$C_L = \frac{\text{lift force per unit length}}{\frac{1}{2}\rho U^2 L} = \frac{-\rho U\Gamma}{\frac{1}{2}\rho U^2 L}$$ where L is the chord length

The leading edge is at $z = -a(1+2c)$, which maps to $\zeta = -a(1+2c) - \dfrac{a^2}{a(1+2c)}$

$\Rightarrow$ The chord length, L, is

$$2a + a(1+2c) + \frac{a}{1+2c} = \frac{(3a+2ac)(1+2c)+a}{1+2c} = \frac{4a+8ac+4ac^2}{1+2c} = \frac{4a(1+c)^2}{1+2c}$$

$$\Rightarrow C_L = \frac{\cancel{\rho U}(\cancel{4}^{2}\pi \cancel{U}\cancel{a}\cancel{(1+c)}\sin\alpha)(1+2c)}{\cancel{\tfrac{1}{2}}\cancel{\rho U^2}\cancel{4a}(1+c)^{\cancel{2}}} = 2\pi\sin\alpha\left(\frac{\overbrace{1+2c}^{=1+c+2c-c}}{1+c}\right) = 2\pi\sin\alpha\left(1+\frac{c}{1+c}\right)$$

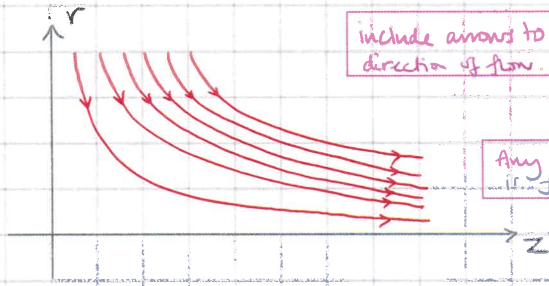
If α is small then $\sin\alpha \simeq \alpha$. If the airfoil is thin then $c \ll 1$ [4]

$$\Rightarrow C_L \simeq 2\pi\alpha$$

Parts (c) and (d) were well answered.

Matthew Juniper 2026

3. a) In the (r,z)-plane, $u_r = -\alpha r$ and $u_z = 2\alpha z$.
This is similar to a 2D stagnation point flow but with u_z doubled.



include arrows to indicate direction of flow.

Any orientation is fine

[2]

Show that $\nabla \cdot \underset{\sim}{u} = 0$ Substitute into the expression for $\nabla \cdot \underset{\sim}{f}$ in the question:

this is zero because v_θ is not a function of θ.

$$\nabla \cdot \underset{\sim}{u} = \frac{1}{r}\frac{\partial}{\partial r}(-\alpha r^2) + \frac{1}{r}\frac{\partial v_\theta(r)}{\partial \theta} + \frac{\partial}{\partial z}(2\alpha z)$$

This was well-answered.

$$= -\frac{\alpha}{r}(2r) + 2\alpha = -2\alpha + 2\alpha = 0$$ as required. [2]

b) $(\underset{\sim}{u} \cdot \nabla)\underset{\sim}{\omega}$ is the velocity in the direction of the gradient of the vorticity, which is the convective derivative of the vorticity. This is the rate of change of vorticity due to convection through the fluid. [1]

$(\underset{\sim}{\omega} \cdot \nabla)\underset{\sim}{u}$ is the vorticity in the direction of the gradient of the velocity. If this is non-zero then there is a source or sink of vorticity due to vortex stretching or vortex tilting. [1]

$\nu \nabla^2 \underset{\sim}{\omega}$ is the viscosity multiplied by the curvature of the vorticity. This is the diffusion of vorticity due to viscous effects. [1]

Only 3/4 of the students knew this.

