EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 3 May 2022 9.30 to 11.10

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A thin filament of stretched material of diameter d is used to measure the temperature of the surrounding fluid flow by detecting the intensity of the radiation emitted by the filament surface.

(a) Under steady-state conditions, the rate of heat convection per unit length of the filament is balanced by radiation loss from the wire to the surroundings, which are at much lower temperature than the gas. Estimate the temperature difference between the filament and the surrounding fluid for a Nusselt number $Nu_d = hd / k_g = 0.35\, Re_d^{\ 0.5}$, where the Reynolds number $Re_d = U_g d / \nu$, h is the heat convection coefficient, U_g the flow velocity, ν the gas kinematic viscosity and k_g the gas thermal conductivity. The emissivity of the filament surface is unity, the fluid temperature T_g = 2000 K and other properties are given in Table 1. [20%]

(b) Under unsteady flow conditions we may neglect heat losses by radiation so that the unsteady rate of energy transfer to the filament is balanced by the rate of change of internal energy. Show that the fluctuations in temperature of the filament are controlled by a time scale $\tau = \rho c d / 4h$ where ρ and c are its density and specific heat capacity respectively. Using this result estimate the maximum frequency that the wire-based sensor could follow for conditions listed in Table 1. [30%]

(c) Wien's approximation to Planck's radiation equation is

$$I_b = \frac{C_1}{\lambda^5 \exp\left(C_2/(\lambda T)\right)}$$

where I_b is the radiation intensity at wavelength λ, and C_1 and C_2 are positive constants. Starting from this equation, show how the ratio of the intensities at two different wavelengths can be used to determine the temperature of the filament (and thus the flow).

[30%]

(d) Denoting the ratio of two wavelengths as R_λ and that of the corresponding intensities R_I , obtain an expression for the fractional uncertainty in temperature as a function of the fractional error in R_I for a fixed R_λ . Comment on how best to choose R_λ for minimum temperature error. [20%]

Table 1: Properties and operating conditions

Property	Symbol	Unit	Value
Wire			
Diameter	d	m	10×10^{-6}
Density	ρ	$\mathrm{kg\,m^{-3}}$	4500
Heat capacity	c	$\mathrm{J\,kg^{-1}K^{-1}}$	500
Thermal conductivity	k	$\mathrm{W\,m^{-1}K^{-1}}$	20
Gas			
Thermal conductivity	k_g	$\mathrm{W\,m^{-1}K^{-1}}$	100
Velocity	U_g	$\mathrm{m\,s^{-1}}$	10
Kinematic Viscosity	ν_g	$\mathrm{m^2s^{-1}}$	10×10^{-5}

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2 A fin with cross-sectional area A and perimeter P extends from its base at axial distance $x = 0$, kept at constant temperature T_0 , to a distance of $x = L$, where it reaches the surrounding temperature T_∞ . Heat is exchanged by convection with the surroundings with an associated convective heat exchange coefficient h , and the thermal conductivity of the material is λ .

(a) Assuming uniform temperature across the cross section and steady-state heat conduction, show that the material temperature along the fin is given by

$$T = T_\infty + (T_0 - T_\infty)\left[\cosh(mx) - \frac{1}{\tanh(mL)}\sinh(mx)\right]$$

where $m^2 = hP/\lambda A$. [30%]

(b) Starting from the steady 3D conduction equation $\nabla^2 T = 0$, and using order of magnitude scaling arguments, show that a sufficient condition for the one-dimensional analysis to be valid is that $L^2 >> A$. Show that, under these conditions, $hP/\lambda << 1$. [25%]

(c) For a given total volume of fin, and a rectangular fin geometry with cross section $L_1 \times L_2$, show that the overall heat transfer is maximised for a fin of square cross section. [25%]

(d) Explain why, in spite of (c), most fin arrays are not usually built as square cross sections considering, for example, fluid mechanics or other relevant factors. Use well labelled sketches to justify your answers. [10%]

(e) Explain why the enhancement in heat transfer by a fin array is not just equal to a simple multiple of the number of fins. Use well labelled sketches to justify your answers. [10%]

3 Consider the steady, incompressible, turbulent flow with free-stream velocity U_∞ and temperature T_∞ over a heated flat plate held at temperature T_w. The free-stream pressure gradient is zero. The effects of the turbulence are represented by writing the velocities and temperature as the sum of mean and fluctuating components:

$$u(t) = U + u' \; ; \; v(t) = V + v' \; ; \; T(t) = T + T'$$

(a) Sketch the wall-normal variation in mean velocity U and mean temperature T. Write down approximations to the fluctuations in terms of the mean gradients of U and T and mixing length ℓ. [20%]

(b) Write down an expression for the turbulent shear stress in terms of the fluctuating velocity components. Explain carefully the physical basis for this expression. Prandtl's mixing length model sets $\ell = \kappa y$ where κ is a constant. Show that this is consistent with a mean velocity profile of the form

$$\frac{U}{U_\infty} = \frac{1}{\kappa}\sqrt{\frac{C_f}{2}}\log y + C$$

where C_f is the skin friction coefficient based on the wall shear stress τ_w and C is a constant. [40%]

(c) Write down an expression for the heat flux in terms of the fluctuating components of velocity and temperature. Using the same mixing length model as in Part (b), show that the profile of $(T - T_w)/(T_\infty - T_w)$ can be expressed in a similar form to that in Part (b) by the inclusion of the Stanton number St. Comment on the result. [40%]

4 Consider the flow in the passages in a catalytic converter. Each passage consists of a long, narrow cylindrical channel with diameter d. The flow is steady and laminar, and the entry length may be considered to be negligibly short.

(a) For a fluid element within the passage from streamwise location x to $x+\Delta x$ derive an expression relating the change in fluid pressure Δp and the wall skin friction τ_w. [15%]

(b) In a similar way derive an expression relating the fluid bulk velocity U_b, bulk temperature T_b and wall heat transfer based on a wall heat transfer coefficient h and wall temperature T_w. [15%]

(c) The chemical reaction at the wall drives a species concentration gradient with wall mass transfer coefficient h_{mi}. Sketch the velocity and species profiles across the passage at a typical streamwise location. Derive an expression relating the streamwise variation of the bulk species concentration ρ_{ib} to the wall concentration ρ_{iw} and mass transfer coefficient. [30%]

(d) At the wall the mass transfer coefficient is related to the reaction rate Ω_i by the following expression

$$h_{mi}(\rho_{ib} - \rho_{iw}) = \eta\Omega_i$$

where η represents surface density and texture factors. Consider two limiting cases: (i) diffusion-limited at the surface; and (ii) constant surface reaction rate. Derive two corresponding expressions for the streamwise variation of the bulk species concentration and comment on the result. When are these two limits likely to arise during the operation of the device? [40%]

END OF PAPER