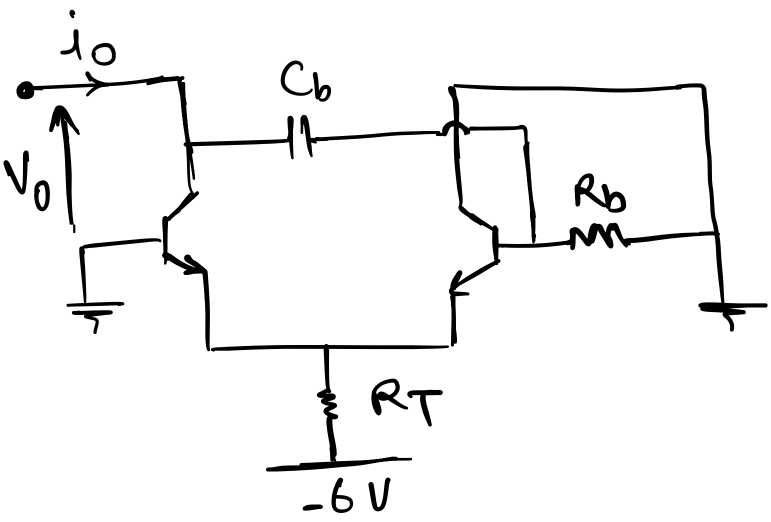
3B1

Q.1)

a)

i)



Assume R_b is large

$$Z_o = R_b \| -2r_e \simeq -2r_e$$

$$r_e = \frac{0.025}{I_C}$$

$$f = \frac{1}{2\pi\sqrt{LC}} = 445\ MHz \Rightarrow C = \frac{1}{(445\times10^6)^2 \cdot (2\pi)^2 \cdot 10\times10^{-9}}$$

$$= 12.8\ pF$$

$$Q = 20 \Rightarrow R_{loss} = 2\pi f \cdot L \cdot Q$$

$$= 560\ \Omega$$

$$\Rightarrow -2r_e \ll R_{loss} \| \underset{75\Omega}{R_L} = 66\ \Omega$$

to enable oscillation $\overset{\text{say}}{\Rightarrow}$ $-2r_e = -50 \Rightarrow r_e = 25\ \Omega$

$$\Rightarrow I_C = 1\ mA$$

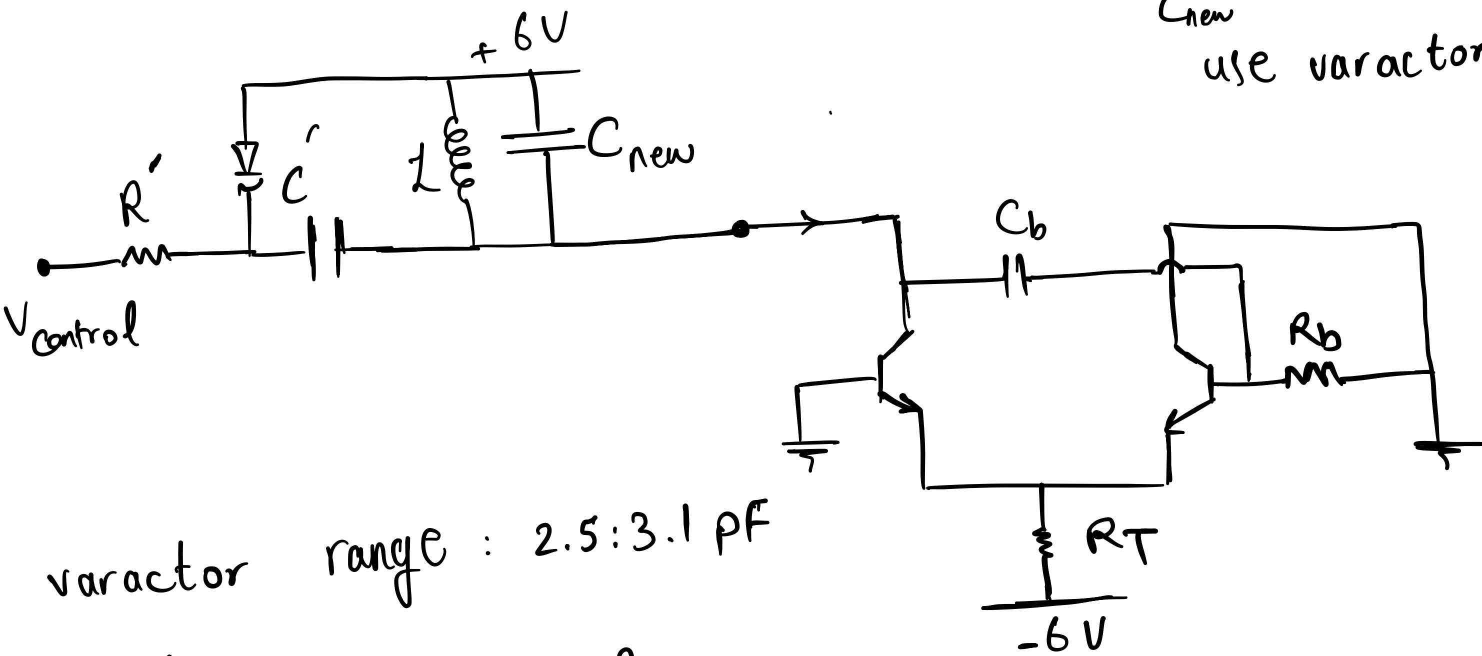
$$\Rightarrow R_T = \frac{0 - 0.65 + 6}{2I_C} = 2.7\ k\Omega$$

