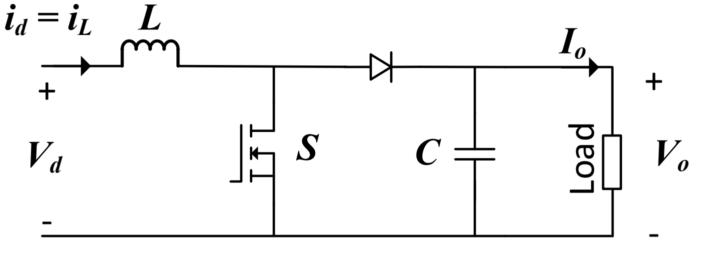
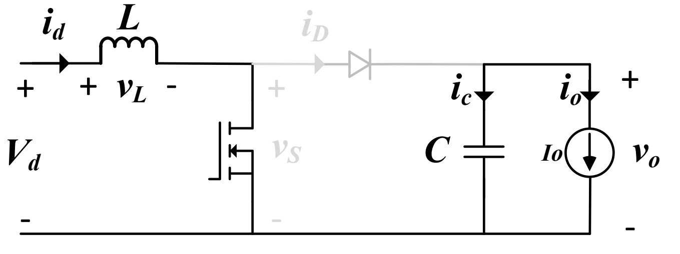
3B3, 2026 Lent

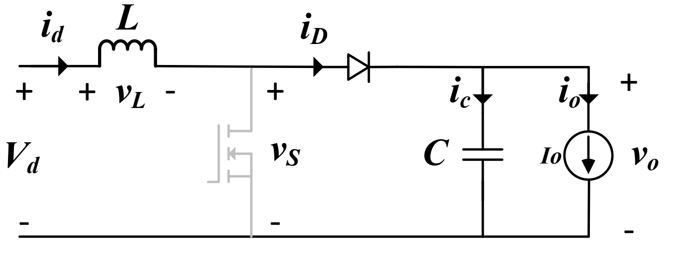
1(a)



S: ON for DT_s



S: OFF for (1-D)T_s



One transistor and complementary diode (or transistor) => two states if transistor and diode states are always inverted (continuous conduction mode), can have three states for discontinuous conduction mode

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1(a)

One transistor and complementary diode (or transistor) => two states if transistor and diode states are always inverted (continuous conduction mode), can have three states for discontinuous conduction mode

Duty cycle $D = T_{\text{on}}/T = T_{\text{on}} f$

steady-state assumption (energy in the coil at the end of a cycle reaches the level at the beginning):

$$\int_0^T v_L = \int_0^{T_{on}} v_L \, \mathrm{d}t + \int_{T_{on}}^{T} v_L \, \mathrm{d}t = 0$$

$$V_{\text{in}} T_{\text{on}} + (V_{\text{in}} - V_{\text{out}}) T_{\text{off}} = 0$$

$$M(D) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{T}{T_{\text{off}}} = \frac{1}{1 - D}$$

1(b)

Boundary between continuous and discontinuous conduction mode: The peak-value (half of the peak-to-peak value) of the ripple current at the inductor L is Δi_L:

$$\Delta i_L = \frac{1}{2} \Delta i_{\text{Lpp}} = \frac{t_{\text{on}}}{2L} (V_{\text{in}} - V_{\text{out}}) =$$

$$= \frac{D T_s}{2L} (V_{\text{in}} - V_{\text{out}}) = \frac{D(1 - D) T_s V_{\text{in}}}{2L}$$

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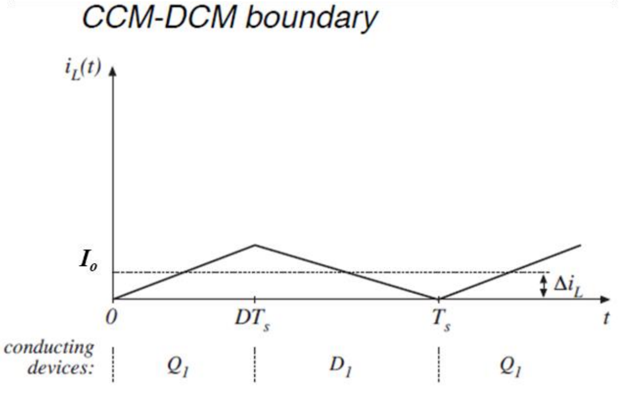
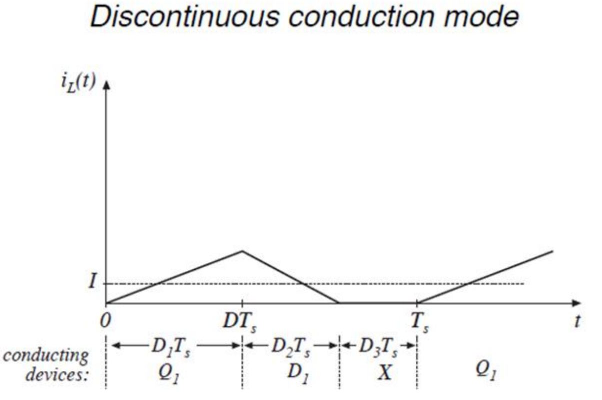
1(b)





