Version AJW/3

EGT2
ENGINEERING TRIPOS PART IIA

Monday 4 May 2026 9.30 to 11.10

Module 3A5

THERMODYNAMICS AND POWER GENERATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 The on-board reformer of a solid-oxide fuel-cell power plant is supplied with methane and steam in the ratio 1 mole of CH_4 to x moles of H_2O, where $x \geq 1$. The reformer operates at pressure p and temperature T and it may be assumed that the exit stream comprises an equilibrium mixture of CO, CO_2, H_2O and H_2 only (i.e., all methane has been reformed).

(a) Write down the chemical reaction governing the equilibrium composition of the exit stream and explain the effects of altering each of p, T and x on the yield of hydrogen. [20%]

(b) Determine the yield of H_2 per mole of CH_4 when $x = 5$, $p = 5$ bar and $T = 800$ K. Calculate also the mole fractions of H_2 and H_2O at this condition. [50%]

(c) By considering the conservation of carbon and oxygen, or otherwise, show that the mole fraction of hydrogen is 3/4 when $x = 1$, irrespective of p and T (provided all CH_4 is reformed). Compare the mole fractions of H_2 and H_2O for this case with those of part (b) and discuss briefly the implications for the operation of the fuel cell. [30%]

2 (a) Starting from the definition of specific Gibbs function, $g = h - Ts$ (where T and h are the temperature and specific enthalpy respectively), determine expressions for the specific entropy, s, and the specific volume, v, in terms of partial derivatives of g. Find also a partial differential relation between v and s. [15%]

(b) At near-ambient pressure and temperature, the characteristic equation of state for a particular (real) gas is given by

$$g = b(p - p_0) + RT\ln\left(\frac{p}{p_0}\right) + \frac{7R}{2}\left\{T - T_0 - T\ln\left(\frac{T}{T_0}\right)\right\}$$

where R is the specific gas constant, p is the pressure and b is a positive constant. T_0 and p_0 are ambient temperature and pressure respectively.

(i) Determine the thermal equation of state (the p-v-T relation) for the gas. [10%]

(ii) Find the isobaric specific heat capacity, c_p, and show that the specific internal energy depends only on temperature. [30%]

(iii) On the basis of your answers to parts (i) and (ii), provide a description of the gas from a molecular perspective, including an explanation for any departures from ideal-gas behaviour. [15%]

(c) A flow of the gas described in part (b) passes through an insulated throttle valve such that its pressure drops from p_1 to p_2. Assuming the equation of state remains valid and that kinetic energy changes are negligible, determine whether the exergetic loss through the valve is less than, more than or the same as that of an ideal (but otherwise identical) gas undergoing the same process. Sketch the process (for the real gas) on a T-s diagram. [30%]

3 *Throughout this question all feed pump (FP) work may be neglected and the turbine (and other relevant components) may be assumed adiabatic.*

Figure 1 shows the simplified steam cycle of a nuclear power plant. Thermodynamic conditions at key locations are given in Table 1. The cycle has n direct-contact feed heaters, labelled FH_1 to FH_n, which are integrated with a thermal energy storage system. While the storage is being charged, excess feedwater is generated such that the outflow from FH_1 is increased to $(1 + f)$ kg per kg of flow through the steam generator, as shown in Fig. 1. The excess, f, is then diverted to the thermal storage, to which it transfers its thermal exergy before being returned to the steam cycle at state 1 (see Table 1). All thermodynamic states around the cycle may be assumed constant, independent of f.

(a) Calculate the net exergy delivered to storage per kg of water sent to storage. [15%]

(b) Show that the flow rate of bled steam, m_1, supplied to FH_1 is proportional to $(1 + f)$. Hence deduce that the total bled steam flow rate is given by

$$m = \sum_{i=1}^{n} m_i = (1 + f)\, m_0$$

where m_0 is the value of m when $f = 0$. [30%]

(c) Sketch a suitable control volume and show that the work output from the steam turbine per kg of flow through the steam generator is given by

$$w = h_3 + (1 - m + f)\, h_1 - (1 - m)\, h_4 - (1 + f)\, h_2$$

where h is specific enthalpy and the numbering is as shown in Fig. 1. Hence show that the reduction in specific work output during charge is proportional to f. Find the constant of proportionality in terms of relevant specific enthalpies and m_0. [30%]

(d) Given that $m_0 = 0.38$, calculate the exergetic charge efficiency, defined by

$$\eta_{\mathrm{chg}} = \frac{\text{rate at which exergy is delivered to storage}}{\text{reduction in power output during charge}}$$

Comment on the value and explain whether it is realistic. [25%]