Apply the vorticity transport equation to $\omega_z(r)$.

$$\Rightarrow (\underset{\sim}{u} \cdot \nabla)\omega_z = u_r \frac{\partial}{\partial r}\omega_z = -\alpha r \frac{d\omega_z}{dr}$$ because ω_z does not depend on θ or z

and $$(\underset{\sim}{\omega} \cdot \nabla)\underset{\sim}{u} = \omega_z \frac{\partial}{\partial z} u_z = 2\alpha\,\omega_z$$ because $\omega_r = \omega_\theta = 0$

and $$\nu \nabla^2 \omega_z = \frac{\nu}{r}\frac{d}{dr}\left(r\frac{d\omega_z}{dr}\right)$$ because ω_z does not depend on θ or z [3]

Many students did excessively long calculations for this simple question.

$$\Rightarrow \underbrace{-\alpha r\frac{d\omega_z}{dr}}_{①} = \underbrace{2\alpha\,\omega_z}_{②} + \underbrace{\frac{\nu}{r}\frac{d}{dr}\left(r\frac{d\omega_z}{dr}\right)}_{③} \quad [1]$$

c) If $\omega_z = \omega_0 \exp\left(-\frac{\alpha r^2}{2\nu}\right)$ then $\frac{d\omega_z}{dr} = -\frac{r\alpha}{\nu}\omega_z$

and $$\frac{d}{dr}\left(r\frac{d\omega_z}{dr}\right) = \frac{d}{dr}\left(-\frac{r^2\alpha}{\nu}\omega_z\right) = -\frac{\alpha}{\nu}\left(2r\,\omega_z + r^2\frac{d\omega_z}{dr}\right) \Rightarrow ③ = -\alpha\left(2\omega_z + r\frac{d\omega_z}{dr}\right)$$

The RHS of [1] is $2\alpha\omega_z - 2\alpha\omega_z - r\alpha\frac{d\omega_z}{dr} = -\alpha r\frac{d\omega_z}{dr}$ = LHS of [1] [1]

A characteristic radius is found when $\frac{\alpha r^2}{2\nu} = 1 \Rightarrow r_c = \left(\frac{2\nu}{\alpha}\right)^{1/2}$ [1]

This was well answered

d) $V_\theta(r) = \frac{1}{r}\int_0^r \rho\,\omega_z(\rho)\,d\rho$ where ρ is a dummy variable to avoid confusion with r.

Note that $\frac{d\omega_z}{d\rho} = -\frac{\alpha}{\nu}\rho\,\omega_z(\rho)$ from previous page

$$\Rightarrow \quad -\frac{\nu}{\alpha}\frac{d\omega_z}{d\rho} = \rho\,\omega_z(\rho)$$

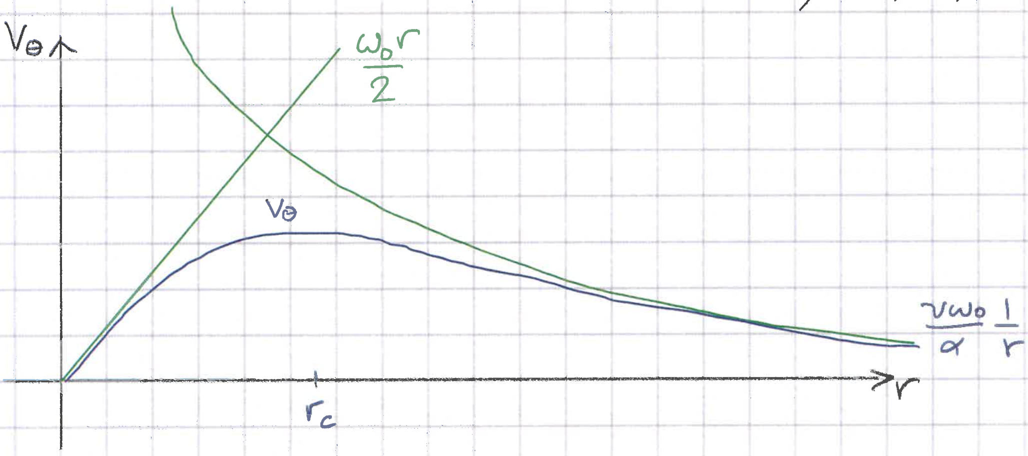
$$\Rightarrow \int_0^r \rho\,\omega_z(\rho)\,d\rho = -\frac{\nu}{\alpha}\Big[\omega_z\Big]_0^r = -\frac{\nu}{\alpha}\Big\{\omega_z(r) - \omega_z(0)\Big\} = \frac{\nu}{\alpha}\Big\{\omega_0 - \omega_z(r)\Big\}$$

$$\Rightarrow V_\theta(r) = \frac{\nu\omega_0}{\alpha}\,\frac{1}{r}\left\{1 - \exp\left(-\frac{\alpha r^2}{2\nu}\right)\right\}$$

In the best answers, the students (i) derived the algebraic expression (ii) calculated its asymptotes (iii) sketched the asymptotes and then (iv) sketched the line.

