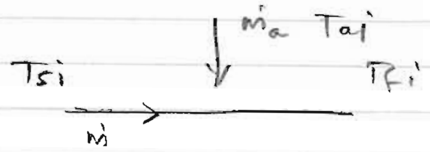
Q1 - HX



(a) $$\dot m c\, dT = UP\, dx\, (T_a - T)$$

$$\frac{dT}{T_a - T} = \frac{UP}{\dot m c} dx$$

$$\ln \frac{(T_{ai} - T_{fi})}{(T_{ai} - T_{si})} = -\frac{UP}{\dot m c} L$$

$$T_{fi} = T_{ai} - (T_{ai} - T_{si}) \exp\left(-\frac{UP}{\dot m c} L\right)$$

(b) $$\varepsilon = \frac{Q}{Q_{MAX}}$$

$$Q_i = \dot m c\,(T_{fi} - T_{si}) = \dot m c \left[T_{ai} - T_{si} - (T_{ai} - T_{si}) \exp\left(-\frac{UPL}{\dot m c}\right)\right]$$

$$= \dot m c\,(T_{ai} - T_{si}) \left[1 - \exp\left(-\frac{UPL}{\dot m c}\right)\right]$$

$$Q_{iMAX} = \dot m c\,(T_{ai} - T_{si})$$

$$\varepsilon = \frac{Q_i}{Q_{iMAX}} = \left[1 - \exp\left(-\frac{UPL}{\dot m c}\right)\right]$$

(c) $$Q_i = \dot m c\,(T_{ai} - T_{si})\,\varepsilon$$

$$Q_{i+1} = \dot m c\,(T_{ai+1} - T_{si+1})\,\varepsilon$$

$$\frac{Q_{i+1}}{Q_i} = \frac{(T_{ai+1} - T_{si+1})}{(T_{ai} - T_{si})}$$

$$\dot m_a c_a (T_{ai} - T_{ai+1}) = \dot m c\,(T_{ai} - T_{si})\,\varepsilon$$

$$T_{ai+1} = T_{ai} - \frac{\dot m c}{\dot m_a c_a}(T_{ai} - T_{si})\,\varepsilon$$

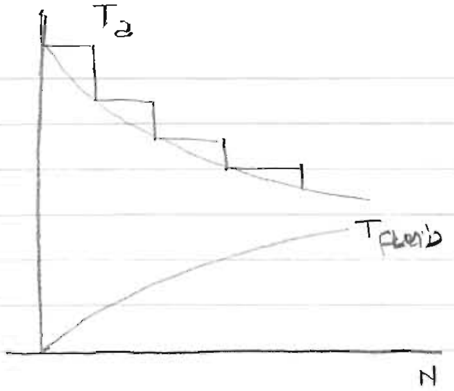
$$T_{si+1} = T_{fi} = T_{ai} - (T_{ai} - T_{si}) \exp\left(-\frac{UPL}{\dot m c}\right)$$

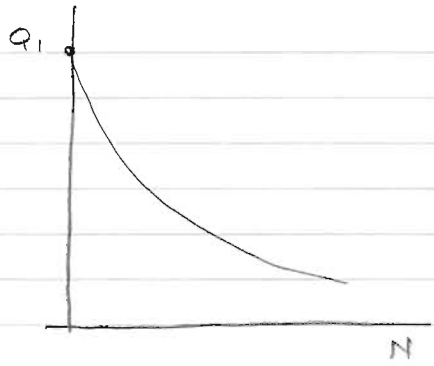
$$(T_{ai+1} - T_{si+1}) = (T_{ai} - T_{si}) \left[\exp\left(\frac{-UPL}{\dot m c}\right) - \frac{\dot m c}{\dot m_a c_a}\varepsilon\right]$$

$$= (T_{ai} - T_{si})\,(1 - \varepsilon - C\varepsilon)$$

$$\boxed{\frac{Q_{i+1}}{Q_i} = 1 - (1 + C)\,\varepsilon}$$

(d)





(e) $Q = \sum Q_i = \dot{m} c \left(T_{a1} - T_{s1}\right) \varepsilon \sum_{i=1}^{\infty} \left(1 - (1+c)\varepsilon\right)^{i-1}$

$$r = 1 - (1+c)\varepsilon$$

$$\sum_{i=1}^{\infty} r^{i} = \frac{1}{1-r} = \frac{1}{1-\left[1-(1+c)\varepsilon\right]} = \frac{1}{(1+c)\varepsilon}$$

$$Q = \dot{m} c \frac{\left(T_{a1} - T_{s1}\right)}{1+c}$$

i.e. for a long enough number of passes, the length of the heat exchanger is equivalent to an infinitely long exchanger, with $\varepsilon = 1$.

