MODULE 3A5

THERMODYNAMICS & POWER GENERATION

CRIB FOR 2026 EXAMINATION

Examiners' comments:

Q1. The question was on chemical equilibrium. Most students understood well Le Chatelier's principle, and the vast majority could perform the water-gas shift equilibrium algorithmic calculation well. The last part, which included some critical thinking about the performance of the fuel cell, was not answered well and proved to be the key differentiator between students.

Q2. This question was on Maxwell relations and equations of state. It was generally well answered, with nearly all candidates correctly deriving the Maxwell relation and using it to determine the thermal equation of state. The best students then found an expression for the enthalpy in order to determine the isobaric heat capacity, but some used a differential approach, which was also fine but more prone to error. Few candidates gave a full molecular description (excluded volume, no attractive intermolecular forces, diatomic). The last section involved determining the exergetic loss through a throttle: there were quite a few near-perfect answers, but many struggled here, not recognising the process as isenthalpic.

Q3. This question was on energy storage integrated into a nuclear power plant. It was the least popular question and not especially well answered. The majority that attempted it wrote the correct expression for the stored exergy, but there was a curious range of numerical answers. The second section involved analysis of the feed heaters; those that drew a control volume fared well on this. Most managed to show the given expression for net work output, but again many students did not draw clearly annotated control volumes. The final part of the question was to find the "effective charge efficiency" (exergy delivered to storage / deficit in work output). There were many numerical errors here, but a handful of students got the correct answer, though they all incorrectly assumed it was unrealistic.

Q4. A question on combined cycle. Few students answered well the first part, which examined qualitative understanding of the cycle's components. The second part was aiming at exploring the efficiency expression, and although most students could put down the original expression, many could not do the differentiation and very few truly explained the underlying physical reason for the trend. Part c(i) was done well, as it was relatively standard calculation for the Joule cycle, but Part (c(ii) was often done poorly (the pressure losses proved problematic). Finally, Part d was not done well in general, with very few candidates truly appreciating the overall energy needs of the two systems compared.

AJW / EM May 2026

Q1 (a) $CO + H_2O \rightleftharpoons CO_2 + H_2$ (THE WATER GAS SHIFT REACTION — R8 in THE DATA BOOK)

- Equimolar: p has no effect
- Exothermic (R8, DATA BOOK) $\Rightarrow K(T)$ decreases with T
  $\therefore$ y decreases with T.
- Increasing x will increase the partial pressure of H_2O, driving the reaction to the right and increasing y. [4]

(b) $CH_4 + x H_2O \rightarrow a CO + b CO_2 + c H_2O + y H_2$

Species conservation

C: $a + b = 1$

O: $a + 2b + c = 1 + b + c = x$

H: $2c + 2y = 4 + 2x$

$\Rightarrow$ $c = 2 + x - y$

$b = x - 1 - (2 + x - y) = y - 3$

$a = 1 - (y - 3) = 4 - y$

$n = a + b + c + d = 1 + (2 + x - y) + y = 3 + x$

Thus
$$K(T) = \frac{(P_{H_2}/P_0)(P_{CO_2}/P_0)}{(P_{CO}/P_0)(P_{H_2O}/P_0)} = \frac{y \cdot b}{a \cdot c} = \frac{y(y-3)}{(4-y)(2+x-y)}$$

From databook @ 800 K & $x = 5$

$\Rightarrow$ $K = e^{1.444} = \frac{y(y-3)}{(4-y)(7-y)}$ $\Rightarrow$ $K(y^2 - 11y + 28) = y^2 - 3y$

$\therefore \quad (K-1)y^2 + (3-11K)y + 28K = 0$

$3.2376\,y^2 - 43.614\,y + 118.65 = 0$

$\Rightarrow \quad y = 9.688$ or $\boxed{y = 3.783}$ (TOTAL $H_2 = 2+x \Rightarrow y<7$) [10]

$X_{H_2} = \frac{y}{n} = \frac{y}{3+x}$; $X_{H_2O} = \frac{e}{n} = \frac{2+x-y}{3+x}$

X_{H_2} @ $x = 5 = 3.783/8 = 0.473$; $X_{H_2O} = \frac{7-3.783}{8} = 0.402$

(c) C: $a + b = 1$

O: $a + 2b + c = x = 1$

$\Rightarrow \quad \cancel{1} + b + c = \cancel{1} \quad \Rightarrow \quad b = -c$