$I_o < \Delta i_L$, for DCM

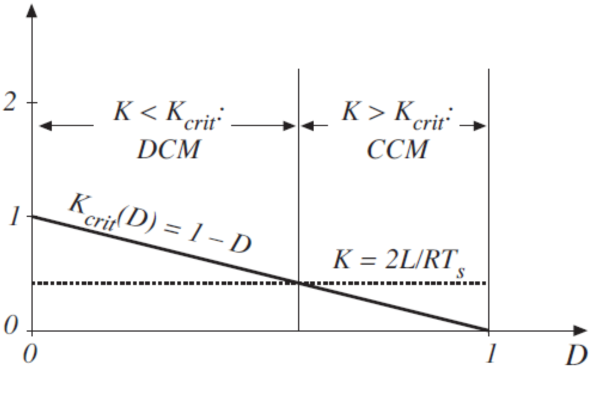
$$\frac{DV_{\text{in}}}{R} < \frac{D(1-D)V_{\text{in}}T_s}{2L}$$

$$\frac{2L}{RT_s} < (1-D)$$

$$K_{\text{crit}}(D) = 1 - D$$

$$K = \frac{2L}{RT_s}$$

for $K < 1$:



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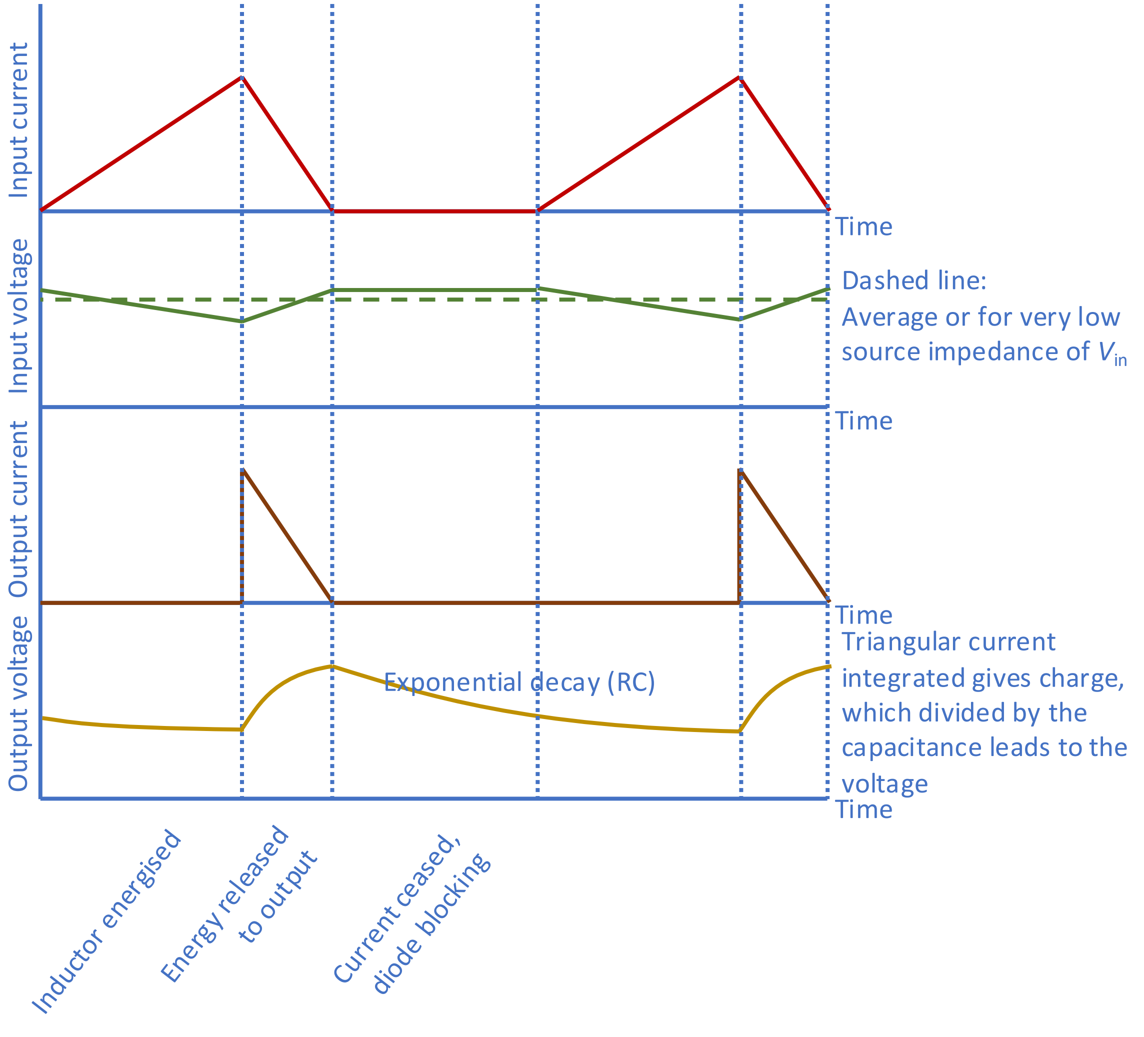
1(b)