A counter-flow system has a higher ΔT_{LMTD} and therefore would yield higher heat transfer rates per unit length.

(a) INSIDE THE ROD, WE HAVE CONDUCTION AND GENERATION:

(i)

$$\nabla^2 T = \dot{g}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_1 r \frac{\partial T}{\partial r}\right) = \dot{g}$$

$$\lambda_1 r \frac{\partial T}{\partial r} = \dot{g}\frac{r^2}{2} + c_1$$

$$\frac{\partial T}{\partial r} = \frac{\dot{g}}{\lambda_1}\frac{r}{2} + \frac{c_1}{\lambda_1 r}$$

$$T = \frac{\dot{g}}{\lambda}\frac{r^2}{4} + \cancel{\frac{c_1}{\lambda}}\ln r \qquad c_1 = 0 \text{ for finite sol.}$$

0

$$T(r) = T(a) + \frac{\dot{g}a^2}{4}\left(1 - \left(\frac{r}{a}\right)^2\right) \qquad T_c = T(0).$$

(ii) ENCASING TUBE : conduction only

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_2 r \frac{\partial T}{\partial r}\right) = 0$$

$$\lambda_2 r \frac{\partial T}{\partial r} = c_2$$

$$\frac{\partial T}{\partial r} = \frac{c_2}{\lambda_2 r}$$

$$T = \frac{c_2}{\lambda_2}\ln r + c_3$$

$$T(a) = \frac{c_2}{\lambda_2}\ln a + c_3$$

$$T(b) = \frac{c_2}{\lambda_2}\ln b + c_3$$

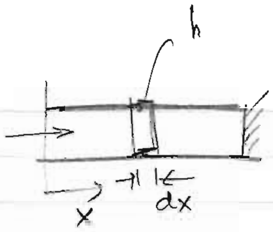
$$c_2 = \frac{\{T(b) - T(a)\}}{\ln(b/a)}\lambda_2$$

$$c_3 = T(a) - \frac{(T(b) - T(a))}{\ln(b/a)}\ln a$$

$$\boxed{T(r) = T(a) + \frac{T(b) - T(a)}{\ln(b/a)}\ln\left(\frac{r}{a}\right)}$$

(b)

(i)



$$t\left\{-\lambda_2\frac{\partial T}{\partial x} - \left[-\lambda_2\frac{\partial T}{\partial x}\right]_{x+dx}\right\} = 2h\,(T - T_\infty)\,dx$$

$$t\lambda_2\frac{\partial^2 T}{\partial x^2} = h\,(T - T_\infty)$$

$$\frac{d^2\theta}{dx^2} - \underbrace{\frac{2h}{\lambda t}}_{m^2}\theta = 0 \qquad \theta = T - T_\infty$$

$$\theta = A e^{mx} + B e^{-mx}$$

$$\theta(0) = T(b) - T_\infty = A + B$$

$$\left.\frac{d\theta}{dx}\right|_L = m\left[A e^{mL} - B e^{-mL}\right] = 0$$

$$\therefore \quad B = A e^{mL}/e^{-mL}$$

$$A + A e^{2mL} = T(b) - T_\infty$$

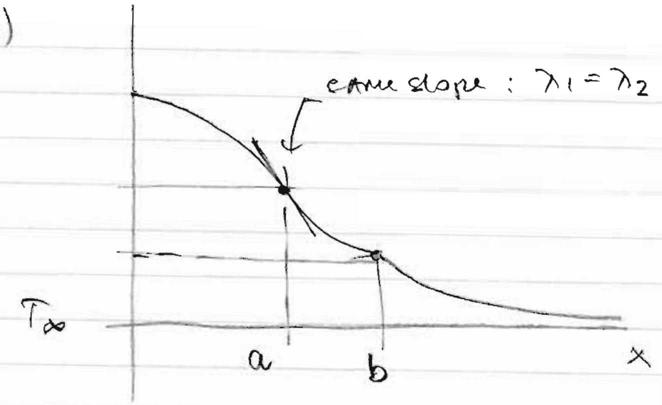
$$A = \frac{T(b) - T_\infty}{1 + e^{2mL}}$$

$$\therefore \quad \theta = T - T_\infty = \frac{T(b) - T_\infty}{1 + e^{2mL}}\left(e^{mx} + e^{-mx}\frac{e^{mL}}{e^{-mL}}\right)$$

$$\boxed{\frac{T - T_\infty}{T(b) - T_\infty} = \frac{e^{+m(x-L)} + e^{-(x-L)}}{e^{-mL} + e^{mL}} = \frac{\cosh[m(L-x)]}{\cosh[mL]}}$$