But neither b nor c can be negative, $\therefore b = c = 0$

THE PURE REFORMING REACTION OCCURS WITH NO H_2O LEFT FOR WGSR

i.e., $CH_4 + H_2O \rightarrow CO + H_2$

$\therefore$ X_{H_2} @ $x = 1 = 3/4 = 0.75$; $X_{H_2O} = 0$

Since the fuel cell reaction is $H_2 + \frac{1}{2}O_2 \rightleftharpoons H_2O$

The lower X_{H_2} and higher X_{H_2O} in (b) impede the forward reaction and therefore lower the Nernst potential, despite the increased yield of H_2 [6]

Q2 (a) $g = h - Ts \Rightarrow dg = dh - Tds - sdT = dh - (dh - vdp) - sdT$

$= vdp - sdT$

$\therefore$ $\boxed{v = \left(\frac{\partial g}{\partial p}\right)_T \; ; \; s = -\left(\frac{\partial g}{\partial T}\right)_p \quad \text{And} \quad \left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T}$ [3]

(b) (i) $v = \left(\frac{\partial g}{\partial p}\right)_T = b + \frac{RT}{p} \Rightarrow \boxed{p(v-b) = RT}$ [2]

(ii) $c_p = \left(\frac{\partial h}{\partial T}\right)_p$ so need to find $h = g + Ts$

$s = -\left(\frac{\partial g}{\partial T}\right)_p = -R\ln\left(\frac{p}{p_0}\right) - \frac{7R}{2}\left(1 - \ln\left(\frac{T}{T_0}\right) - 1\right)$

$= \underline{\frac{7R}{2}\ln\left(\frac{T}{T_0}\right) - R\ln\left(\frac{p}{p_0}\right)}$ (A)

$\therefore \; h = b(p - p_0) + RT\ln\left(\frac{p}{p_0}\right) + \frac{7R}{2}\left(T - T_0 - T\ln\left(\frac{T}{T_0}\right)\right)$

$+ \frac{7R}{2}T\ln\left(\frac{T}{T_0}\right) - RT\ln\left(p/p_0\right)$

$\Rightarrow \; \underline{h = b(p - p_0) + \frac{7R}{2}(T - T_0)}$ (B)

$\Rightarrow \; \boxed{c_p = 7R/2}$;

$\boxed{u = h - pv = h - (RT + bp) = \frac{5RT}{2} - \frac{7RT_0}{2} - bp_0 = \frac{5RT}{2} + \text{const.}}$ [6]

(iii) $u = f(T)$ only $\Rightarrow$ no intermolecular forces, but the $p(v-b) = RT$ accounts for the "excluded volume" of the molecules. $c_p = \frac{7R}{2}$ (and $c_v = \frac{5R}{2}$) imply diatomic molecules (5 D.O.F.). It's an "almost perfect" gas. [3]

(c) Throttle flow is isenthalpic, so exergetic loss $= b_1 - b_2 = T_0(s_2 - s_1)$

From (A) $s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$

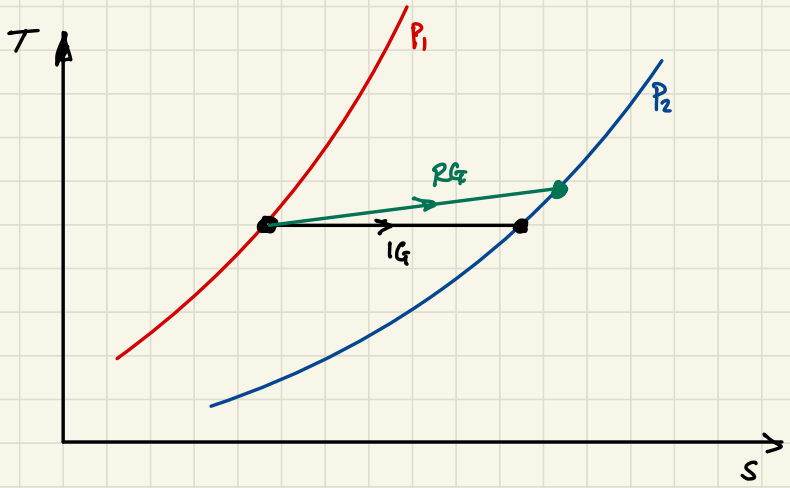
for an IG $h = \text{const.} \Rightarrow T = \text{const.}$