$R_b = 10k\Omega$ (to be large enough)

ii) need a variable capacitor in the range 13.1 : 12.5 pF

$\Rightarrow$ Chang C to $\underset{C_{new}}{10\,pF}$ + (2.5 : 3.1) pF

use varactor



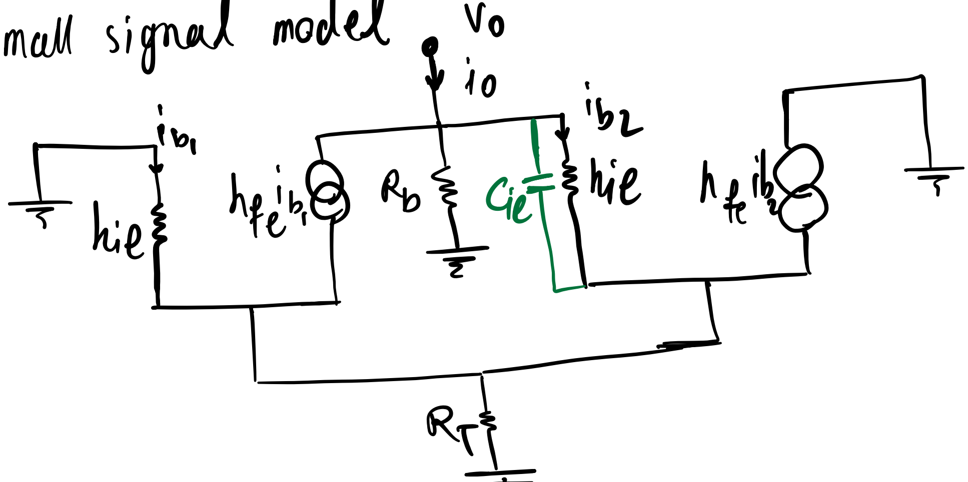
varactor range : 2.5 : 3.1 pF

R' : large say 1 MΩ

C' : dc blocking say 1nF

iii)

small signal model



We need to find when $i_b \rightarrow \frac{1}{\sqrt{2}} i_b$

Looking into i_o and "no gain" transistor (no Miller effect)

we see that C_{ie} is the main factor with hie

(note that the referred value of R_T to the emitter will be much larger than hie)