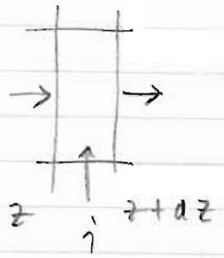
(ii) h is affected by the flow pattern above the fin, whether natural or forced convection, and the properties of the fluid.

(c)



Q2

(a)



$$\dot m Y_b\big|_z - \dot m Y_b\big|_{z+dz} + j_s P\,dz = 0$$

$$\dot m Y_{b,z} - \dot m\left(Y_{b,z} + \frac{\partial Y_b}{\partial z}dz\right) + j_s 2\pi R\,dz = 0$$

$$j_s = \frac{\dot m}{2\pi R}\frac{dY_b}{dz} = \frac{\rho u_b}{2}\frac{dY_b}{dz}$$

SINCE $\frac{\partial Y}{\partial z} = \text{const}$, $\frac{dY_b}{dz}$ IS CONSTANT AND SO IS j_s.

(b) $$u\frac{\partial Y}{\partial z} + \underbrace{\cancel{v\frac{\partial Y}{\partial r}}}_{0} = D\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Y}{\partial r}\right) + \underbrace{\cancel{\frac{\partial^2 Y}{\partial z^2}}}_{0}\right]$$

$$u\overbrace{\frac{\partial Y}{\partial z}}^{\text{const.}} = \frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Y}{\partial r}\right)$$

$$2u_b\left(1-\left(\frac{r}{R}\right)^2\right)\frac{\partial Y}{\partial z} = \frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Y}{\partial r}\right)$$

SINCE $\frac{\partial Y}{\partial z}$ IS CONSTANT, WE CAN INTEGRATE THE EQUATION IN r:

$$C_1 + \frac{2u_b}{D}\frac{\partial Y}{\partial z}\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right) = r\frac{\partial Y}{\partial r}$$

$$\frac{2u_b}{D}\frac{\partial Y}{\partial z}\left(\frac{r^2}{4} - \frac{r^4}{16R^2}\right) + C_1 \ln r + C_2 = Y$$

USING THE FACT THAT $Y(0) = Y_c$, WE HAVE:

$$Y - Y_c = \frac{u_b}{2D}\frac{\partial Y}{\partial z}r^2\left(1 - \frac{1}{4}\left(\frac{r}{R}\right)^2\right)$$

(c) $\dot m Y_b = \int^R \rho u Y \, 2\pi r \, dr$

$$\dot m (Y_b - Y_c) = \int_0^R \rho u \,(Y - Y_c)\, 2\pi r\, dr$$

$$\rho u_b \pi R^2 (Y_b - Y_c) = \rho \, 2u_b \int_0^R \left(1 - \left(\frac{r}{R}\right)^2\right) \frac{u_b}{2D} \frac{\partial Y}{\partial z} r^2 \left(1 - \frac{1}{4}\left(\frac{r}{R}\right)^2\right) 2\pi r dr$$

$$Y_b - Y_c = \frac{2}{\pi R^2} \frac{u_b}{2D} \frac{\partial Y}{\partial z} \int_0^R \left(r^2 - \frac{r^4}{R^2}\right)\left(1 - \frac{1}{4}\left(\frac{r^2}{R^2}\right)\right) 2\pi r\, dr$$

$$= \frac{2u_b}{DR^2} \frac{\partial Y}{\partial z} \int_0^R \left[r^3 - \underbrace{\frac{r^5}{R^2} - \frac{r^5}{4R^2}}_{5/4} + \frac{r^7}{4R^4} \right] dr$$

$$= \frac{\cancel{2} u_b}{DR^2} \frac{\partial Y}{\partial z} \underbrace{\left[\frac{R^4}{4} - \frac{5}{4} \frac{R^6}{6R^2} + \frac{R^8}{32 R^4} \right]}_{\frac{24 - 20 + 3}{96} = \frac{7}{96}} =$$

$$\boxed{Y_b - Y_c = \frac{7}{48} \frac{u_b R^2}{D} \frac{\partial Y}{\partial z}}$$

