Version wnd_2

EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 4 May 2021 9.00 to 10.40

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 (a) The two-dimensional momentum and energy equations for steady, incompressible, laminar flow are given by:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\nabla^2 u \qquad\qquad u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\nabla^2 T$$

Identify the quantities appearing in the expressions and briefly explain the physical significance of each of the terms. [20%]

(b) Consider fully developed flow in the bearing shown in Fig.1. The two bearing pads are separated by a small distance H. The lower pad is stationary; the upper pad moves at constant speed U and has temperature $T_s(x)$. Derive an expression for the velocity profile across the bearing gap when the flow experiences a streamwise pressure gradient ($\mathrm{d}p/\mathrm{d}x$). Sketch the profile and explain the physical significance of the terms. [20%]

(c) For the same case, derive an expression for the temperature profile across the bearing gap. You may assume that the streamwise temperature gradient ($\mathrm{d}T/\mathrm{d}x$) is constant and that the lower bearing pad is adiabatic. Sketch the profile and explain the physical significance of the terms in your expression. [20%]

(d) Hence, derive an expression for the heat flux to the moving bearing pad. Explain the effect of the streamwise pressure gradient. [20%]

(e) For the special case with ($\mathrm{d}p/\mathrm{d}x$)=0 derive an expression for the heat transfer coefficient and comment on the physical significance of the result. [20%]

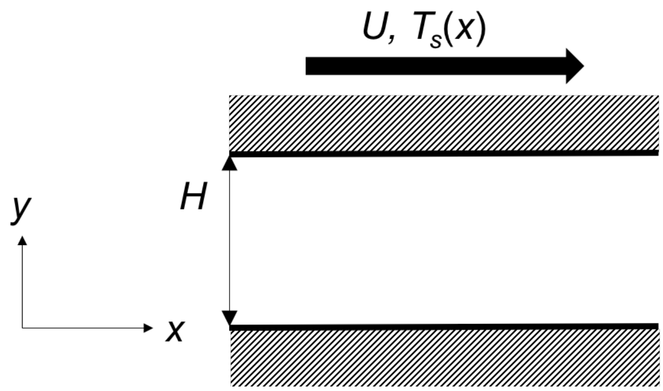

Fig.1

2 The air next to a vertical, heated, flat plate is driven by natural convection. The characteristic scale of the plate is H and of the induced velocity U. With the plate in the x-direction and y the plate-normal distance, the two-dimensional mass continuity, momentum and energy equations for steady, incompressible, laminar flow are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+g \qquad u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\nabla^2 T$$

(a) Sketch the plate, boundary layer edge and velocity profile of the induced flow. With the coefficient of expansion of the air defined as $\beta=-(1/\rho)(\partial\rho/\partial T)_p$ and T_∞ the free stream temperature, derive the modified form of the momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T_\infty-T)+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \quad .$$

[40%]

(b) Using classical, order-of-magnitude boundary layer analysis derive a relationship between the Nusselt number Nu and the Raleigh number Ra of the form:

$$Nu \sim Ra^n \; .$$

Assume that the thicknesses of the flow and thermal boundary layers are comparable. [40%]

(c) Using the results from Part (b) show that the induced velocity has the form

$$U \sim (H\Delta T)^{1/2}$$

where ΔT represents the wall-to-freestream temperature difference. Describe some practical situations where this may have significance. [20%]

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3 A vessel is used to collect condensate from a saturated vapour as shown in Fig.2. The outer surface of the vessel bottom is subject to convective cooling with a constant heat transfer coefficient h and ambient temperature T_∞. The side walls of the vessel are perfectly insulated resulting in one dimensional heat transfer in the x direction. The vessel is initially empty and the condensing vapour results in a time-dependent increase in the liquid-vapour interface given by $X[t]$.

(a) Applying conservation of energy to a differential element, derive the one-dimensional, non-steady, partial differential conduction equation applicable to the condensed liquid in terms of temperature T, thermal conductivity λ, specific heat capacity c_p, density ρ, time t and spatial coordinate x. You may neglect any convection within the condensed fluid. [15%]

(b) Using an energy balance at the vapour-condensate interface, derive the following equation:

$$-\lambda_v \frac{\partial T_v(X[t])}{\partial x} = -\lambda_l \frac{\partial T_l}{\partial x} + \rho_l l_C \frac{\partial X[t]}{\partial t}$$

where l_C is the latent heat of condensation and the vapour temperature T_v is greater than the liquid temperature T_l away from the interface. [20%]

(c) Provide three equations for the boundary conditions. You may assume that the vapour is at constant saturation temperature T_S and has a constant latent heat of condensation l_C. The initial condition is given as $T(x,0) = T_S$. [15%]

(d) It is often reasonable to assume that the time rate of change of internal energy of the liquid condensate is negligible. In this case the energy equation can be reduced to $d^2T/dx^2 = 0$. Assuming the vapour temperature is constant in time and space at T_s, determine an expression for $T_l(x)$ as a function of T_S, T_∞, λ, h and $X[t]$. [25%]

(e) Determine a relationship for $X[t]$ and sketch the form as λ_l becomes large.
Hint: the solution to an ODE of the form $A\,x\,x' + B\,x + C = 0$ is given by:
$x[t] = (-B \pm \sqrt{(B^2 + 2A(D - Ct))})/A$ where D is a constant.