$\therefore \Delta s - \Delta s_{IG} = c_p \ln\left(\frac{T_2}{T_1}\right)$

From (B) $h_2 - h_1 = b(P_2 - P_1) + c_p(T_2 - T_1) = 0$

$\Rightarrow (T_2 - T_1) = b(P_1 - P_2) > 0$

$\Rightarrow$ **$\Delta s > \Delta s_{IG}$** (THE LOSS IS BIGGER THAN I/G CASE)



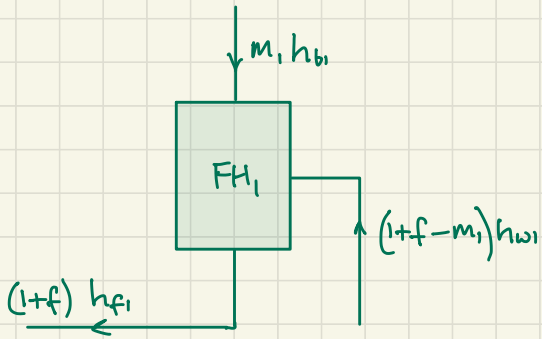
[6]

Q3 (a) $\Delta b = b_2 - b_1 = (h_2 - h_1) - T_0(s_2 - s_1)$

$= (1008.3 - 121.5) - 298\,(2.646 - 0.422)$

$= \boxed{224 \text{ kJ/kg}}$ ($\simeq 62.2$ kWh/m^3 — NOT BAD!) [3]

(b)



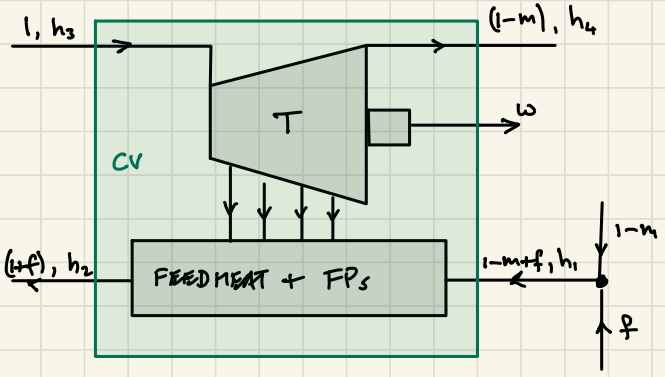
NOTE: h_{b1}, h_{w1} & h_{f1} are not specified, but all we require is that they do not change with f, as stated in the question

SFEE: $m_1 h_{b1} + (1+f-m_1)h_{w1} = (1+f)h_{f1} \Rightarrow$ $\boxed{m_1 = (1+f)\left(\dfrac{h_{f1} - h_{w1}}{h_{b1} - h_{w1}}\right)}$

The flow rate out of FH$_2$ $(1+f-m_1)$ is thus proportional to $(1+f)$ so that m_2 and by induction all bleed flows, m_i, must be proportional to $(1+f)$

$\therefore$ $\boxed{m = \sum m_i = \sum (1+f)m_i^\circ = (1+f)m_0}$ [6]

c)



SFEE: $\boxed{\omega = h_3 - (1-m)h_4 + (1-m+f)h_1 - (1+f)\,h_2}$

$\therefore$ $\omega_0 = h_3 - (1-m_0)h_4 + (1-m_0)h_1 - h_2$

$\Rightarrow$ $\Delta\omega = \omega_0 - \omega = (m_0 - m)h_4 + (m - m_0 - f)h_1 + f h_2$

$\boxed{\Delta\omega = f\{-m_0 h_4 + (m_0 - 1)h_1 + h_2\}}$ [6]

(d) Clearly $\eta_{CHG} = \dfrac{f(b_2 - b_1)}{\Delta w}$

But $\Delta w = f\left(-0.38 \times 1975.5 + (0.38 - 1)\,121.5 + 1008.3\right)$

$= f \times 182.3$ kJ/kg

$\therefore$ $\boxed{\eta_{CHG} = \dfrac{224}{182.3} = 123\ \%}$

It seems surprising to have a charge efficiency greater than 100%, but this is in fact feasible because energy is being put into storage before conversion to electricity, so turbine losses are effectively being deferred. "Real world" losses (e.g., heat transfer losses in the TES) will reduce η_{CHG}, but it can still be over 100%. [5]

(4) (a) (i) True - multiple pressure levels reduce the HX loss in the HRSG [2]

(ii) False - Feed heating would increase exhaust temperature from the HRSG, thereby giving a larger exhaust loss. [2]

(b) $\eta = 1 - \frac{1}{r_T}$ NOTE THIS IS JUST "$1 - T_C/T_H$" BECAUSE THE TEMPERATURE RATIO r_T IS THE SAME OVER ALL BITS OF THE ISC.