(c) $$j = \rho D \frac{\partial Y}{\partial r}\bigg|_{r=R} = \rho D \frac{2 u_b}{D} \frac{\partial Y}{\partial z} \left(\frac{r}{2} - \frac{r^3}{4R^2} \right)_{r=R} = \frac{\rho u_b}{2} \frac{\partial Y}{\partial z}$$

($R/4$)

SINCE WE ARE IMPOSING A CONSTANT RATE OF CHANGE ON Y, THIS AUTOMATICALLY TIES DIFFUSION TO THE RATE OF CHANGE

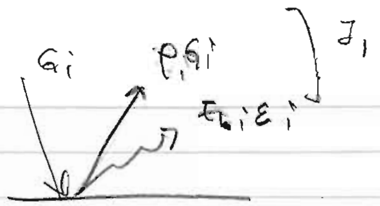
(d) $$j_s = -\rho h_s (Y_b - Y_s) = -h_s \left[(Y_b - Y_c) - (Y_s - Y_c) \right] = \frac{\rho u_b}{2} \frac{\partial Y}{\partial z}$$

$$Y_s - Y_c = \frac{3}{8} \frac{u_b R^2}{D} \frac{\partial Y}{\partial z}$$

$$-h_s \underbrace{\left[\frac{7}{48} - \frac{3}{8} \right]}_{\frac{11}{48}} \frac{u_b R^2}{D} \frac{\partial Y}{\partial z} = \frac{u_b}{2} \frac{\partial Y}{\partial z} \longrightarrow h_s = \frac{48}{11} \frac{D}{2R}$$

$$\therefore \boxed{Sh = \frac{h_s (2R)}{D} = \frac{48}{11}}$$

4(a)



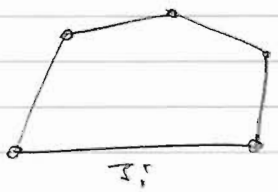
Radios. $J_i = \rho_i G_i + E_{bi}\varepsilon_i = (1-\varepsilon_i) G_i + E_{bi}\varepsilon_i$

$$\to G_i = \frac{J_i - E_{bi}\varepsilon_i}{1-\varepsilon_i}$$

Rad. heat loss:

$$-Q_{ii} = (J_i - G_i) A_i$$
$$= A_i \left[J_i - \left(\frac{J_i - \varepsilon_i E_{bi}}{1-\varepsilon_i} \right) \right] = A_i \varepsilon_i \frac{(E_{bi} - J_i)}{1-\varepsilon_i}$$

(b)



$$J_i = (1-\varepsilon_i) G_i + \varepsilon_i E_{bi}$$

$$A_i G_i = \sum_{i \neq j} F_{ji} A_j J_j + F_{ii} A_i J_i$$

$$G_i = \sum_{i \neq j} F_{ji} \frac{A_j}{A_i} J_j + F_{ii} J_i$$

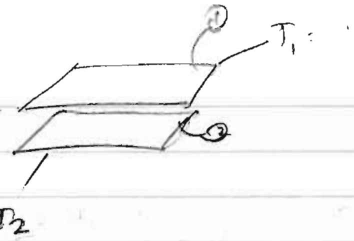
$$J_i - (1-\varepsilon_i) F_{ii} = (1-\varepsilon_i) \sum_{i \neq j} F_{ji} \frac{A_j}{A_i} J_j + \varepsilon_i E_{bi}$$

but $F_{ji} A_j = F_{ij} A_i$

$$\therefore J_i = \frac{1}{1-(1-\varepsilon_i) F_{ii}} \left[(1-\varepsilon_i) \sum_{i \neq j} F_{ij} J_j + \varepsilon_i E_{bi} \right]$$

AS REQUIRED.