[25%]

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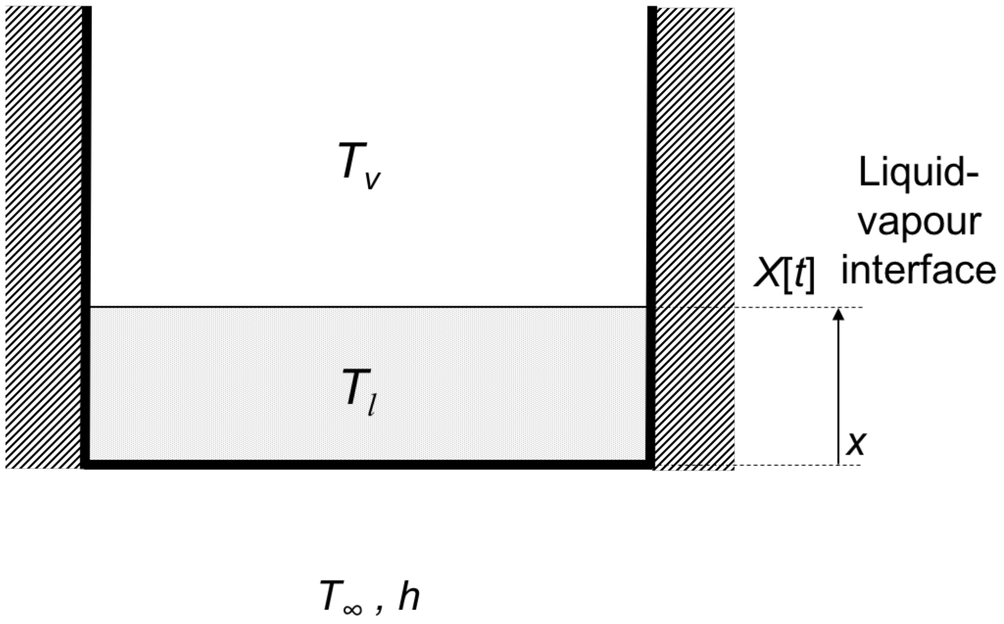

. **Fig.2**

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4 A long diffuse, grey, thin-walled tube of diameter D = 2 m is contained inside a long black duct of square cross section as shown in Fig. 3. The top of the tube is open with semi angle θ = 30° forming an inner wall that has the same area $A_1 = A_2$ and temperature $T_1 = T_2$ at the outer wall of the tube. The tube has an inner and outer emissivity of ϵ_1 = 0.1 and ϵ_2 = 0.2, respectively.

(a) Sketch the appropriate heat transfer equivalent circuit, and write expressions for each of the network resistances. [15%]

(b) Calculate the appropriate view factors for each surface with the others and self-view factors if appropriate. [15%]

(c) Calculate $\dot{q}_1$, the heat transfer per unit length from the inner tube surface, and $\dot{q}_3$, the total heat transfer per unit length to the duct surface. [25%]

(d) The effective emissivity $\varepsilon_{\mathrm{eff}}$ of the opening is defined as the ratio of $\dot{q}_1$, the radiant heat flow leaving the opening, to that of a blackbody having the area of the opening A_4 and a temperature of the inner surface of the cavity T_1. Determine $\varepsilon_{\mathrm{eff}}$. [20%]

(e) Sketch the heat loss $\dot{q}_1$ and effective emissivity as a function of the angle as θ varies from 0° to 180°. [25%]

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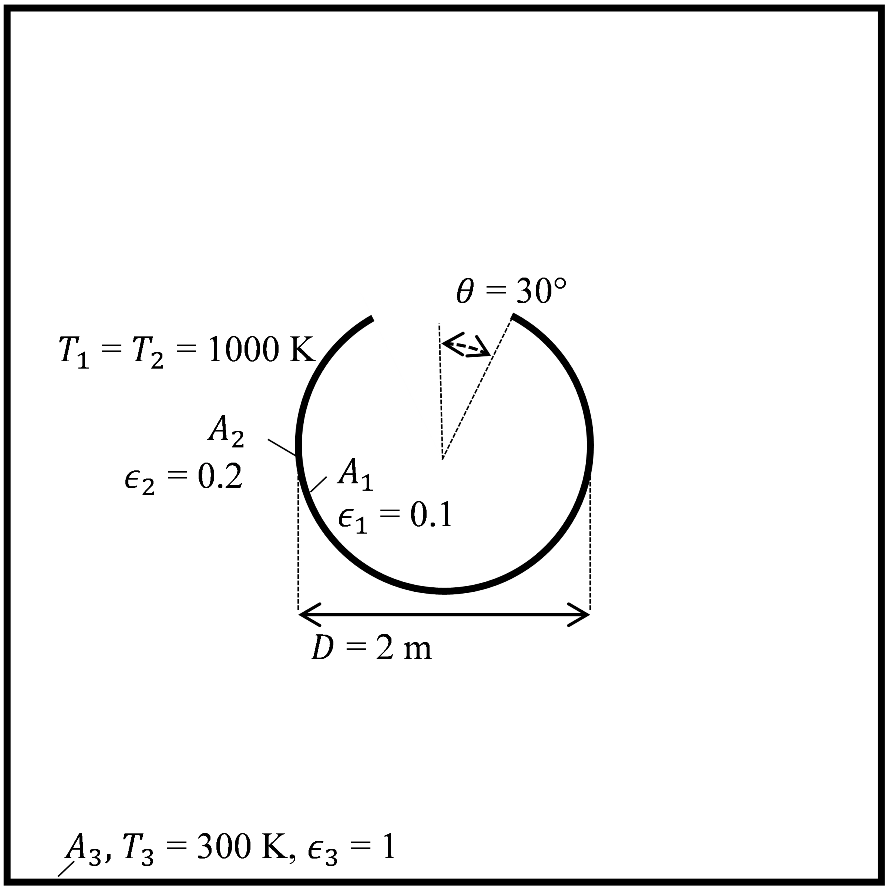

Fig.3

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