$\Rightarrow h_{ie} = h_{fe} r_e$

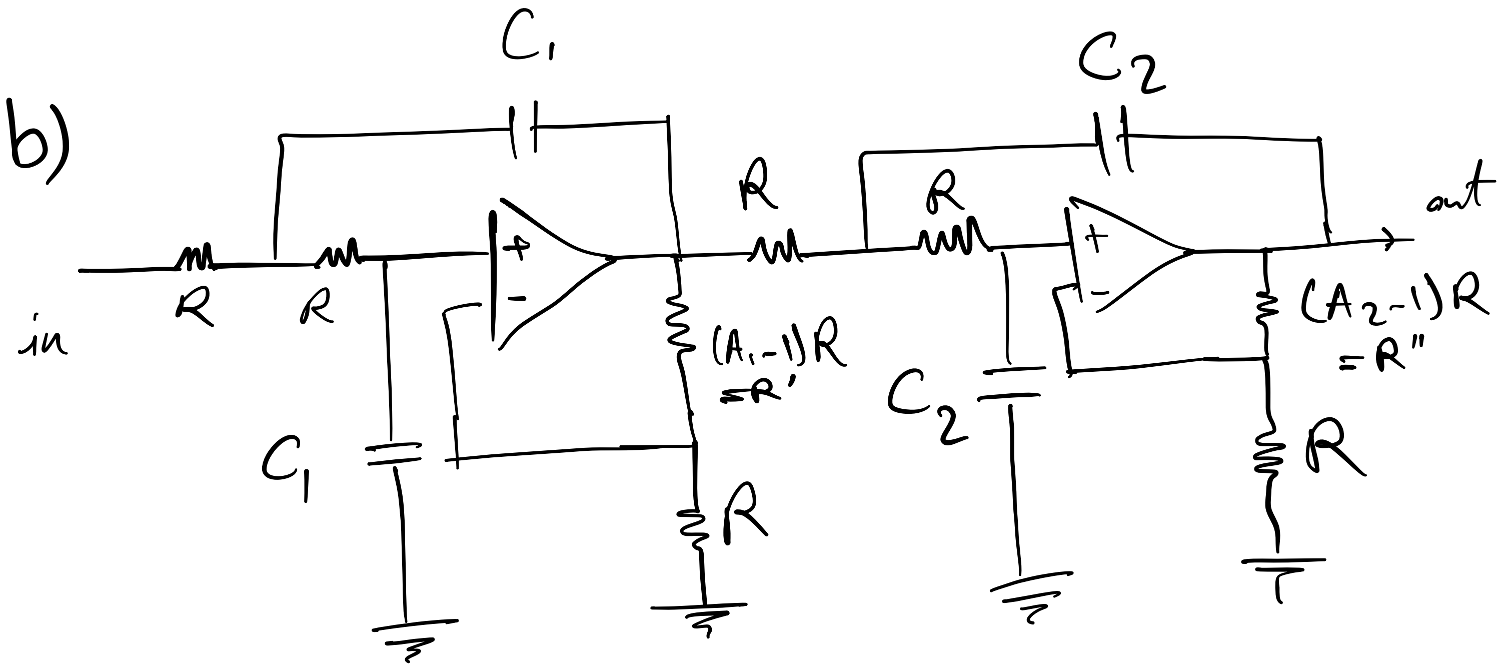
$$f_T = \frac{1}{2\pi C_{ie} r_e}$$

$\Rightarrow$ cut-off frequency $\simeq \dfrac{1}{2\pi C_{ie} h_{ie}} = \dfrac{1}{2\pi C_{ie} r_e h_{fe}}$

$$= \frac{1}{h_{fe}} f_T$$

$\Rightarrow$ for our design : 3dB cut-off is 500 MHZ which is ok and the designed oscillator will marginaly work

b)



$R = 100\,\Omega$

$R' = (A_1 - 1)R = (1.582 - 1) \cdot 100 = 58.2\ \Omega$

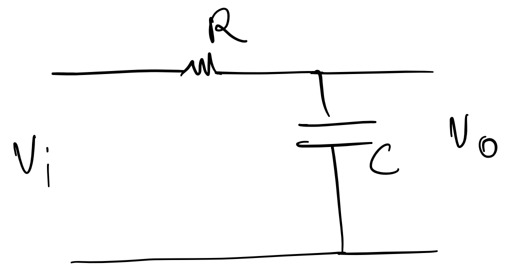
for C_1 : $f_c = \dfrac{1}{2\pi R C_1 f_n} \Rightarrow C_1 = 5.8\ pF$

$f_n \swarrow 0.597$

$R'' = (A_2 - 1) R = (2.660 - 1) \cdot 100 = 166\ \Omega$

$C_2 = \dfrac{1}{2\pi R f_c f_{n_2}} = 3.36\ pF$

Similarly a highpass filter can be designed with a chosen cut-off freq < 430 using the additional two op-amps



low pass RC

$f_c = \dfrac{1}{2\pi RC}$

choose $R = 100\ \Omega$ $\Rightarrow C = 3.5\ pF$

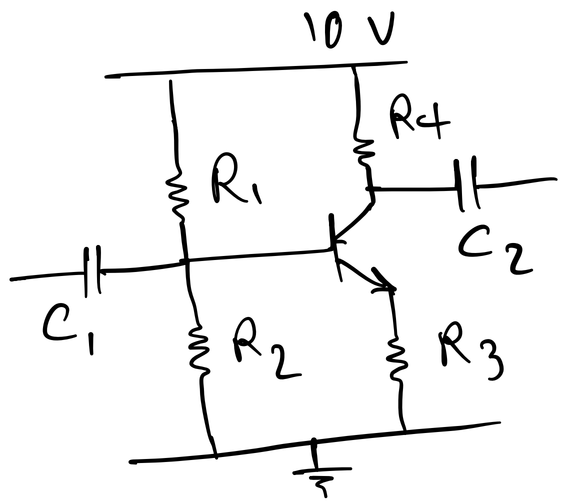
(can be other values too)

$\boxed{\text{Both can be chosen if properly justified}}$

	VCVS	RC
pros	- steeper roll off compared to RC so better in removing high-freq noise - generally better filter parameters	simple, passive components
cons	- works as long as the op-amp is operational, which is challenging in 450 MHz. - overshoot in time domain	- performance depends on the source/load - weak rejection of high-freq noise

Q.2)

i)



20 dB gain $\rightarrow$ x100

output power : 100 mW

$\Rightarrow V_{rms}^2 = 50\,\Omega \times 100\,mW$

$V_{rms}^{out} = 2.24\,V$

$\Rightarrow V_{pp}^{out} = 6.3\,V$

ok for supply

input power = 1 mW $\Rightarrow V_{rms}^{in} = 0.22\,V \Rightarrow V_{pp}^{in} = 0.63\,V$

voltage gain: $\frac{1}{0.5} \times \frac{6.3}{0.63} = 20$

$R_4 = 50\,\Omega$ to match output

$V_C = \frac{V_S}{2} = 5\,V$

$20 = \frac{50}{R_3 + r_e} \Rightarrow R_3 = 2.25\,\Omega$

$V_C = 5\,V \quad R_4 = 50\,\Omega \Rightarrow I_C = 0.1\,A$

$\Rightarrow r_e = \frac{25}{100} = 0.25\,\Omega$

$V_B = (0.65 + 0.1 \times 2.25)(\times \text{margin}) \simeq 1\,V$

$\Rightarrow 10 \dfrac{R_2}{R_1+R_2} = 1$ ✓ (including 20% margin)

