EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 5 May 2021 9.00 to 12.10

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
You are allowed access to the Compressible Flow Data Book, in electronic or hard-copy form.
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 30 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 (a) A large reservoir contains air at pressure 5 bar and temperature 583 K. From the reservoir, the air passes into a frictionless convergent-divergent nozzle, where it accelerates to a speed of 603 m s^{-1}. It then enters a cylindrical duct of length 1.2 m and diameter 0.08 m. At the end of the duct there is a normal shock wave. Immediately upstream of the shock, the temperature is 69 K higher than at the duct entry. The flow is adiabatic throughout, and exhibits perfect-gas behaviour.

(i) Calculate the Mach number of the flow when it enters the cylindrical duct. [10%]

(ii) What is the value of the skin-friction coefficient in the duct? [20%]

(iii) Find the ambient pressure in the atmosphere surrounding the duct exit. [20%]

(b) The ambient pressure at the duct exit is now gradually raised from the value you found in part (a) up to 5 bar.

(i) Describe, qualitatively, the changes in the flow during this process, and explain why they occur. [20%]

(ii) At what value of the ambient pressure has the mass flow rate reduced by 1% relative to the condition of part (a)? [30%]

2 (a) Consider the steady one-dimensional flow of a perfect gas in a constant-area duct.

(i) Show that infinitesimal changes in the flow conditions are linked by the relation

$$\mathrm{d}p = \rho \left[R\mathrm{d}T - \frac{a^2}{\gamma} \frac{\mathrm{d}V}{V} \right],$$

where V is the flow velocity, a is the speed of sound, and the other symbols have their usual meanings. [20%]

(ii) If the duct is frictionless, with changes of state brought about by heat transfer, show that the maximum possible temperature is obtained when the Mach number is $1/\sqrt{\gamma}$. [20%]

(b) A perfect gas with the properties of air flows steadily and one-dimensionally in a frictionless cylindrical duct of cross-sectional area 5×10^{-3} m^2. At entry to the duct, the stagnation pressure is 2 bar, the stagnation temperature 273.4 K, and the density 2.356 kg m^{-3}.

(i) Calculate the Mach number and flow speed at the duct entry. [15%]

(ii) Find the direction and magnitude of the heat transfer (in Watts) required to choke the flow. [20%]

(iii) Starting from the choked condition, what further heat transfer is needed to maximise the flow temperature? [10%]

(iv) What alternative flow state might be obtained via the application of the heat transfer of part (iii)? [15%]

3 Figure 1 shows a long duct filled with stationary air at a temperature of 300 K and a pressure of 1 bar. A piston is located at $x = 0$ in the laboratory frame of reference. At time $t = 0$ it is impulsively given a constant speed to the left.

(a) Consider the expansion-wave system on the right-hand side of the piston.

(i) What is the velocity (magnitude and direction) of the head of the wave system with respect to the laboratory frame of reference? [10%]

(ii) It is observed that the tail of the wave system is stationary at $x = 0$. In the x-t plane, sketch: the wave system; the trajectory of the piston; and the trajectory of a fluid element. [20%]

(iii) What is the speed of the piston in the laboratory frame of reference? [10%]

(b) In a second experiment with the same configuration, the piston is given a different speed, again to the left.

(i) A microphone placed at $x = 2$ m detects variations in the air pressure over a duration of 0.03 s. Calculate the new piston speed, and the pressure on its face. [35%]

(ii) The piston stops abruptly at $x = -5$ m. The microphone detects a change in pressure at $t = 0.0504$ s. What is the pressure on the piston face after it stops? [25%]

Note: across a one-dimensional right-running sound wave in a perfect gas, the Riemann invariant is

$$V - \frac{2}{\gamma - 1} a = \text{constant},$$

where V is the flow velocity (positive to the right), a the speed of sound, and γ the ratio of specific heats.