As $r \to \infty$, $V_\theta \to \frac{\nu\omega_0}{\alpha}\frac{1}{r}$

As $r \to 0$, $\exp\left(-\frac{\alpha r^2}{2\nu}\right) \to 1 - \frac{\alpha r^2}{2\nu}$ so $V_\theta(r) \to \frac{\cancel{\nu}\omega_0}{\cancel{\alpha}}\,\frac{1}{\cancel{r}}\,\frac{\cancel{\alpha} r^{\cancel{2}}}{2\cancel{\nu}} = \frac{\omega_0 r}{2}$



[6]

e) Vorticity is diffused through viscous effects, which continuously dissipates energy. However, vorticity is generated continuously through vortex stretching. In the steady state found in the Burgers vortex, these two effects balance everywhere in the fluid. Note that work needs to be done on the fluid to enable vortex stretching. [2]

Matthew Juniper 2026

(a) For high Reynolds number flows, $Re \gg 1$

$$\delta \ll L, \quad \text{or} \quad \frac{\delta}{L} \ll L$$

(b) horizontal velocity u : Scale U.
horizontal distance : x : Scale L.
vertical distance y : Scale δ.

$$\Rightarrow \frac{\partial u}{\partial x} \sim \frac{U}{L}, \quad \frac{\partial u}{\partial y} \sim \frac{U}{\delta}$$

(c) $\frac{\partial u}{\partial x} \sim \frac{U}{L}$, $\frac{\partial v}{\partial y} \sim \frac{\alpha}{\delta}$ where α is the scale of v.

Continuity eqn: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$O\left[\frac{U}{L}\right] \quad O\left[\frac{\alpha}{\delta}\right]$$

α must be of order $\alpha \sim \frac{\delta}{L} U$

(d) x - momentum eqn

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$O\left[\frac{U^2}{L}\right] + O\left[\frac{\delta}{L} U \frac{U}{\delta}\right] = -\frac{1}{\rho}\frac{\partial p}{\partial x}\frac{L}{U^2} + O\left[\frac{\nu U}{L^2}\right] + O\left[\frac{\nu U}{\delta^2}\right]$$

Divided by U^2/L, $O[1] + O[1] = -\frac{1}{\rho}\frac{\partial p}{\partial x}\frac{L}{U^2} + O\left[\frac{1}{Re}\right] + O\left[\left(\frac{\delta}{L}\right)^{-2}\frac{1}{Re}\right]$

The last term is indetermined $\frac{1/Re}{(\delta/L)^2} \sim \frac{0}{0} \rightarrow ?$

① If $\frac{1/Re}{(\delta/L)^2} \rightarrow 0$ as $Re \rightarrow +\infty$, we are left with the same momentum equation as Euler Eqn. Not good

② $\frac{1/Re}{(\delta/L)^2} \rightarrow \infty$, we deduce Stokes Eqn. in contradiction with high Re

③ The last possibility is the ratio $\frac{1/Re}{(\delta/L)^2}$ is finite. that is $\delta/L \sim \sqrt{1/Re}$ Therefor the approximate eqn is:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

(e) We use the complex potential in Polar coordinates: $f(z) = Bz^n = Br^n[\cos n\theta + i\sin(n\theta)]$

where B and n are constants.

Then stream function & velocity:

$\psi = Br^n \sin(n\theta)$, $u_r = Bnr^{n-1}\cos n\theta$, $u_\theta = -Bnr^{n-1}\sin(n\theta)$

$\theta = 0$ and $n\theta = \pi$ are straight streamline.

For a corner of 240 degree, we can

choose $n = \frac{3}{2}$, hence $\theta = \frac{3\pi}{2}$.