$R_{in} = R_1 \parallel R_2 \parallel h_{fe} R_3 \approx 50\,\Omega$

$\boxed{\text{if larger } R_2 \text{ was chosen, } R_1 \,\&\, R_2 \text{ may needed reduction to satisfy } R_{in}}$

$h_{fe} R_3 = 250 \times 1 = 250\,\Omega$

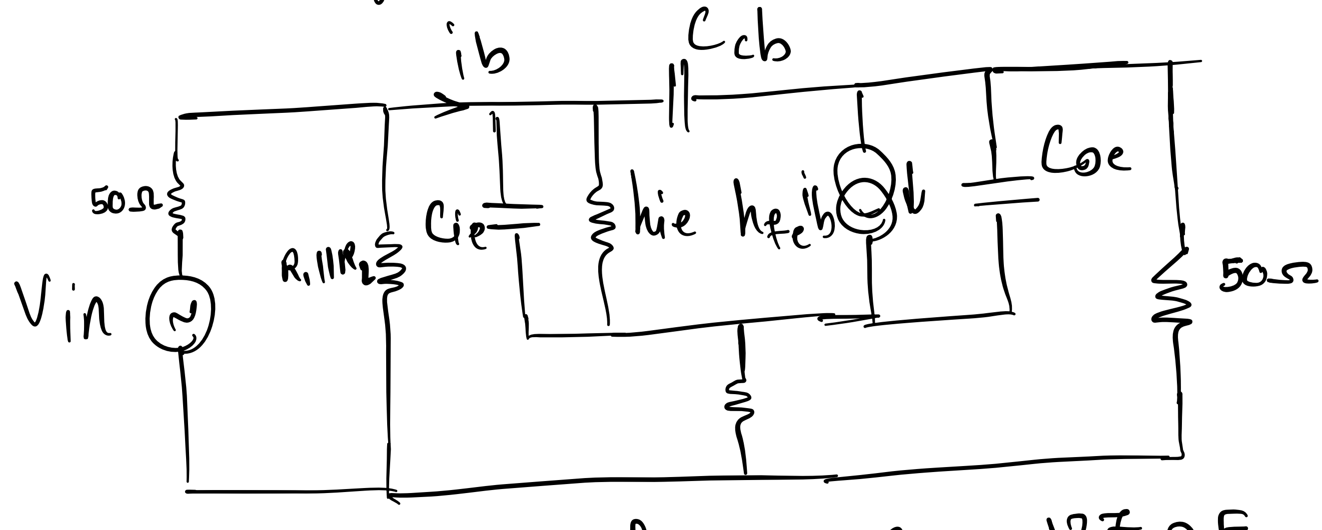
$R_2 = 1.5 R_{in} = 75\,\Omega$ $\Rightarrow$ check $R_{in} \approx 53\,\Omega$ OK

$\Rightarrow R_1 \approx 675\,\Omega$

choose large $C_1, C_2 \rightarrow$ say $10\,nF$

ii)

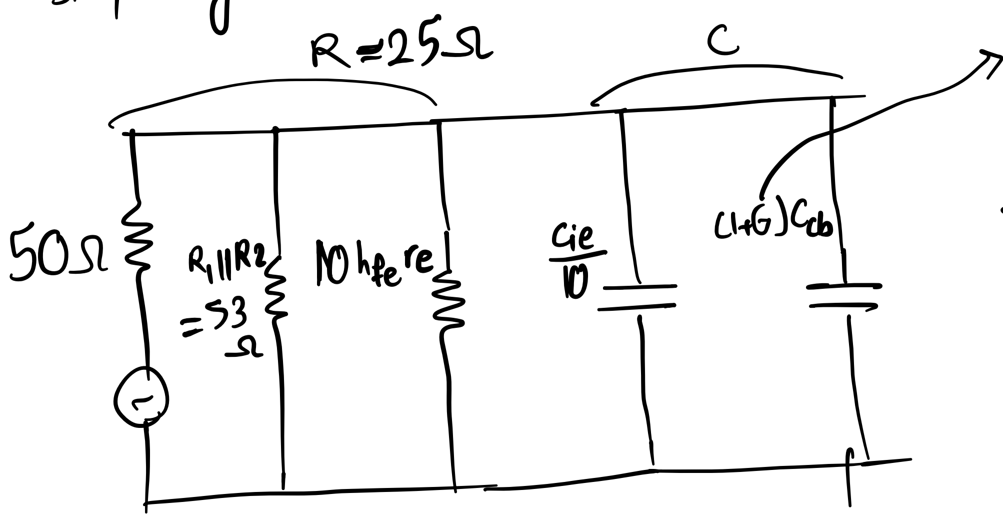
small signal model



$f_T = \dfrac{1}{2\pi\, C_{ie} r_e}$ $\left\{ \begin{array}{l} T_1: \; C_{ie} = 127\,pF \\ T_2: \; C_{ie} = 64\,pF \end{array} \right.$

