Version JVT/8

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 6 May 2026 9.30 to 12.40

**Module 3A3**

**FLUID MECHANICS II**

*Answer not more than **five** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Engineering Data Book
Attachments:
Compressible Flow Data Book (38 pages);

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationery from the Examination Room.**

1 (a) Describe the various flow regimes within a converging-diverging nozzle as the static pressure at the exit is reduced relative to the inlet total pressure. Include in your description sketches of the Mach number and static pressure distributions through the nozzle. Assume the flow to be both adiabatic and inviscid. [30%]

(b) Air flows steadily through an adiabatic frictionless convergent-divergent nozzle with a throat area of $0.025\,\mathrm{m}^2$ and an exit area of $0.04\,\mathrm{m}^2$. The stagnation pressure and stagnation temperature at inlet to the nozzle are 200 kPa and 283 K respectively. The nozzle throat is choked and the static pressure at exit is 135 kPa.

(i) Find the mass flow rate and the Mach number at the nozzle exit. [20%]

(ii) Show that there must be a shock wave in the diverging part of the nozzle. [10%]

(iii) Calculate the Mach number and nozzle area at the plane where the shock is located. [20%]

(iv) Find the net force exerted on the diverging part of the nozzle by the flow and indicate its direction. [20%]

2 (a) For frictionless flow in a constant area duct, a small change in stagnation enthalpy can be expressed as

$$\delta h_0 = h\left(1 - M^2\right)\frac{\delta V}{V}$$

where $h$ is the specific enthalpy, $M$ is the Mach number and $V$ is the velocity. Using this expression, or otherwise, show that in a frictionless constant area duct, heating will accelerate a subsonic flow and decelerate a supersonic flow. Illustrate each of the two situations on a $T - s$ diagram and label the point where the flow is choked. [30%]

(b) Air flows steadily through a frictionless cylindrical duct. At entry to the duct the velocity is $150\,\mathrm{m\,s^{-1}}$, the stagnation pressure is 2.5 bar and the stagnation temperature is 300 K. Heat is added along the length of the duct giving choked flow at the exit.

(i) Calculate the Mach number at the duct inlet. [10%]

(ii) Determine the heat added per kg of air flow. [20%]

(iii) Find the stagnation pressure and the velocity at the duct exit. [30%]

(c) Explain how heat transfer can be used to accelerate a flow in a frictionless constant area duct from a subsonic to a supersonic Mach number. [10%]

3 (a) Show that the Riemann invariant for a left-running infinitesimal wave in a compressible gas flow is given by

$$V + \frac{2a}{\gamma - 1} = \text{constant}$$

where $V$ is the velocity of the gas, $a$ is the speed of sound and $\gamma$ is the ratio of the specific heat capacities for the gas. [30%]

(b) A large pressure vessel containing compressed air at atmospheric temperature is fitted with a relief valve at the downstream end of a long straight pipe. The valve is designed to open instantaneously when the pressure in the vessel reaches 4.5 bar allowing the compressed air to vent to the atmosphere. The pressure and temperature of the atmosphere are 1 bar and 15°C respectively.

(i) Draw a space-time diagram and label the wave pattern that forms after the valve opens. [20%]

(ii) Show that the flow at exit from the valve is choked immediately after opening. [20%]

(iii) Calculate the pipe diameter required to vent a mass flow rate of $100\,\mathrm{kg\,s^{-1}}$. [30%]

4 Figure 1 shows a profile section through a small two-dimensional unswept control vane fitted to an aircraft. The vane is mounted on a horizontal pivot at its centre, about which it can be rotated. The vane is a symmetrical 12° double-wedge design with a side length of 15 cm. The aircraft is flying at a Mach number of 2.0 at an altitude of 24 km.

(a) For the case where the centreline of the vane is aligned with the flow direction:

(i) Draw a carefully labelled sketch of the shock and expansion system. [20%]

(ii) Calculate the section pressure drag coefficient. [20%]

(b) For the case where the vane is set at an angle of 14° to the flow direction:

(i) Draw a carefully labelled sketch of the shock and expansion system. [20%]

(ii) Calculate the vertical section force on the pivot. [30%]

(iii) Estimate the angle that the wake downstream of the trailing edge makes to the flow direction. [10%]