And the free stream velocity U on the x-axis is.

$$\boxed{U(x) = Ax^{1/2}, \quad A = \frac{3}{2}B.}$$

since $\frac{\delta}{x} \sim \sqrt{\frac{1}{Re_x}}$ where $Re_x = \frac{Ux}{\nu}$

$$\boxed{\delta \sim \left(\frac{\nu x}{U}\right)^{\frac{1}{2}} = \left(\frac{\nu x}{Ax^{1/2}}\right)^{\frac{1}{2}} \sim x^{\frac{1}{4}}}$$

$$\frac{u}{U} = 1 - e^{-\eta}$$

(a)

$$\theta = \int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \delta^* \int (1-e^{-\eta})e^{-\eta}\,d\eta = \delta^* \int_0^\infty e^{-\eta} - e^{-2\eta}\,d\eta = \frac{\delta^*}{2}$$

$$H = \frac{\delta^*}{\theta}$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \mu \frac{\partial}{\partial y}\left(U(1-e^{-\eta})\right)\Big|_{y=0} = \frac{\mu U}{\delta^*}$$

(b) Momentum integral equation

$$\frac{d\theta}{dx} + \frac{\theta}{U}\frac{dU}{dx}(H+2) = \frac{C_f'}{2}$$

$$\frac{1}{2}\frac{d\delta^*}{dx} + \frac{\delta^*}{2U}\frac{dU}{dx}(4) = \frac{\nu}{U\delta^*}, \quad \frac{d\delta^*}{dx} + \frac{4\delta^*}{U}\frac{dU}{dx} = \frac{2\nu}{U\delta^*}$$

(c) $\frac{dP}{dx} = 0 \Rightarrow \frac{dU}{dx} = 0$, $\frac{d\delta^*}{dx} = \frac{2\nu}{U\delta^*}$, $\delta^*\frac{d\delta^*}{dx} = \frac{2\nu}{U}$

$$\frac{1}{2}(\delta^*)^2 = \frac{2\nu}{U}x + \text{constant}$$

say $\delta^* = 0$ at $x = 0$, $\delta^* = 2\sqrt{\frac{\nu x}{U}}$, U is constant

(d) want constant δ^*, $\frac{d\delta^*}{dx} = 0$, $\frac{2\delta^*}{U}\frac{dU}{dx} = \frac{\nu}{U\delta^*}$

$$\frac{dU}{dx} = \frac{\nu}{2(\delta^*)^2}, \quad U(x) = \frac{\nu x}{2(\delta^*)^2} + \text{constant}.$$

One realistic flow is the flow around a stagnation point where the freestream velocity field $\underline{u} = \alpha(x, -y)$, α is a constant.

(d) $\frac{dP}{dx} = A x$ A : constant

$$\frac{d(U^2)}{dx} = 2U \frac{dU}{dx} = -2 \frac{1}{\rho} \frac{dP}{dx} = \frac{-2A}{\rho} x$$

$$U^2 = Bx^2 + \text{constant} \ , \quad B = -\frac{A}{\rho}$$

Without loss generality, set $U(0) = 0$, hence

$$U^2 = Bx^2 \ , \text{ or } \boxed{U = \alpha x \ , \quad \alpha = \sqrt{B}}$$

The momentum Eqn hence becomes.

$$\frac{d\delta^*}{dx} + \frac{4\delta^*}{\cancel{\alpha} x} \times \cancel{\alpha} = \frac{2\nu}{\alpha x \delta^*}$$

or $$\frac{1}{2} x \frac{d(\delta^{*2})}{dx} + 4(\delta^{*2}) = \frac{2\nu}{\alpha}$$

By inspection, if δ^{*2} = constant, there is a solution

$$\boxed{\delta^{*2} = \frac{\nu}{2\alpha}} \ , \text{ or } \boxed{\delta^* = \sqrt{\frac{\nu}{2\alpha}}}$$

One realistic flow is the flow around a stagnation point where the freestream velocity field $\underline{u} = \alpha(x, -y)$.