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1(c)

Charge provided by the inductor in a cycle is distributed between the capacitor and the load (R). In steady state, the net capacitor charge over a cycle is 0 so that only the load matters:

$$\int_0^{T_S} i_L(t)\,\mathrm{d}t = \int_0^{T_S} i_c\,\mathrm{d}t + \int_0^{T_S} \frac{V_o}{R}\,\mathrm{d}t = \int_0^{T_S} \frac{V_o}{R}\,\mathrm{d}t = T_S\frac{V_o}{R}$$

For the discontinuous conduction mode, there is the interval during which the transistor is on (D), the one during which the diode conducts (D_2) and the remainder during which no current flows:

$$T_s\frac{V_o}{R} = \frac{1}{2}\left(\frac{V_{\text{in}} - V_{\text{out}}}{L}\right) DT_s(D + D_2)T_s$$

$$T_s\frac{V_o}{R} = \frac{1}{2}\left(\frac{V_{\text{in}} - V_{\text{out}}}{L}\right) DT_s(D + D_2)T_s$$

$$V_{\text{out}} = V_{\text{in}}\frac{D_1}{D_1 + D_2}$$

$$M(D, K) = \frac{2}{1 + \sqrt{1 + \frac{4K}{D^2}}} \qquad K = \frac{2L}{RT_s}$$

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1(d)

$$\Delta i_{\mathrm{Lpp}} = \frac{V_{\mathrm{d}} t_{\mathrm{on}}}{L} = \frac{-(V_{\mathrm{in}} - V_{\mathrm{out}}) t_{\mathrm{off}}}{L}$$

$$T_s = t_{\mathrm{on}} + t_{\mathrm{off}} = \frac{\Delta i_{\mathrm{Lpp}} L}{V_{\mathrm{d}}} + \frac{\Delta i_{\mathrm{Lpp}} L}{V_{\mathrm{out}} - V_{\mathrm{in}}} = \frac{\Delta i_{\mathrm{Lpp}} L V_{\mathrm{out}}}{V_{\mathrm{d}}(V_{out} - V_{\mathrm{in}})}$$

$$\Delta i_{\mathrm{Lpp}} = \frac{V_{\mathrm{in}} D}{f_{\mathrm{sw}} L}$$

$$f_{\mathrm{sw}} = \frac{V_{\mathrm{in}}}{L \Delta i_{\mathrm{Lpp}}} D = \frac{725\ \mathrm{V}}{1\ \mathrm{mH}\ 2\ \mathrm{A}} D = 362.5\ \mathrm{kHz}\ D$$

Boundary conduction mode => average output current is half the ripple, i.e., 1 A

$$\frac{2L}{RT_s} = (1 - D)$$

$$D = 1 - \frac{2L f_{\mathrm{sw}}}{R} = 1 - \frac{2\ \mathrm{mH}}{R} f_{\mathrm{sw}}$$

$$f_{\mathrm{sw}} = 362.5\ \mathrm{kHz} \left(1 - \frac{2\ \mathrm{mH}}{R} f_{\mathrm{sw}}\right)$$