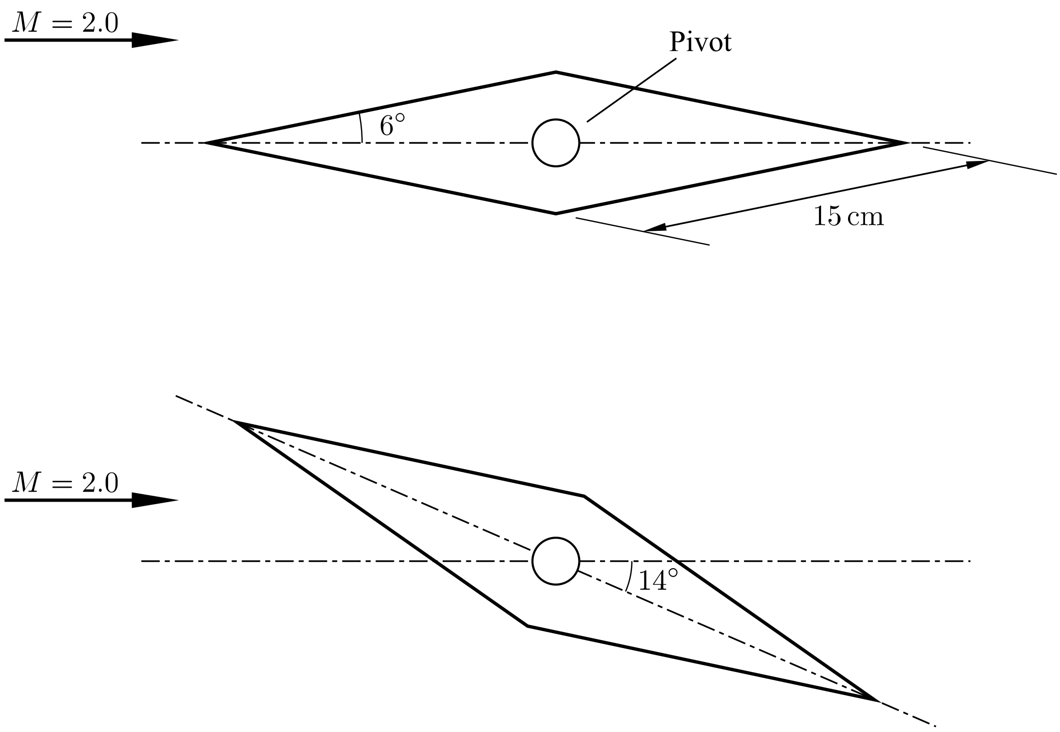


Fig. 1: Control vane section, note not drawn to scale

5 (a) By considering the conservation of mass, momentum and energy of a steady, compressible, two-dimensional, isentropic flow, show that

$$[M^2 - 1]\frac{1}{V}\frac{\partial V}{\partial s} - \frac{\partial \theta}{\partial n} = 0$$

and

$$V\frac{\partial \theta}{\partial s} = \frac{\partial V}{\partial n}$$

where $V$ is the flow velocity, $M$ is the Mach number of the flow, $\theta$ is the flow deflection angle, $s$ is the streamwise direction and $n$ is the direction normal to the flow. [60%]

(b) Hence show that

$$(\nu - \theta) = \text{constant}$$

along a line that makes an angle of

$$\frac{1}{\sqrt{M^2 - 1}}$$

to the flow direction, where $\nu$ is the Prandtl-Meyer function. [30%]

(c) Explain the physical significance of the angle given in part (b). [10%]

6 The partial differential equation (PDE)

$$\frac{\partial u}{\partial t} + F\big(u(x,t),x,t\big) = 0$$

is to be numerically integrated with time step $\Delta t$. The discretisation $t = N\Delta t$ and $x = i\Delta x$ is to be used. To improve the order of accuracy with respect to time, information at both the current timestep, $N$, and previous timestep , $N-1$, can be used to move forward as

$$u_i^{N+1} = u_i^N - \Delta t\left(\alpha\, F_i^N + \beta\, F_i^{N-1}\right) \tag{1}$$

(a) Determine the values of $\alpha$ and $\beta$ that will lead to an approximation of the original PDE to an accuracy of $O(\Delta t^2)$ [40%]

(b) Given that

$$F\,(u,x,t) = 2u\,\frac{\partial u}{\partial x} - x$$

with a boundary condition of $u(0,t) = 0$:

(i) Derive an update for $u_i^{N+1}$ in terms of $u$ at timesteps $N$ and $N-1$ at neighbouring grid points. Use Eq. (1) with the values of $\alpha$ and $\beta$ you have found and a suitable first order estimate for the spatial derivative. [10%]

(ii) Calculate the steady state solution to the PDE using the approximation with 3 grid points across the domain $x = [0, 4]$. [30%]

(iii) Determine the error between the exact steady state solution and the approximation from part (b)(ii). Comment on your result. [20%]

 (TURN OVER

7 A small low cost radial inflow turbine is tested experimentally. Axial and meridional views of the geometry and stations are shown in Fig. 2. At station 1, air enters a stator radially with a Mach number of 0.1, a stagnation temperature of 800 K and a stagnation pressure of 200 kPa. At station 2, which is aligned with the rotor leading edge, the stator has turned the flow to give a swirl angle of 76.5°. To reduce cost in this design the rotor uses straight radial blades. At the design operating point the rotor spins at 91,000 RPM, the flow into the rotor at station 2 has zero incidence and at the outlet the flow at station 3 has negligible deviation. The axial velocity is zero throughout the entire machine.