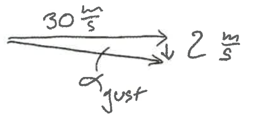
PS: General solution of the above equation is

$$\delta^{*2} = \frac{\nu}{2\alpha} + \frac{\beta}{x^8} \ , \quad \beta \text{ is a constant}$$

we take $\beta = 0$

a) $c_\ell = 2\pi\alpha = 0.329$

$\ell = \frac{1}{2}\rho U^2 c_\ell c = 178 \frac{N}{m}$

b) 

$\alpha_{gust} = -3.8^\circ$

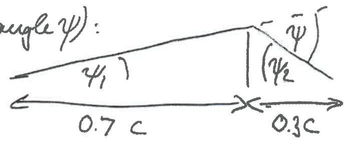
Change in U negligible : $U_{eff} = \sqrt{30^2 + 2^2} = 30.07 \frac{m}{s}$

New α : -0.8°

$\ell = -48\ N$ Downforce !

$\Delta C_\ell = 2\pi\alpha_{gust} = 0.417$

c) with flap (angle ψ):

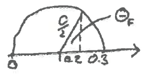


Assuming small angles : $\psi_1 c = \psi\, 0.3c \therefore \psi_1 = 0.3\psi$

$\psi_2\, 0.3c = \psi\, 0.7c \therefore \psi_2 = 0.7\psi$

Deploying the flap increases angle of attack by ψ_1

New chord line : $\frac{dy_c}{dx} = \psi_1 \quad 0<x<\frac{2}{3}c \quad \pi > \theta > \theta_F$



$\frac{dy_c}{dx} = -\psi_2 \quad \frac{2}{3}c<x<c \quad \theta_F > \theta > 0$

where $\cos\theta_F = 0.4 \quad \theta_F = 1.16$

$g_0 = \frac{1}{\pi}\int_0^\pi \left(-2\frac{dy_c}{dx}\right) d\theta = -\frac{2}{\pi}\left[-\psi_2\theta_F + \psi_1(\pi - \theta_F)\right]$

$g_0 = \frac{2}{\pi}\theta_F(\psi_2 + \psi_1) - 2\psi_1 = \psi\left[\frac{2}{\pi}\theta_F - 0.6\right] = 0.138\psi$

$g_1 = \frac{2}{\pi}\int_0^\pi \left(-2\frac{dy_c}{dx}\right)\cos\theta\, d\theta = -\frac{4}{\pi}\left[-\psi_2\int_0^{\theta_F}\cos\theta\, d\theta + \psi_1\int_{\theta_F}^{\pi}\cos\theta\, d\theta\right]$

$g_1 = \frac{4}{\pi}\left[\psi_2 \sin\theta_F + \psi_1 \sin\theta_F\right] = \frac{4}{\pi}\psi \sin\theta_F = 1.17\psi$

ΔC_ℓ due to flap :

$\Delta C_\ell = \underbrace{\psi_1 2\pi}_{\text{incidence}} + \pi\left(g_0 + \frac{g_1}{2}\right)$

$= 0.6\pi\psi + 0.138\pi\psi + \frac{1.17}{2}\pi\psi$

$= 1.323\pi\psi = 4.156\psi$

To compensate for gust requires

$\Delta C_\ell = 0.417 = 4.156\psi$

$\psi = 0.1 \approx 5.7^\circ$

c) At the reduced speed, the change in angle of attack due to the gust is much greater (11.3 deg). This means that the flap has to deploy at a much greater angle. This is likely to lead to flow separation over the flap - which in turn can trigger complete airfoil stall. The likelihood for stall is further exacerbated because the main airfoil has to operate at higher Cl at the lower flight speed (and is thus already closer to stall). Stall during flat deployment is particularly dangerous because it causes the opposite of what is required, i.e. a loss in lift instead of an increase.

$$A_R = \pi = \frac{4s^2}{S} = \frac{2s}{c} \qquad \Gamma(\theta) = Us\left(G_1 \sin\theta + G_3 \sin 3\theta\right)$$

$$y = -s\sin\theta \qquad G_3 = \tfrac{1}{8}G_1$$

a)

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 S} = \frac{\rho U}{\frac{1}{2}\rho U^2 S}\int_{-s}^{s}\Gamma(y)\,dy$$

$$= \frac{2}{US}\int_0^{\pi} Us\left(G_1\sin\theta + G_3 \sin 3\theta\right) s\sin\theta\, d\theta$$