emitter gain $\dfrac{v_e}{v_i} = \dfrac{R_3}{R_3 + r_e} = \dfrac{2.25}{2.25 + 0.25} = 0.9 \Rightarrow$ refer base-emitter component to GND $\times \dfrac{1}{1-0.9} = \times 10$

simplify for the input side



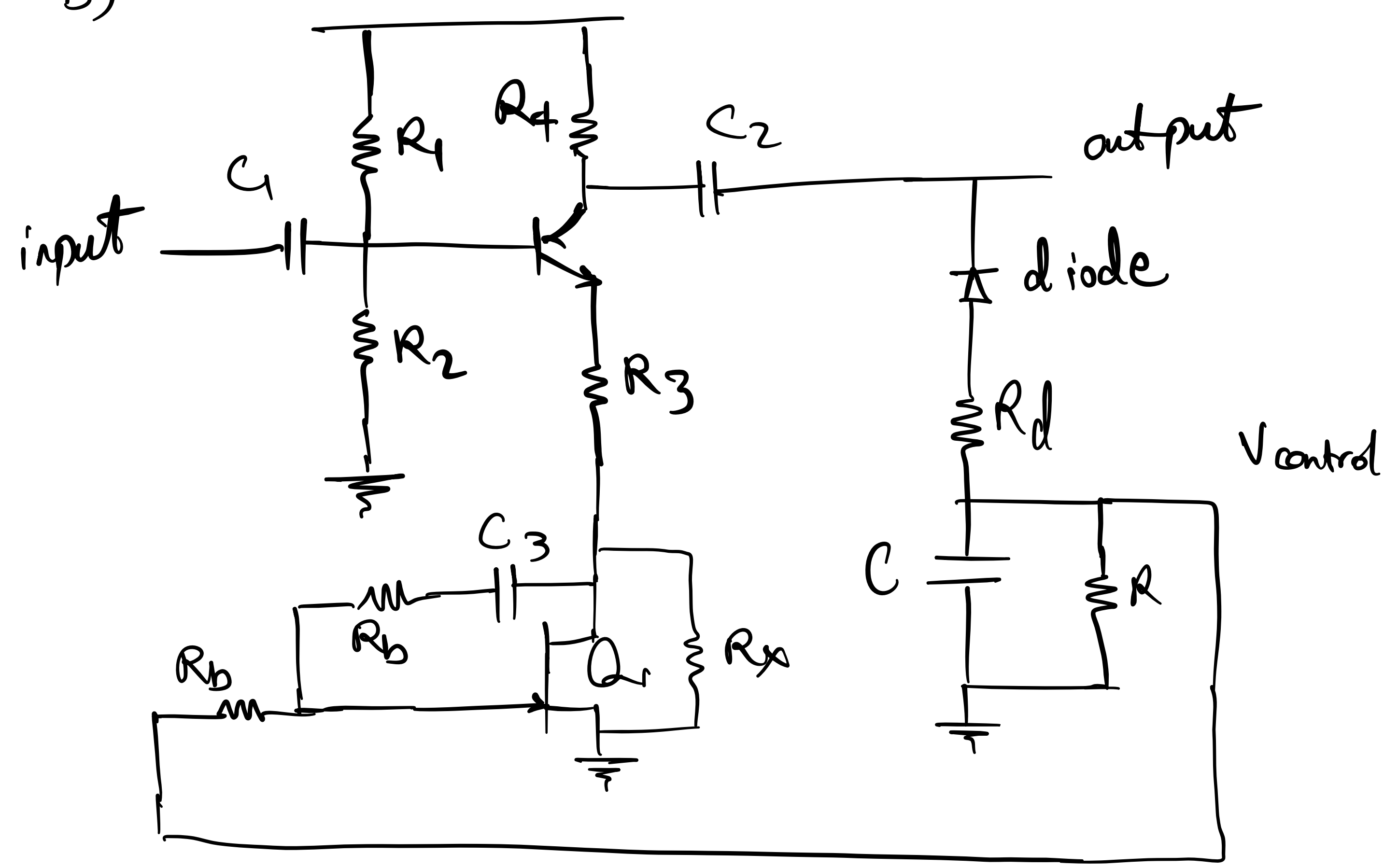
loaded gain =10

$$f_{3dB} = \frac{1}{2\pi RC}$$

$$\Rightarrow f_{3dB} = \begin{cases} T_1 : \dfrac{\ }{2\pi \times 25 \times (12.7+3.3)\times 10^{-12}} > 200\ \text{MHz} \\ T_2 : \dfrac{1}{2\pi \times 25 \times (6.4+2.2)\times 10^{-12}} > 200\ \text{MHz} \end{cases}$$

based on this **both** will be suitable.

b)



* The MOSFET works as a voltage controled variable resistor.
* R_x sets the minimum gain

