EGT2
ENGINEERING TRIPOS PART IIA

Friday 1 May 2026   9.30 to 11.10

Module 3A6

HEAT AND MASS TRANSFER

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Figure 1a. illustrates highly-ordered graphite comprising stacked, two-dimensional graphene layers. Carbon atoms in each layer are held together by covalent bonds, whereas the stacked layers are held together by weak van der Waals forces. This gives rise to thermal anisotropy. The in-plane thermal conductivity, $\lambda_{||}$, is 2000 $\mathrm{Wm^{-1}K^{-1}}$ and its out-of-plane (layer to layer) thermal conductivity, $\lambda_{\perp}$, is 5 $\mathrm{Wm^{-1}K^{-1}}$. The graphite has a density ρ and specific heat capacity c.

(a) Using the coordinate system in Fig. 1a, define an appropriate 3-dimensional element and show that the energy governing equation of the system is

$$\frac{\partial T}{\partial t} = \alpha_x \frac{\partial^2 T}{\partial x^2} + \alpha_y \frac{\partial^2 T}{\partial y^2} + \alpha_z \frac{\partial^2 T}{\partial z^2}.$$

Obtain the expressions for α_x, α_y and α_z in terms of $\lambda_{||}$, $\lambda_{\perp}$, ρ and c. [20 %]

(b) At steady state, the stack has three sides held at $T = T_0$, and its top side at $T = T_0 + \Delta T$, as shown in Fig. 1b. The stack can be assumed to be infinitely long in the y-direction. Introduce the reduced temperature function $\Theta(x, z) = X(x)Z(z)$ and use separation of variables to show that the solution is

$$\Theta(x, z) = \sum_n b_n \sin(\beta_n x) \sinh(\gamma_n z)$$

State the boundary conditions used. Obtain expressions for β_n and γ_n. [30 %]

(c) Use the inhomogeneous boundary condition to obtain an expression for b_n. [20 %]

(d) Sketch the temperature contours for the system. [15 %]

(e) Describe the change in the temperature contours if the stacking direction is along the x-axis. Which plane provides the better thermal conductivity in this new system? [15 %]

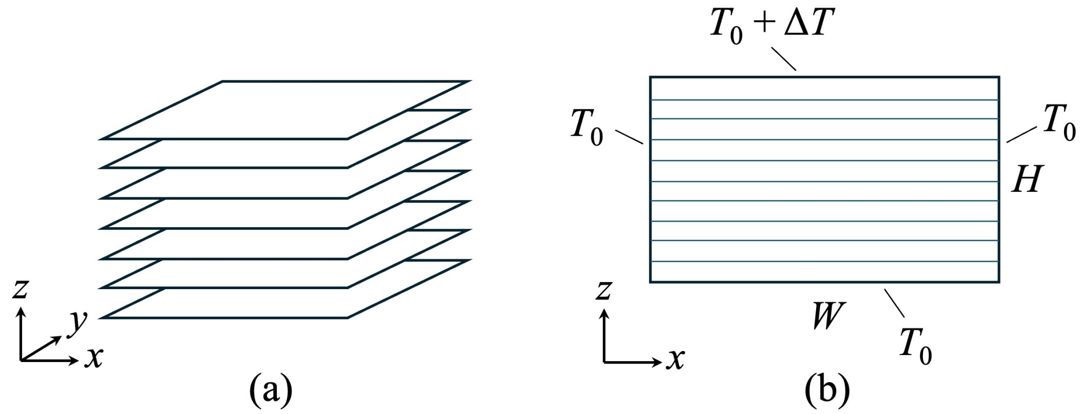

Fig. 1

2 Two stationary parallel plates with a spacing of $2H$ form a channel, along which an incompressible fluid flows, as shown in Fig. 2. The fluid has a constant value for density ρ, viscosity μ, specific heat capacity c_p, and thermal conductivity λ. The fully developed laminar flow is driven by a constant pressure gradient, dp/dx, and the system is at steady state. The heat fluxes directed into the top and bottom plates are constant at $\dot{q}_w$ and are uniform along the channel. The coordinate system is defined in Fig. 2, and there is negligible flow into the depth of the channel.

(a) Obtain the velocity u_x of the fluid given that the Navier–Stokes equation is

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u}$$

State any further assumptions and state the boundary conditions. [20 %]

(b) Determine the bulk mean velocity U_b of the fluid. [10 %]

(c) Given the energy equation of the fluid is

$$\rho c_p\left(\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T\right) = \nabla\cdot(\lambda\nabla T)$$

Show that the governing equation for the temperature in the flow field is

$$\frac{3}{2}U_b\left[1-\left(\frac{y}{H}\right)^2\right]\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2}$$

where α is $\lambda/\rho c_p$. [10 %]

(d) The temperature at the centre line is T_0. Obtain an expression for the temperature profile in terms of $\dot{q}_w$ given that $\partial T/\partial x$ is constant. State any further assumptions and state the boundary conditions. [30 %]