(a) At the experimentally tested design point calculate:

(i) The mass flow through the machine. [15%]

(ii) The stator exit Mach number. [15%]

(iii) The stator total pressure loss coefficient. [15%]

(b) For this simple rotor, with zero tangential velocity in the relative frame, show that:

$$\frac{\Delta h_0}{U_2^2} = 1 - \left(\frac{r_3}{r_2}\right)^2$$

where $\Delta h_0$ is the stage stagnation enthalpy drop and $U_2$ is the blade speed at station 2. [15%]

(c) The rotor exit stagnation pressure is found to be 146 kPa. Calculate the total-total isentropic efficiency. [15%]

(d) The radial velocity at rotor exit is $142\,\mathrm{m\,s^{-1}}$. Calculate the total-static isentropic efficiency and comment on the value. [15%]

(e) If this turbine is used as a single stage, exhausting to atmosphere, state which efficiency is most appropriate and suggest an improvement to the design. [10%]

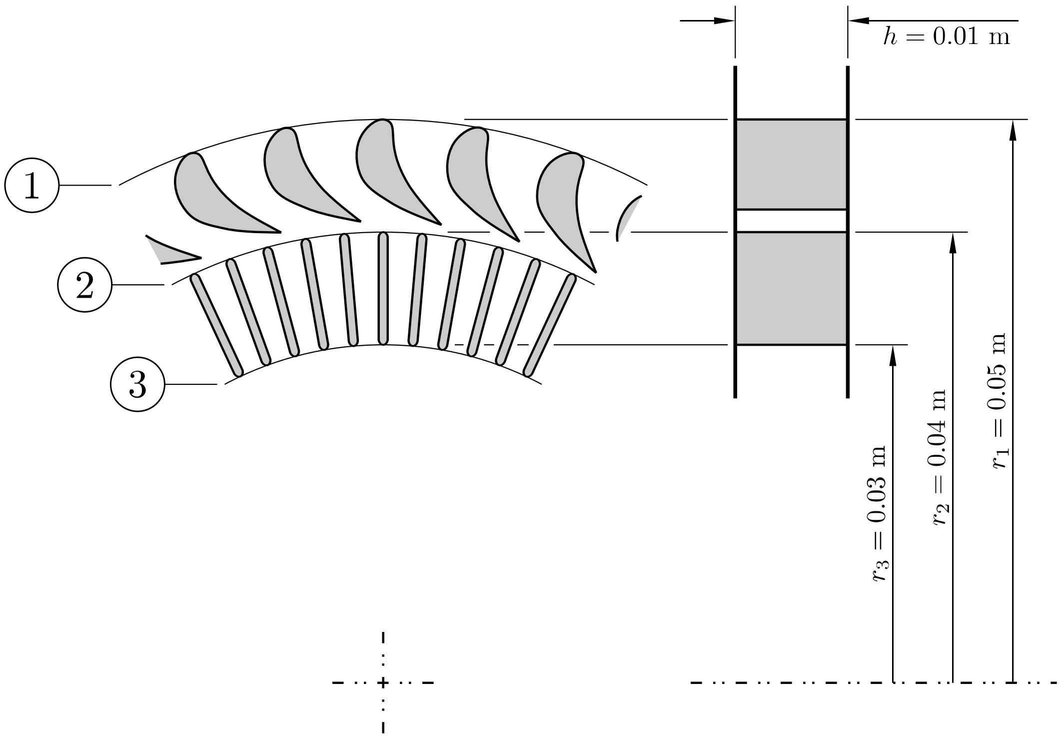


Fig. 2: Radial inflow turbine

8 (a) The partial differential equation

$$\frac{\partial u}{\partial t} + \left(x - \frac{1}{2}\right)\frac{\partial u}{\partial x} = 0$$

is to be solved over the domain $x = [0, 1]$, for an initial condition of $u(x, 0) = (x - \frac{1}{2})^2$.

(i) Explain why no boundary conditions are needed. [10%]

(ii) Determine the algebraic form of the characteristic curves and sketch their shape on a plot of $t$ vs. $x$. [15%]

(iii) The solution is to be obtained numerically on an equally spaced spacial grid such that $x = i\Delta x$. Derive updates in the form

$$u_i(t + \Delta t) = f\left(u_{i-1}(t), u_i(t), u_{i+1}(t)\right)$$

that is stable and first order accurate in both time and space. Suggest the maximum stable time step. [15%]

(iv) Without computation, sketch the evolution of the solution with time. [10%]

(b) A repeating stage axial turbine has constant axial velocity and mean radius, the flow coefficient is 0.4 and the stage loading coefficient is 1.8.

(i) The inlet swirl angle is $-45°$ in the absolute frame. Draw the velocity triangles of a stage and calculate the remaining swirl angles in both absolute and relative frames. Determine the reaction of the turbine. [25%]

(ii) Calculate the inlet swirl angle required to produce an impulse turbine with reaction of zero, whilst maintaining the stage loading and flow coefficients. [15%]

(iii) Comment on a benefit and a drawback of impulse turbines compared to reaction turbines. [10%]

**END OF PAPER**

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