When Q_1 is fully on $\rightarrow$ low resistance ($V_{control}$ large)

$\Rightarrow$ gain as before - assuming R_{DS} is $\ll R_3$

when Q_1 is off R_x limits the gain

$$\Rightarrow \text{gain} = \frac{R_4}{R_3 + R_x + r_e} \Rightarrow 50\% \text{ reduction in gain } R_x = R_3 + \underbrace{r_e}_{\text{small}}$$

$\simeq 5k\Omega$

choose R_b and C large $\xrightarrow{\text{say}}$ $R_b = 1M\Omega$, $C = 100\ nF$

* C: blocks d.c.
* R_b: Set the gate bias
  (we add another one to mitigate nonlinearity)

* diode : to rectify the output signal to generate $V_{control}$

  RC : to make the rectified signal into a smoother one (lowpass)

  $R_d \rightarrow$ to limit the charge rate (sensitivity)
  to also limit the diode current

Solutions Question 3

Assuming free-space propagation conditions and that the antenna is aligned for maximum gain throughout

- f =9.4 ×10^9 Hz, therefore,

$$\lambda = \frac{3 \times 10^8}{9.4 \times 10^9} = 0.0319 \text{ m}$$

- $P_t = 10 \times 10^3$ W
- $G = 34$ dB $= 10^{3.4} = 2512$

Question 3(a)

Gain (G): The ratio of the radiation intensity in a given direction to the radiation intensity that would be obtained if the power supplied to the antenna were radiated isotropically. It accounts for ohmic losses in the antenna structure.

Directivity (D): The ratio of the radiation intensity in a given direction (usually the maximum) to the radiation intensity averaged over all directions. It depends solely on the shape of the radiation pattern.

Effective Aperture (A_e): A measure of the effective area of the antenna for intercepting incident electromagnetic waves. It relates the received power to the incident power density ($P_r = S \times A_e$).

Gain vs Directivity: They are related by the radiation efficiency (e), such that $G = e\, D$. If the antenna is 100% efficient, Gain equals Directivity.

Gain vs Effective Aperture: They are directly proportional via the wavelength (λ). The relationship is

$$G = \frac{4\pi A_e}{\lambda^2}$$

Question 3b

$$A_e = \frac{G\lambda^2}{4\pi} = 0.204 \text{ m}^2$$

Physical area of dish, A, given efficiency of the aperture is 60%:

$$A = \frac{A_e}{0.6} = 0.339 \text{ m}^2$$

Find diameter d, knowing that the cross-section is a circle, $A = \pi\left(\frac{d}{2}\right)^2$

$$d = 0.66\ \mathrm{m}$$

Question 3c

Beam angle estimation. Step 1 Calculate directivity.

$$D = \frac{G}{e} = \frac{2512}{0.75} = 3349$$

Step 2

$$\text{Directivity } D = \frac{\text{Isotropic area}}{\text{Beam area}} = \frac{4\pi r^2}{\int_0^{2\pi}\int_0^{\theta} r\sin\theta\,\mathrm{d}\theta\, r\,\mathrm{d}\phi} = \frac{4\pi r^2}{2\pi r^2(1-\cos\theta)}$$
$$= \frac{2}{(1-\cos\theta)}$$

N.B. The beam area is estimated as a surface integral over a small portion of the sphere, limited within the beam angle 2θ.

$$\cos\theta = 1 - 2/D$$

$$\theta = 0.035\ \mathrm{rad} = 1.98°$$

And the beam angle is $2\theta = 3.96°$

Question 3d

Power density at 12 km

$$r = 12 \times 10^3\ \mathrm{m}$$

Power density $S = \frac{P_t\,G}{4\pi r^2} = \frac{10\times10^3 \times 2512}{4\pi(12\times10^3)^2} = 0.01388\ \mathrm{W/m^2}$
$= 13\,.88\ \mathrm{mW/m^2}$

Electric field $E = \sqrt{2\eta_0 S} = \sqrt{2 \times 377 \times 0.01388} = 3.23\ \mathrm{V/m}$
Magnetic field $H = \frac{E}{\eta_0} = \frac{3.23}{377} = 0.00858\ \mathrm{A/m}$
=8.58 mA/m

Question 3e
Step 1. Assume antenna oriented towards vessel for maximum gain. Received power density at the vessel, as calculated earlier: $S = 0.01388\ \mathrm{W/m^2}$.