Inductor loss for triangular current:

$$P_{\mathrm{L}}(t) =$$

$$\frac{1}{2} R_{\mathrm{L}}\, I_{\mathrm{in}}{}^2(t) = \frac{R_{\mathrm{L}}}{2} \begin{cases} \left(2\ \mathrm{A} \dfrac{t}{(DT)}\right)^2 & \text{for } t < DT \\ \left(2\ \mathrm{A} \dfrac{t - DT}{(T - DT)}\right)^2 & \text{for } DT \le t < T \end{cases}$$

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1(d)

Transistor loss only for the rising period:

$$P_{\mathrm{FET}}(t) =$$

$$= \frac{1}{2} R_{\mathrm{ds,on}}\, {I_{\mathrm{FET}}}^2(t) = \frac{R_{\mathrm{ds,on}}}{2} \begin{cases} \left(2\ \mathrm{A} \dfrac{t}{(DT)}\right)^2 & \text{for } t < DT \\ 0 & \text{for } DT \le t < T \end{cases}$$

Diode loss on the falling part of the current:

$$P_{\mathrm{D}}(t) =$$

$$V_{\mathrm{fd}}\, I_{\mathrm{D}}(t) = \begin{cases} 0 & \text{for } t < DT \\ V_{\mathrm{fd}} 2\ \mathrm{A} \dfrac{t - DT}{(T - DT)} & \text{for } DT \le t < T \end{cases}$$

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2(a)

Constant-power load:

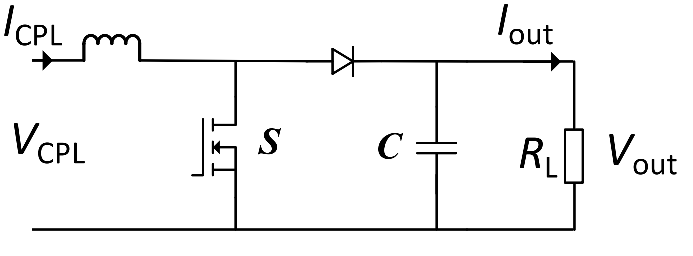
$$P_{\mathrm{CPL}} = V_{\mathrm{CPL}}\, I_{\mathrm{CPL}}$$

$$V_{\mathrm{CPL}}(I_{\mathrm{CPL}}) = \frac{P_{\mathrm{CPL}}}{I_{\mathrm{CPL}}} \qquad R_{\mathrm{CPL}}(V_{\mathrm{CPL}}) = \frac{V_{\mathrm{CPL}}}{I_{\mathrm{CPL}}} = \frac{V_{\mathrm{CPL}}^2}{P_{\mathrm{CPL}}}$$

2(b)

Implementation with a fast-switching DC-DC converter, e.g., boost or buck converter, with a resistor R_{L} as a load. In principle, any converter possible. Boost converter has the advantage of a smoother current towards the terminals of the constant-power load for the same switching rate and capacitor size.

The constant power load has to measure its input voltage and control the duty cycle to the following equation in dependence of the measured input voltage, the circuit parameter RCPL and the desired power P_{CPL}:



$$P_{\mathrm{CPL}} = I_{\mathrm{out}} V_{\mathrm{out}} = \frac{V_{\mathrm{out}}^2}{R_{\mathrm{L}}} = const$$

$$\frac{V_{\mathrm{out}}^2}{R_{\mathrm{L}}} = \frac{V_{\mathrm{in}}^2}{(1-D)^2\, R_{\mathrm{L}}}$$

$$\Rightarrow D(V_{\mathrm{CPL}}) = 1 - \sqrt{\frac{V_{\mathrm{CPL}}^2}{R_{\mathrm{CPL}} P_{\mathrm{L}}}}$$

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2(c)

Voltage at the input side:

$$V_{\mathrm{AC}}(t) = \hat{V}\sin(2\pi f t) \qquad f = 50\ \mathrm{Hz}$$

Voltage at the output side:

$$V_{\mathrm{CPL}}(t) = \mathrm{abs}(V_{\mathrm{AC}}) = \left|\hat{V}\sin(2\pi f t)\right|$$

Current at the output side:

$$I_{\mathrm{CPL}}(t) = \frac{P_{\mathrm{CPL}}}{\left|\hat{V}\sin(2\pi f t)\right|}$$