Note: $\int_0^{\pi} \sin 3\theta \sin\theta \, d\theta = 0$

$$C_L = \frac{2s^2}{S}\int_0^{\pi} G_1 \sin^2\theta\, d\theta = \frac{A_R}{2} G_1 \frac{\pi}{2} = \frac{\pi^2}{4} G_1$$

b)

$$C_{D_i} = (1+\delta)\frac{C_L^2}{\pi A_R}$$

Here, C_L and A_R same for both wings.

Twisted wing: $\delta = 0$, Rect. wing: $\delta = 3\left(\frac{1}{8}\right)^2 = 0.047$

∴ 4.7% drag saving

Both wings operate at same Lift and C_L

$$L = \tfrac{1}{2}\rho U^2 C_L S = \frac{\pi}{2}\rho U \Gamma_0 s \quad (\text{for elliptical } \Gamma(y))$$

$$\Gamma_0 s = \frac{U}{\pi} C_L \frac{S}{s} = \frac{U}{\pi}\frac{\pi^2}{4} G_1 \frac{4s^2}{A_R s} = G_1 U s$$

At root: $\Gamma(y=0) = \Gamma_0$

Lifting line equ.: $\frac{\Gamma(y=0)}{\pi U c} = \alpha_{root} - \alpha_d$ $\left(\text{from: } \rho U \Gamma = \underbrace{2\pi\alpha_{eff}}_{c_l} \tfrac{1}{2}\rho U^2 c\right)$

for elliptic distr.: $\alpha_d = \frac{\Gamma_0}{4Us}$

$$\therefore \alpha_{root} = \frac{\Gamma_0}{\pi U c} + \frac{\Gamma_0}{4Us} \qquad \text{with } c = \frac{2s}{A_r} = \frac{2s}{\pi}$$

$$\alpha_{root} = \frac{\Gamma_0}{U}\left(\frac{1}{2s} + \frac{1}{4s}\right) = \frac{3}{4}\frac{\Gamma_0}{Us} = \frac{3}{4}G_1$$

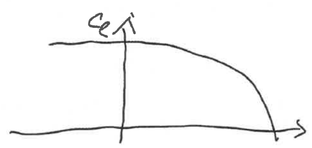
At tip: $\Gamma(y=s) = 0$

$$\alpha_{tip} = \alpha_d = \frac{G_1}{4}$$

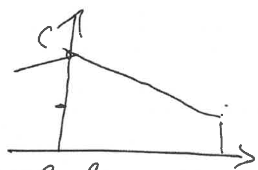
Twist angle: $\frac{G_1}{2}$

c) See notes. Rectangular wing has benign stall characteristics because wing loading is approximately elliptical. This means that the local lift, and thus local cl is highest in the centre, so there is no danger of tip stall. Twisting the wing reduces the geometric angle of attack at the tips and thus reduces the likelihood of tip stall even further, the img is therefore even more benign.

d) A wing with a taper ratio of 40% has an induced drag coefficient that is only marginally greater than an elliptic wing (see notes). This therefore improves the wings performance by reducing the induced drag (by around 4%). Tapering the wing also reduces wetted area which lowers the viscous drag.