(c)



③ T_3

$F_{12} = F_{21} = 0.2$

(i) $\cancel{F_{11}} + F_{12} + F_{13} = 1$, with $F_{11} = 0$

$F_{13} = 0.8$

Similerly, $F_{23} = 0.8$

$F_{13} A_1 = F_{31} A_3$ (with $A_3 \to \infty$) $\longrightarrow F_{31} = 0$

$F_{32} = 0$

$F_{31} + F_{32} + F_{33} = 1 \longrightarrow F_{33} = 1$

$\varepsilon_3 = 1$ (large)

(ii)

$$J_1 = \frac{1}{1 - (1-\varepsilon_1)\cancel{F_{11}}}\left[(1-\varepsilon_1) F_{12} J_2 + (1-\varepsilon_1) F_{13} J_3 + \varepsilon_1 E_{b1}\right], \quad F_{11} = 0$$

$$J_2 = \frac{1}{1 - (1-\varepsilon_2)\cancel{F_{22}}}\left[(1-\varepsilon_2)(F_{21} J_1 + F_{23} J_3) + \varepsilon_2 E_{b2}\right], \quad F_{22} = 0$$

$$J_3 = \frac{1}{1 - \cancel{(1-\varepsilon_3)} F_{33}}\left[\cancel{(1-\varepsilon_3)}\left(\quad\right) + \varepsilon_3 E_{b3}\right], \quad 1-\varepsilon_3 = 0$$

WE HAVE :

$$\underline{J_1} - (1-\varepsilon_1) F_{12} \underline{J_2} - (1-\varepsilon_1) F_{13} \underline{J_3} = \varepsilon_1 E_{b1}$$

$$-(1-\varepsilon_2) F_{21} \underline{J_1} + J_2 - (1-\varepsilon_2) F_{23} \underline{J_3} = \varepsilon_2 E_{b2}$$

$$\underline{J_3} = E_{b3}$$

$$A = \begin{bmatrix} 1 & -(1-\varepsilon_1) F_{12} \\ -(1-\varepsilon_2) F_{21} & 1 \end{bmatrix} = \begin{bmatrix} 1 & -(0.2)(0.2) \\ -(0.5)(0.2) & 1 \end{bmatrix} = \begin{bmatrix} 1 & -0.04 \\ -0.1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} \varepsilon_1 E_{b1} + (1-\varepsilon_1) F_{13} E_{b3} \\ \varepsilon_2 E_{b2} + (1-\varepsilon_2) F_{23} E_{b3} \end{bmatrix} = \begin{bmatrix} 0.8 E_{b1} + (0.2)(0.8) E_{b3} \\ 0.5 E_{b2} + (0.5)(0.8) E_{b3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.04 \\ -0.1 & 1 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} = \begin{bmatrix} 0.8\,E_{b1} + 0.16\,E_{b3} \\ 0.5\,E_{b2} + 0.4\,E_{b3} \end{bmatrix}$$

① × 0.1 + ② :

$$\begin{bmatrix} 1 & -0.04 \\ 0 & 1-0.004 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} = \begin{bmatrix} 0.8\,E_{b1} + 0.16\,E_{b3} \\ (0.5+0.08)E_{b1} + (0.4+0.016)\,E_{b3} \end{bmatrix}$$

$\underline{J_2} = 0.510\,E_{b1} + 0.417\,E_{b3}$

$\underline{J_1} = 0.8\,E_{b1} + 0.16\,E_{b2} + 0.04\,J_2 = 0.82\,E_{b1} + 0.16\,E_{b2} + 0.0167\,E_{b3}$

$\underline{J_3} = E_{b3}.$

(iii) $$\frac{Q}{A_1} = \frac{\varepsilon_1}{1-\varepsilon_1}\left(E_{b1} - J_1\right) = \frac{0.8}{0.2}\left((1-0.82)E_{b1} - 0.16\,E_{b2} - 0.0167\,E_{b3}\right)$$

$$\frac{Q}{A_1} = 0.72\,E_{b1} - 0.64\,E_{b2} - 0.067\,E_{b3}$$

(d) THE HEAT LOSS BY RADIATION DOES NOT CHANGE,

BUT THE HEAT LOSS BY CONVECTION DOES.

$$\frac{Q_1}{A_1} = \frac{\varepsilon_1}{1-\varepsilon_1}\left(E_{b1} - J_1\right) - h\left(T_1 - T_\infty\right)$$

THE MATRIX FOR $AJ = C$ IS THE SAME, AND T_1 IS UNKNOWN. NOW WE HAVE ONE MORE EQUATION FOR T_1, AS ABOVE. THE NON-LINEAR SYSTEM CAN BE SOLVED FOR T_1 AND J_1.