Step 2. Total power reflected from the vessel = $P_{ref} = S \times 50 = 0.694$ W.
Step 3. This reflected power spreads out isotropically (assumption) as it travels back to the dish, so Power density at dish $S_{dish} = \frac{P_{ref}}{4\pi r^2} = 3.84 \times 10^{-10}\ \mathrm{W/m^2}$
Step 4: Antenna captures this power depending on its effective aperture

$$A_e = \frac{G\lambda^2}{4\pi} = 0.204\ \mathrm{m^2}$$

$$P_r = S_{dish}A_e = 7.81 \times 10^{-11}\ \mathrm{W} = 71\ \mathrm{dBm}$$

Master equation would be:

$$\frac{S \times 50}{4\pi r^2}\frac{G\lambda^2}{4\pi}$$

<u>Question 3f</u>

As a dipole antenna, length is $\frac{\lambda}{2} = \frac{1}{2}\frac{c}{f} = \frac{1}{2}\frac{3\times10^8}{150\times10^6} = 1$ m
Taking the electrical resistivity of copper to be 1.72 $\times10^{-8}$ Ω/m, as in the databook, skin depth is:

$$\delta = \sqrt{\frac{\rho}{\pi f \mu}} = \sqrt{\frac{1.72 \times 10^{-8}}{\pi \times 150 \times 10^6 \times 4\pi \times 10^{-7}}} = 5.39 \times 10^{-6}\mathrm{m} = 5.39\ \mu\mathrm{m}$$

Resistance is approximately $\rho\frac{L}{2\pi a\delta} = 1.72 \times 10^{-8} \times \frac{1}{2\pi\times0.5\times10^{-3}\times5.39\times10^{-6}} = 1.01\ \Omega$

Radiation efficiency $= \frac{73}{73+1.01} = 98.6\%$

Solutions Question 4

<u>Question 4a</u>

Capacitance per unit length is approximately (as given in lectures):

$$C = \frac{w + 2d}{d}\varepsilon_r\varepsilon_0$$

Also note

$$v = \sqrt{\frac{1}{LC}} = \frac{c}{\sqrt{\varepsilon_r\mu_r}} = \frac{c}{\sqrt{\varepsilon_r}} = 1.59 \times 10^8\ \mathrm{m/s}$$

assuming $\mu_r = 1$ as non-magnetic

(1)

Inductance per unit length is

$$L = \frac{1}{v^2 C}$$

(2)

Combining with (1) and (2) with $Z_0 = \sqrt{\frac{L}{C}}$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{1}{v^2 C^2}} = \frac{1}{vC} = \frac{\sqrt{\varepsilon_r}}{c}\frac{1}{\varepsilon_r \varepsilon_0}\frac{d}{w+2d} = \frac{1}{c\varepsilon_0\sqrt{\varepsilon_r}}\frac{d}{w+2d} = 50$$

Solving for w gives

$$w = \frac{d}{Z_0 c\varepsilon_0\sqrt{\varepsilon_r}} - 2d = \frac{1}{50 \times c \times \varepsilon_0 \times \sqrt{3.55}} - 2 = 2\text{ mm}$$

Question 4b

$$Z_L = 75 + \text{j}20\ \Omega$$

$$Z_0 = 50\ \Omega$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 + \text{j}20 - 50}{75 + \text{j}20 + 50} = \frac{25 + \text{j}20}{125 + \text{j}20} = 0.2200 + \text{j}0.1248 = 0.259\angle 29.6°$$

RL = 20 $\log_{10}(0.253)$ =11.9 dB (could also read this off the Smith Chart)

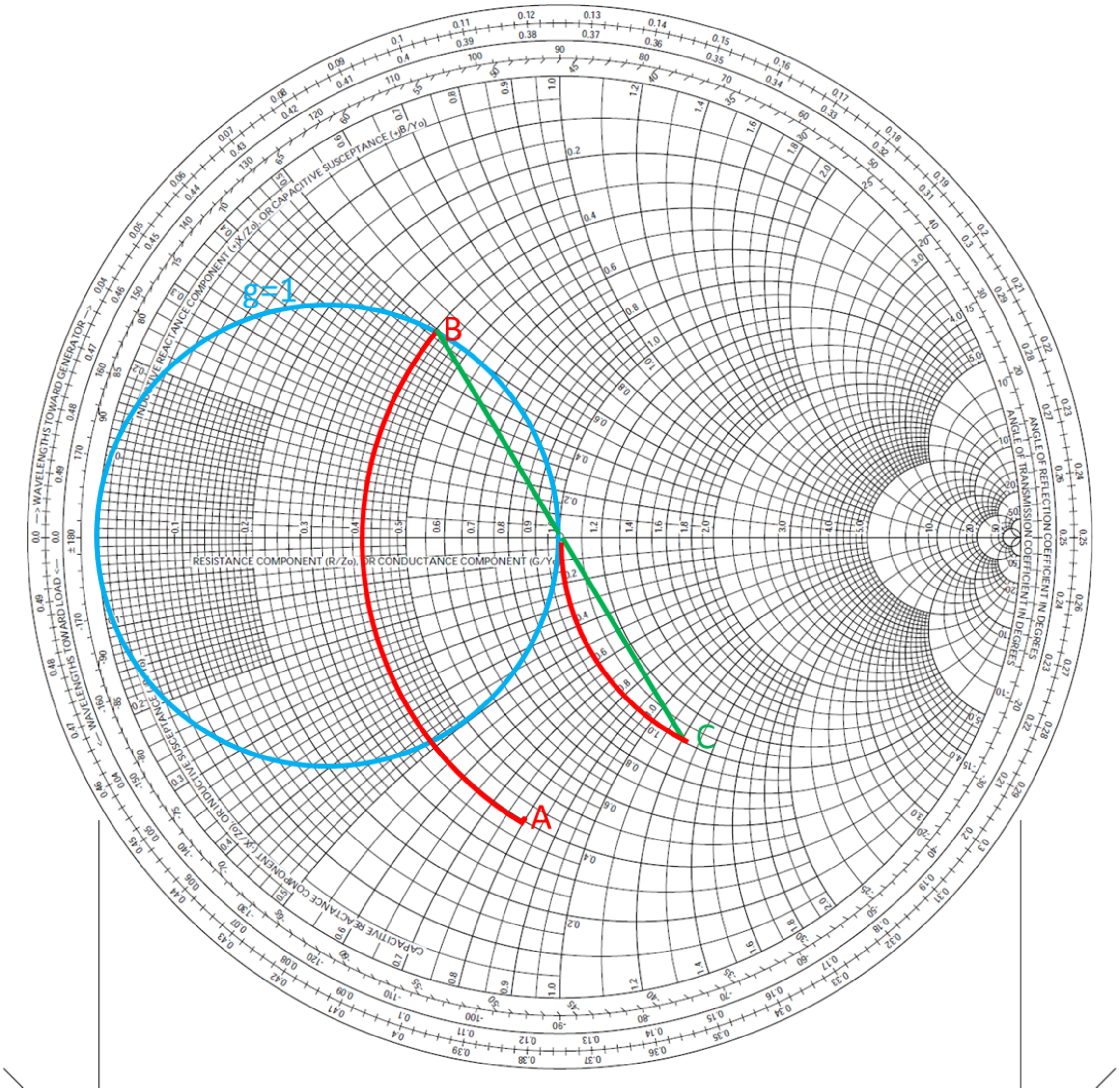
Students who used the Smith chart method, with $Z_{L,norm} = 1.5 + \text{j}0.4\ \Omega$*, obtained sufficiently accurate numerical answers and were awarded full marks.*