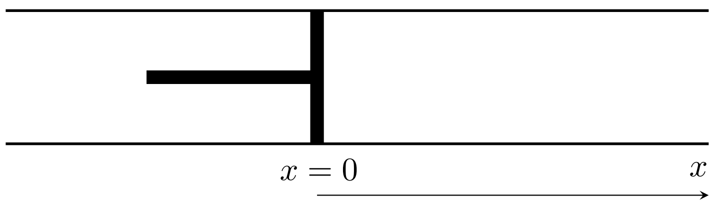


Fig. 1

4 (a) Figure 2(a) shows a simple supersonic diffuser, in which oblique shock waves generated by wedges are employed to decelerate a flow of air from an initial Mach number of 2.60. Conditions are uniform both upstream and downstream of the shock system. The angle δ is $9.33°$. Find:

(i) the Mach numbers in regions 2 and 3; [20%]

(ii) the streamwise length of the diffuser, in terms of the upstream height h; [15%]

(iii) the static-pressure ratio across shock S_{12}. [10%]

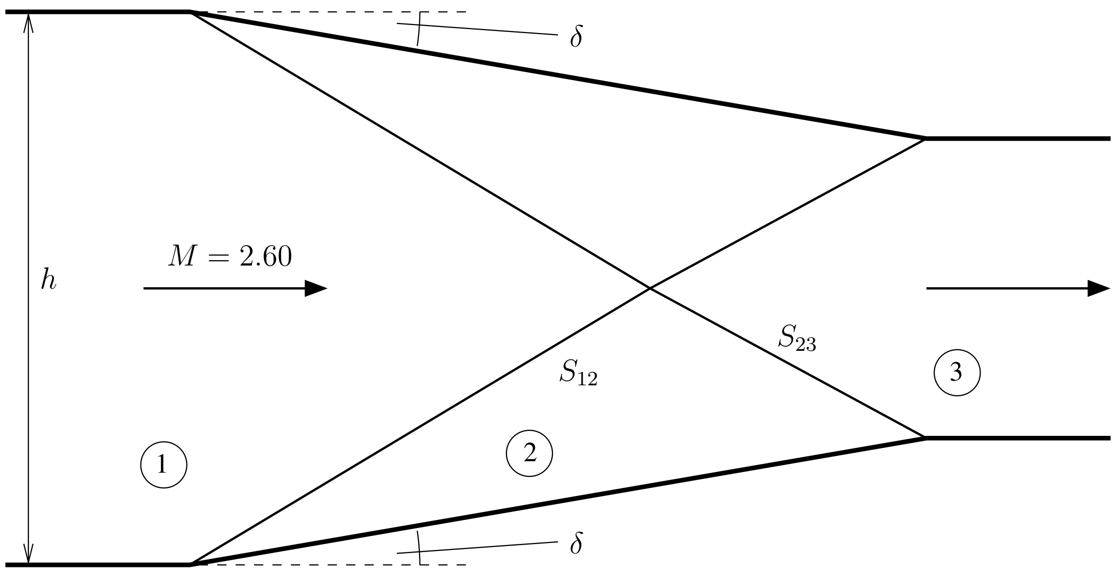
(b) Describe, with the aid of labelled sketches, the wave patterns that would be observed if the upstream Mach number were slightly:

(i) larger; [15%]

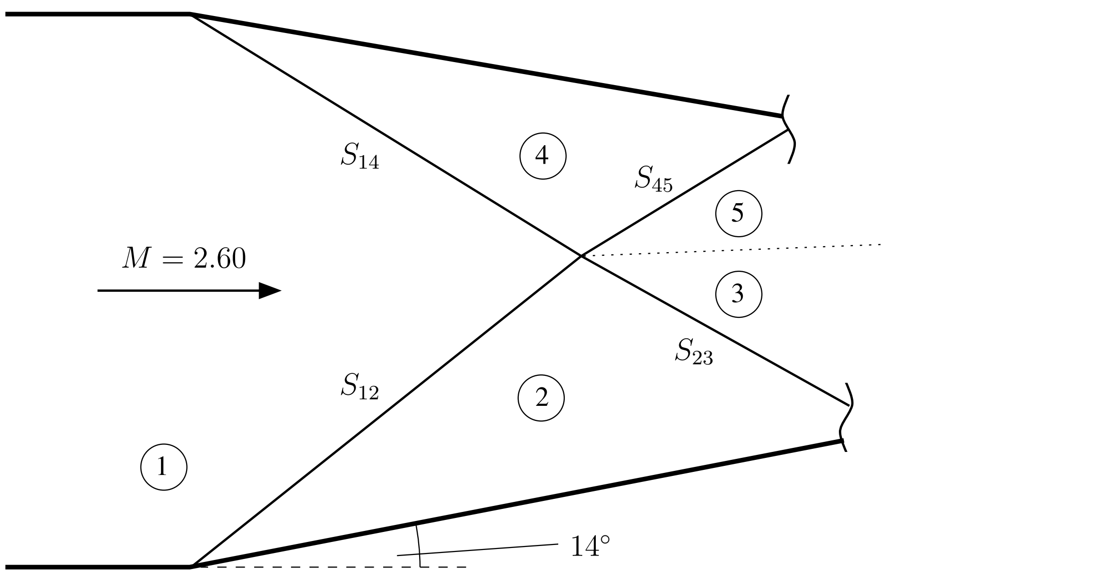
(ii) smaller [15%]

than its design value of 2.60.