Current at the input side:

$$I_{\mathrm{AC}}(t) = \frac{P_{\mathrm{CPL}}}{\hat{V}\sin(2\pi f t)}$$

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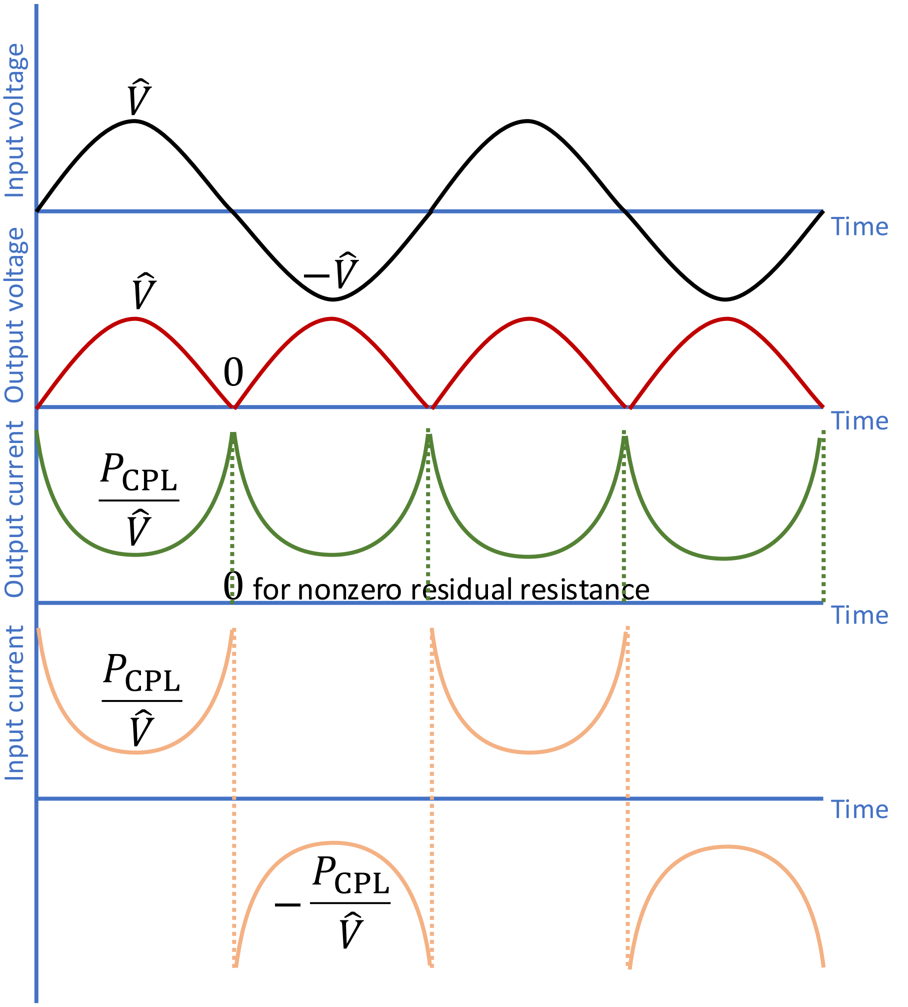
2(c)



The current is very non-sinusoidal. The largest current flows during low-voltage periods. With a non-zero residual resistance, the current at least does not spike to infinity. That is still worse than even just constant current on the DC side (and rectangular current on the AC side).

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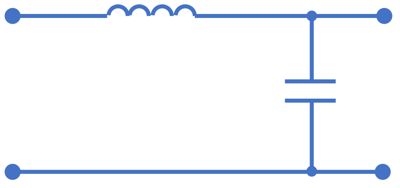
2(d)

A second-order filter needs a capacitor parallel and a series inductor. There are two options, specifically inductor first or capacitor first. To avoid the problems of a capacitor connected right to the output of the rectifier (spikes around the voltage maximum), inductor first appears more appropriate.