(e) Given that the bulk mean temperature of the system is

$$T_b = T_0 - \frac{\dot{q}_w H}{\lambda}\frac{39}{280}$$

Determine the Nusselt number, Nu. [20 %]

(f) Describe how the Nusselt number at the channel entrance differs from that in the fully developed region of the channel. [10 %]

Fig. 2

3 Consider a static film of thickness H on a solid sheet of thickness H with the coordinate system as shown in Fig. 3. This system is assumed to be one-dimensional, with no concentration gradients in the y and z directions. Species A diffuses through the film with a diffusion coefficient D_A. The concentration of A on the outer film boundary at $x = -H$ is C_{AS}. At the interface between the film and the sheet, species A is irreversibly converted to species B by a first-order surface reaction with a rate constant k. Species B diffuses only through the sheet, and it has a diffusion coefficient of D_B. At the outer surface of the sheet, species B is removed by convection into the surroundings with convective mass-transfer coefficient h_m. The far field concentration of species B in the surroundings is $C_{B,\infty}$. The mass conservation equation for species i is

$$\frac{\partial C_i}{\partial t} = D_i \left[\frac{\partial^2 C_i}{\partial x^2} + \frac{\partial^2 C_i}{\partial y^2} + \frac{\partial^2 C_i}{\partial z^2} \right] + R_i$$

where D_i is the diffusivity of species i, R_i is the rate of formation per unit volume of species i.

(a) Write the governing mass conservation equations for species A and B in the film and the sheet respectively, stating all the boundary conditions for each species. Briefly explain the physical meaning of each boundary condition. [15 %]

(b) Obtain steady-state expressions for the concentrations C_A in the film and C_B in the sheet. Sketch the concentration profiles. [30 %]

(c) Determine the molar flux, J_A, of species A through the film to the interface between the film and the sheet. [20 %]

(d) Determine the molar flux, J_B, of species B from the interface between the film and sheet to the surrounding. [20 %]

(e) Consider the two limiting cases of $k \to 0$ and $k \to \infty$. For each case, describe how k affects the molar fluxes and concentration profiles of the system. [15 %]

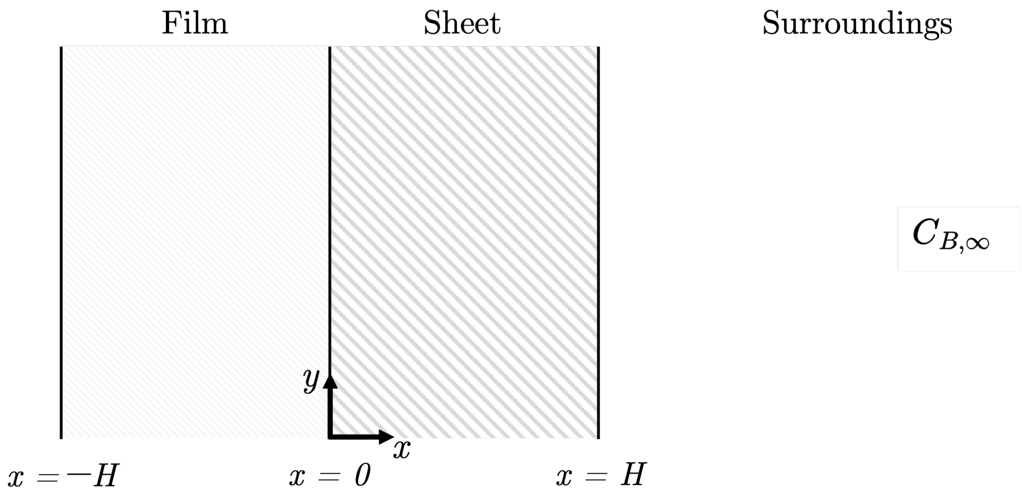

Fig. 3

4 Three horizontal, parallel plates separated by a finite distance are contained in an insulated enclosure. The top and bottom plates are grey, diffuse, and opaque surfaces, while the middle plate is a semi-transparent plate that emits and transmits radiation. The system operates at steady state, and radiation is the only mode of heat transfer. The plates can be assumed to be much longer than the spacing between them. The top plate is maintained at 1200 K, and the bottom plate at 500 K. The middle plate reaches a steady-state temperature, T_m. The emissivities of the top, middle, and bottom plates are 0.4, 0.7, and 0.6 respectively. The transmissivity of the middle plate is 0.3. All three plates have a planar surface area of 0.25 m^2.

(a) Sketch the radiation circuit and write expressions for each radiative resistance component in the circuit. [20 %]

(b) Determine the view factors between all surfaces in this system. [10 %]

(c) Determine the total effective radiation resistance between the top and bottom plate. [15 %]

(d) Determine the steady-state temperature T_m of the middle plate. [25 %]

(e) Calculate the net radiative heat transfer rate between the top and bottom plates. [15 %]

(f) Describe the changes to the radiation circuit and net radiative heat transfer rate if the middle plate is replaced by an opaque shield with the same emissivity. [15 %]

END OF PAPER