(c) Due to a manufacturing error, the lower wedge angle increases from $9.33°$ to $14.00°$, creating the asymmetric flow depicted in Fig. 2(b). At the design condition, a researcher estimates the flow angle in region 3 to be $4.00°$ upwards from the horizontal. Assess the theoretical consistency of this value, and thereby state whether it is too high or too low. [25%]



(a)



(b)

Fig. 2

5 The high-altitude manoeuvrability of a supersonic aircraft is being assessed at a flight Mach Number, M, of 2.20. The aircraft wing section is sketched in Fig. 3. It has $4°$ leading- and trailing-edge wedges of length 0.5 m, with a 1 m parallel-sided central part. The underside is flat, and is at $6°$ incidence to the oncoming flow. Three-dimensional effects are negligible.

(a) Draw a labelled sketch of the wave system around the wing. [30%]

(b) On the basis of small-perturbation (linearised) theory, calculate:

(i) the pressure-drag coefficient; [30%]

(ii) the pitching-moment coefficient, evaluated about the leading edge. [10%]

(c) The manoeuvrability is found to be inadequate. It is proposed to improve the lift performance of the wing by deflecting the rear wedge of the aerofoil downwards by $8°$ to form a flap. Calculate the resulting change in pitching-moment coefficient. (You may retain the small-perturbation assumption.) [10%]

(d) Identify two ways in which the proposal of part (c) is likely to affect overall aspects of the aircraft design. [20%]

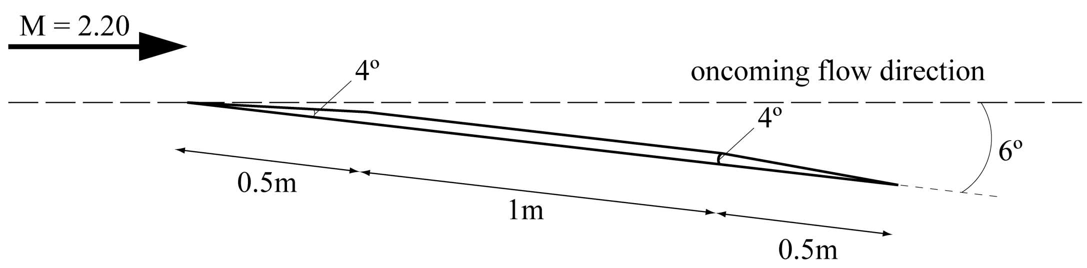


Fig. 3

6 Consider the linear convection equation,

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0,$$

with initial condition $u(x,0) = \cos(kx)$ and a a positive constant.

(a) Show that $u = \cos\left[k(x - at)\right]$ is the solution. [10%]

(b) An approximate solution at times $t = n\Delta t$ and positions $x = i\Delta x$ is to be found by solving the difference equation

$$u_i^{n+1} = u_i^n - \alpha\left(u_i^n - u_{i-1}^n\right),$$

where $\alpha = a\Delta t/\Delta x$. The equivalent PDE is

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = \nu_{\text{eff}}\frac{\partial^2 u}{\partial x^2} + \epsilon,$$

in which

$$\nu_{\text{eff}} = \frac{a\Delta x}{2}(1 - \alpha)$$

and ϵ represents the higher-order error terms.

(i) When ϵ is neglected, $u = A(t)\cos\left[k(x - at)\right]$ is the solution to the equivalent problem. Find $A(t)$. [20%]

(ii) In the light of your answer to part (i), discuss the stability of the numerical scheme, and its accuracy when stable. [20%]

(c) A better approximate solution is obtained from

$$u_i^{n+1} = u_i^{n-1} - \alpha\left(u_{i+1}^n - u_{i-1}^n\right).$$

Following the same approach as part (b), it can be shown that $u = \cos\left[k(x - ct)\right]$ is a solution to the new truncated equivalent PDE. Here c is a constant close to, but not exactly equal to, a.