$\therefore \left(\frac{\partial \eta}{\partial \gamma}\right)_\beta = \frac{1}{r_T^2}\left(\frac{\partial r_T}{\partial \gamma}\right)_\beta$

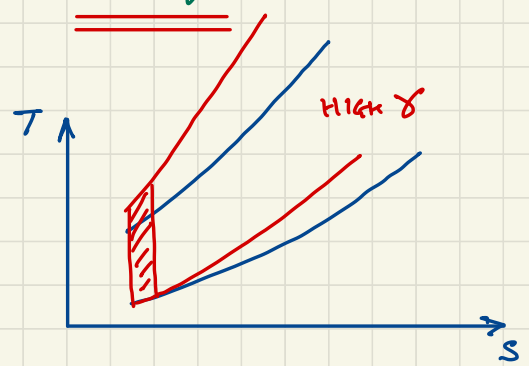
$r_T = \beta^{(\gamma-1)/\gamma} \Rightarrow \ln r_T = \frac{\gamma-1}{\gamma}\ln\beta$

$\Rightarrow \frac{1}{r_T}\left(\frac{\partial r_T}{\partial \gamma}\right)_\beta = \frac{1}{\gamma^2}\ln\beta.$

$\therefore \boxed{\left(\frac{\partial \eta}{\partial \gamma}\right)_\beta = \frac{1}{\gamma^2}\ln\beta \cdot \frac{1}{\beta^{(\gamma-1)/\gamma}}} \Rightarrow A = \frac{1}{\gamma^2}$

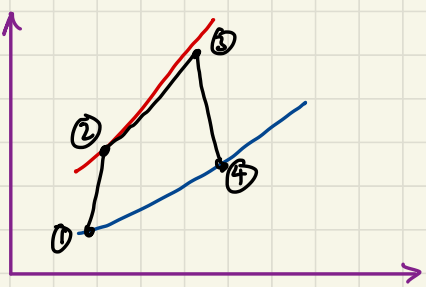
η clearly increases with γ

Increasing γ increases r_T (i.e. ratio of heat addition to rejection temperatures.



[5]

(c) (i)



$\eta = \frac{w_T - w_C}{q_{in}} = \frac{(T_3 - T_4) - (T_2 - T_1)}{T_3 - T_2}$

$T_2 = T_1 \beta^{\frac{\gamma-1}{\eta\gamma}} = 288 \times 24^{\frac{0.4}{0.9 \times 1.4}} = 789.9\ K$

$T_4 = T_3 \times \left(0.95\beta\right)^{-\left(\frac{\eta\gamma-1}{\gamma}\right)} = \frac{1700}{(0.95 \times 24)^{\frac{0.9 \times 2}{7}}} = 760.8\ K$

$\therefore \eta = \frac{(1700 - 760.8) - (789.9 - 288)}{1700 - 789.9} = 48.05\ \%$

[5]

(ii) With extra pressure loss $P_4 = 1.15 P_a$, $P_3 = 0.95 P_2$ $\Rightarrow \beta_T = \frac{\beta_c \times 0.95}{1.15}$

$\therefore T_4 = 1700 \Big/ \left(\frac{0.95 \times 24}{1.15}\right)^{0.9 \times 2/7} = 788.6\,K$

$\therefore \eta = \frac{(1700 - 788.6) - (789.9 - 288)}{1700 - 789.9} = 45.0\,\%$

REDUCTION OF ~ 3%

[3]

(d) SMR: $\frac{W_{CCGT}}{W_E} = \frac{0.63 \times 120}{2.7 \times 3.6} = 7.78$

ELECTROLYSIS: $\frac{W_{CCGT}}{W_E} = \frac{0.63 \times 120}{47 \times 3.6} = 0.447$

For electrolysis this is really a "round-trip efficiency" – because electricity is the only energy input

For SMR, the energy input from the fuel needs to be taken into account in order to make a fair comparison.

[3]