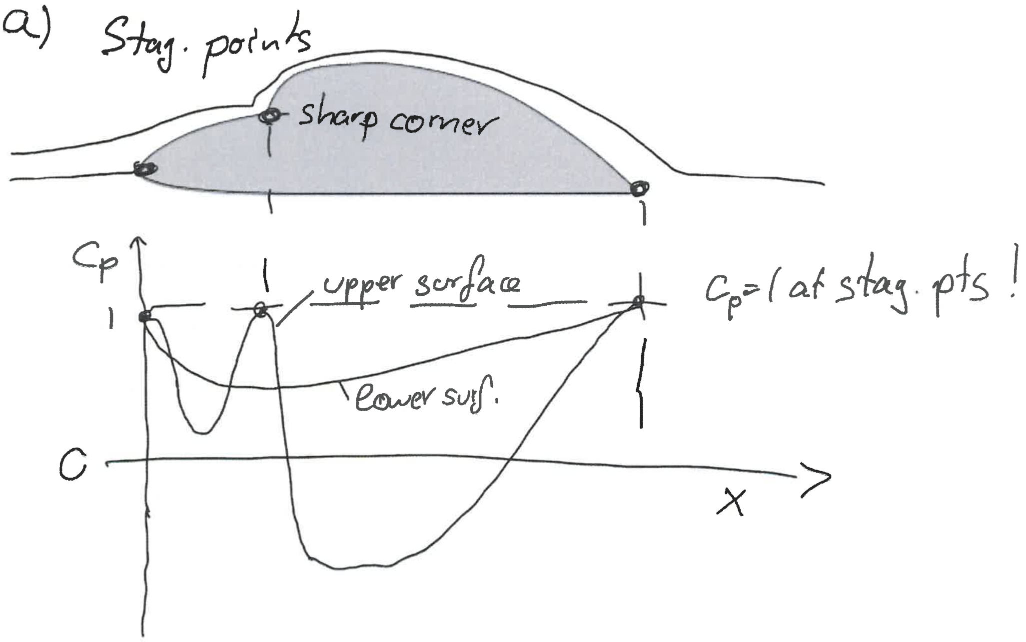
Wing loading



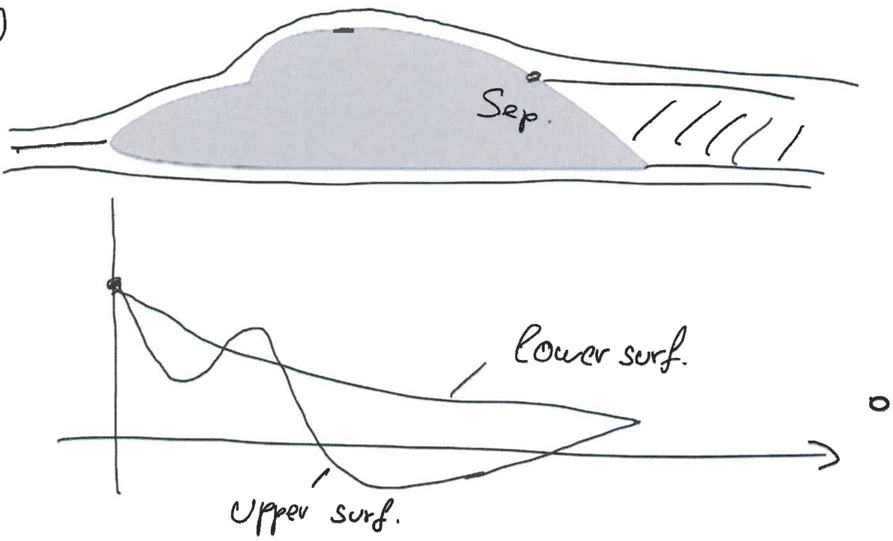
planform

Tapering the wing reduces wing area near the tips. The wing loading reminds approximately elliptical and this therefore increases the lift coefficient towards the tips. This can introduce dangerous tip stall (see notes).

a)



b)



c)

The sharp corner at the start of the bonnet is very likely to cause flow separation (as sketched). The rounded nose prevents this which reduces drag. Also, the rounder shape has some low pressure regions facing forwards, which generates some thrust. Note that the round nose raises the stagnation point, sending more flow towards the underbody (relevant for e)).

d)

The front spoiler reduces flow into the gap between the underbody and the road. Because this is an old car, there is likely to be significant roughness along the underbody which generates drag when exposed to fast moving air. Also, the interaction with wheels and wheel arches generates drag. Reducing the flow speeds underneath the vehicle therefore reduces drag (even though the spoiler itself introduces a drag penalty)

e)

Both devices in isolation generate additional drag in other areas. The rounded nose moves the stagnation point further up and is thus likely to 'push' more flow into the underbody - creating more drag there. Likewise, the front spoiler diverts more flow above the vehicle, exacerbating the problem described in c). Therefore, either device in isolation does not bring the full benefit. Combining both devices is therefore more effective.