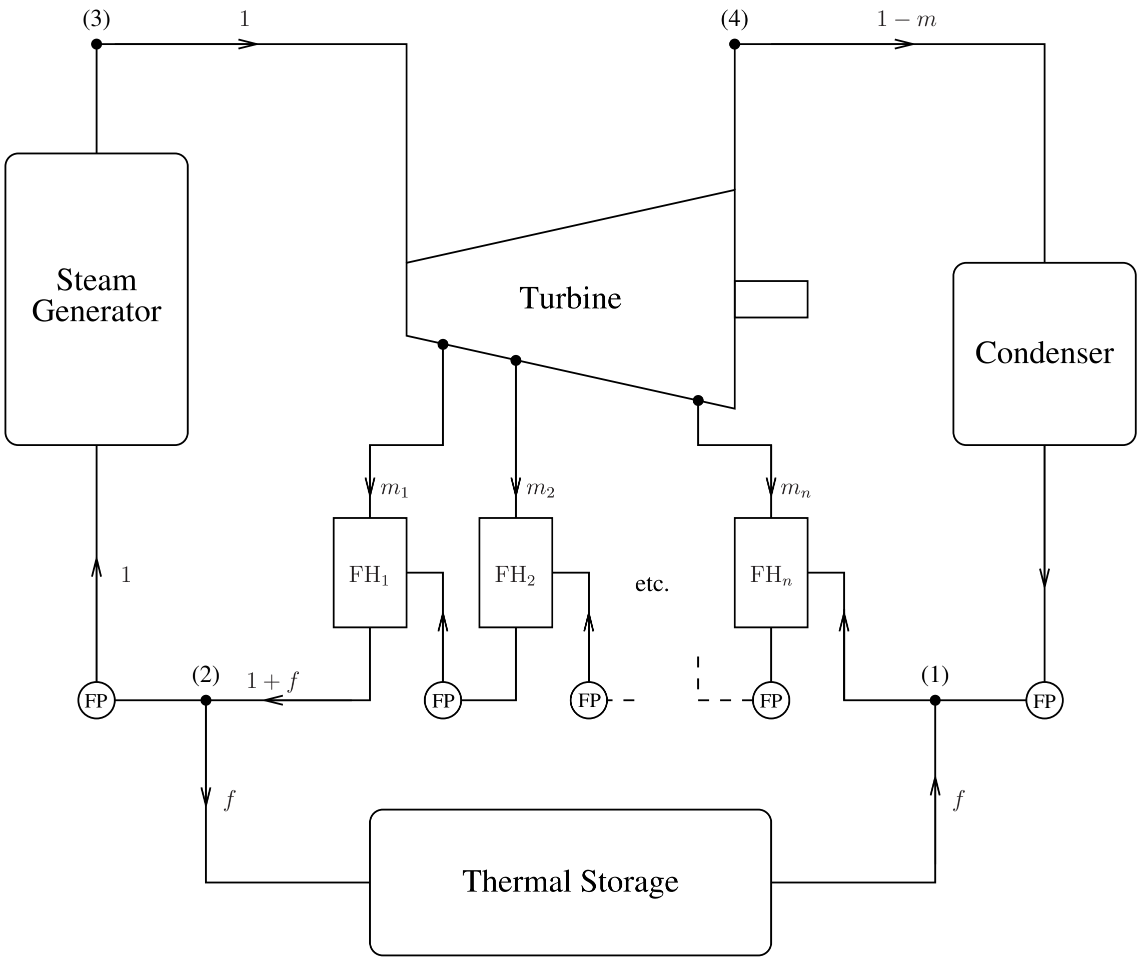

Fig. 1

Table 1

	p, bar	T, °C	h, kJ/kg	s, kJ/kg K
(0) ambient	1	25	–	–
(1) storage exit	1	29	121.5	0.422
(2) storage inlet	30	234	1008.3	2.646
(3) turbine inlet	60	276	2784.6	5.890
(4) turbine exit	0.04	29	1975.5	6.560

4 (a) State whether each of the following propositions is true or false, giving a brief explanation for your answer in each case.

(i) Combined-cycle gas turbine (CCGT) power plants may have multiple steam pressure levels within the heat-recovery steam generator. [10%]

(ii) CCGT power plants usually include feedheating within the steam cycle. [10%]

(b) Show that for an Ideal Joule Cycle

$$\left(\frac{\partial \eta}{\partial \gamma}\right)_{\beta} = A \frac{\ln \beta}{\beta^{(1-1/\gamma)}}$$

where η is the efficiency, β is the pressure ratio, γ is the ratio of specific heat capacities and A is a function of γ to be determined. Does the cycle efficiency increase or decrease with γ? Give a thermodynamic interpretation of this trend. [25%]

(c) A simple gas-turbine operates with a pressure ratio of 24. The compressor and turbine both have a polytropic efficiency of 0.9. The compressor inlet temperature is 288 K and the turbine inlet temperature is 1700 K. The combustor pressure loss is 5% of the combustor inlet pressure. Assume that the working fluid may be modelled as a perfect gas with the properties of air in all components.

(i) Stating any further assumptions, calculate the efficiency. [25%]

(ii) The addition of a post-combustion carbon-capture system causes a pressure loss downstream of the turbine exit equal to 15% of atmospheric pressure. Determine the change in efficiency due to the addition of this system. [15%]

(d) An alternative to post-combustion carbon capture is to use hydrogen rather than natural gas as the fuel. Table 2 shows the electricity requirements for two different hydrogen production methods. Assuming that the hydrogen is used to fuel a CCGT plant with an overall efficiency of 63%, find the ratio of the CCGT work output to the electricity requirements for each production method. Explain briefly why the values are so different and whether they provide a fair comparison of the H_2 production methods.
Take the lower calorific value for H_2 as 120 MJ kg^{-1}. [15%]

Table 2

Production method	Electricity input per kg H_2
SMR*:	2.7 kWh
Electrolysis:	47 kWh

* SMR = Steam methane reforming with carbon capture.

END OF PAPER

THIS PAGE IS BLANK