Question 4c

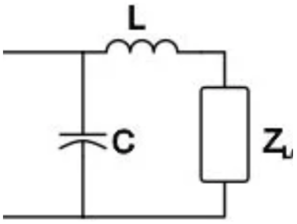
Part i

1. Normalise the impedance to 50 Ω, i.e. 0.4 – j 0.8

2. Series Inductor (L): Add series reactance to move from 0.4 – j 0.8 to the circle where g = 1. (or G = 1/50 S).
3. Shunt Capacitor (C): Add shunt susceptance to cancel the imaginary part and reach the centre.



Part ii



<u>Using the Q-factor method</u>

$$Q = \sqrt{\frac{50}{20} - 1} = 1.225$$

Required series reactance $X_{total} = Q \times R_{low} = 1.225 \times 20 = 24.5\Omega$

Inductor $X_L = 24.5 - (-40) = 64.5\Omega$ therefore $L = \frac{64.5}{2\pi f} = 4.19$ nH

Required shunt susceptance $B = \frac{Q}{R_{high}} = \frac{1.225}{50} = 0.0245$ S therefore $X_c = 40.8\Omega$ therefore $C = \frac{0.0245}{2\pi f} = 1.59$ pF

<u>Using Smith chart method:</u>

1. Normalise the impedance to 50 Ω, i.e. 0.4 – j 0.8
2. Series Inductor (L): Add series reactance to move from 0.4 – j 0.8 to the circle where g = 1. (or G = 1/50 S). This requires an additional j0.8 + j0.5 of series reactance.

So, a normalised series reactance of j0.8 + j0.5 = j1.3 must be added.

In terms of actual reactance, it is 50 × 1.3j = 65j Ω

In terms of inductance, it is 65 /ω = 4.2 nH

3. Shunt Capacitor (C): Add shunt susceptance to cancel the imaginary part and reach the centre. This requires an additional 1.25j susceptance.

So, a normalised shunt susceptance of 1.22j must be added. This corresponds to 1/1.22 = -0.82j shunt reactance.

In terms of actual reactance, -0.82j corresponds to 41j Ω reactance.

In terms of actual capacitance, it is 1/(41 ω)=1.6 pF.

Question 4d

Match 12.5 Ω to 50 Ω.

(i)

$$Z_T = \sqrt{50 \times 12.5} = 25\ \Omega$$

(ii) $l = \frac{\lambda}{4} = \frac{c}{4f\sqrt{\varepsilon_r}} = \frac{3\times10^8}{4\times2.45\times10^9\sqrt{3.55}} = 16.2\text{ mm}$

Question 4e

A patch antenna will normally radiate linearly polarised radiation. It is normally designed to be of dimension $\lambda/2$ where the wavelength is that in the PCB material. A notch allows reflection of propagating waves within the patch at a 90° angle, which creates the emission of radiation of the opposite polarisation. The phase difference between the two propagating waves must also be 90° for circularly polarised radiation to be emitted. The coupling between the two modes must be equal, otherwise the radiation will be elliptical.