(i) To leading order, what is the difference between c and a? [35%]

(ii) Comment on the implications for the numerical behaviour of this scheme. [15%]

7 (a) The simple radial equilibrium equation for axisymmetric, incompressible, isentropic flow in a turbomachine is:

$$\frac{1}{\rho}\frac{\mathrm{d}p_0}{\mathrm{d}r} = V_x\frac{\mathrm{d}V_x}{\mathrm{d}r} + \frac{V_\theta}{r}\frac{\mathrm{d}\,(rV_\theta)}{\mathrm{d}r},$$

where ρ is the (uniform) density and, at radius r: the stagnation pressure is p_0; the axial velocity is $V_x = V\cos\alpha$; and the tangential velocity is $V_\theta = V\sin\alpha$.

A particular flow, in an annulus with hub radius r_H and tip radius r_T, has both p_0 and α independent of r, with $\alpha = 45°$. Determine:

(i) how the flow speed $V(r)$ varies with radius from its value V_H $(= V(r_H))$ at the hub; [30%]

(ii) the total mass flow rate through the annulus. [20%]

(b) A variable u is a function of x and t. For a small time increment δt, write down an expression for the change in u along the specific x-t plane trajectory $x = X(t)$. [10%]

(c) The general form of the convection equation is

$$\frac{\partial u}{\partial t} + f(u)\frac{\partial u}{\partial x} = 0,$$

where $f(u)$ is a function of u alone. The 'characteristics' of the equation are the trajectories in the x-t plane along which u is constant. Determine the characteristics, and thereby sketch the evolution of an initial profile $u(x, 0) = \sin^2(\pi x/L)$, for

(i) $f(u) = a$, a positive constant; [20%]

(ii) $f(u) = u$. [20%]

8 The stagnation temperature and stagnation pressure at the inlet to a single-stage axial-flow turbine were 1500 K and 10 bar respectively when it suffered a catastrophic failure. Fortunately, some preliminary measurements were recorded: the stagnation pressure at stator exit (p_{02}) was 9.505 bar , the stagnation pressure at the outlet of the turbine was 5.903 bar, and the power output was 7.24 MW. Eventually, it was determined that the failure had resulted from a complete lack of the cooling air that normally bleeds out of small holes in the stator and rotor blades.

(a) An engineer used a numerical calculation to predict that, when operating without coolant, the stator had an exit flow angle of 70° and a stagnation-pressure loss of $0.1(p_{02} - p_2)$, with p_2 the static pressure at stator exit. The hub and tip radii at this plane were 0.27 m and 0.33 m respectively.

(i) Estimate the mass flow rate through the turbine. [35%]

(ii) Comment on the reliability of your answer, in the context of its dependence on the quantities from the numerical calculation. [15%]

(b) (i) Estimate the isentropic efficiency of the turbine before failure. [20%]

(ii) Discuss whether the efficiency would increase or decrease if the cooling air flow were present. [10%]

(c) Explain, including relevant equations where necessary, what additional information could have been determined had the rotational speed been recorded. [20%]

You may assume that the working fluid is a perfect gas with $R = 287$ J kg^{-1} K^{-1}, $\gamma = 1.333$, and $c_p = 1149$ J kg^{-1} K^{-1}.

END OF PAPER

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ENGINEERING TRIPOS PART IIA

Module 3A3: FLUID MECHANICS II

ANSWERS

1(a) (i) 1.500 (ii) 0.00213 (iii) 2.47bar

(b) (ii) 3.68bar

2(b) (i) 0.400, 130.5m/s (ii) 376kW, heating (iii) 16kW, cooling

3(a) (i) 347m/s, to right (iii) 289m/s

(b) (i) 243m/s, 0.348bar (ii) 0.982bar

4(a) (i) $M_2 = 2.20$, $M_3 = 1.85$ (ii) $1.41h$ (iii) 1.83

5(b) (i) 0.0248 (ii) 0.120

(c) 0.062

6(b) (i) $A(t) = \mathrm{e}^{-\mathrm{k}^2 \nu_{\mathrm{eff}} t}$

(c) (i) $c - a \approx \frac{(k\Delta x)^2 a}{6}(\alpha^2 - 1)$

7(a) (i) $V = V_H \left(\frac{r_H}{r}\right)^{1/2}$ (ii) $\frac{2\sqrt{2}\pi}{3}\rho V_H r_H^{1/2}\left(r_T^{3/2} - r_H^{3/2}\right)$

(b) $\delta u = \left[\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\dot{X}\right]\delta t$

(c) (i) $x = x_0 + at$ (ii) $x = x_0 + u(x_0, 0)t$

8(a) (i) 37.4kg/s

(b) (i) 0.910