preferred

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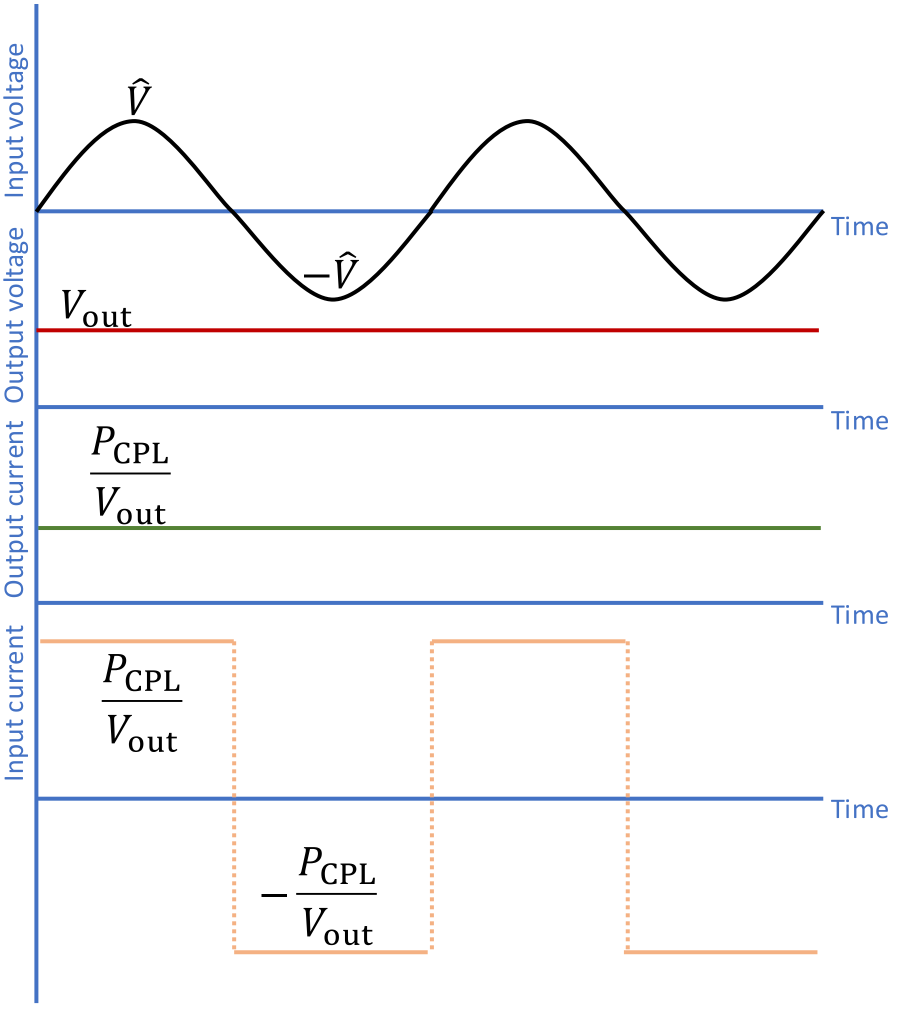
2(e)



For very large components, both output voltage and current are practically flat. If they are smaller, both will include a residual near-sinusoidal component at the mains frequency.

Q4. (i) $V_2 = \frac{N_2}{N_1}\frac{D}{(1-D)}V_1$

When switched on, $V_1 = \frac{\Delta i_s}{DT} L_m$, $\Delta i_s L_m = V_1 DT$

$\Delta\phi \cdot N_1 = \Delta i_s L_m = V_1 DT$

When switched off, $\Delta\phi \cdot N_2 = \frac{\Delta i_d}{(1-D)T}\frac{L_m}{N_1^2/N_2^2} = V_2(1-D)T$

$$\frac{V_1 DT}{N_1} = \frac{V_2(1-D)T}{N_2}, \quad \frac{V_2}{V_1} = \frac{N_2}{N_1}\frac{D}{(1-D)}$$

Continuous flux $N_1 V_1 DT = N_2 V_2 (1-D)T$

$$\frac{V_2}{V_1} = \left(\frac{D}{1-D}\right)\frac{N_2}{N_1}$$

(ii) The minimum L_m is at critical conduction of flux in the transformer core, i.e. no offset of flux. Due to lossless systems

$I_1 V_1 = I_2 V_2$, $\quad I_1 = \frac{40 \times 10}{400} = 1A$

When critical conduction of flux the input current is also in critical conduction

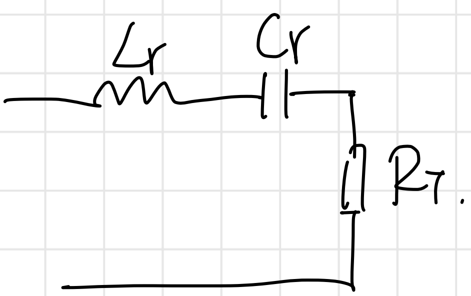
Therefore for DT, the average input current I_1 is half of the peak current in $\underline{DT}$, $I_{pk} = 4A$

$$L_m = \frac{V_1 D T}{I_{1pk}} = \frac{400 \times 0.5 \times \frac{1}{100 \times 10^3}}{4} = 500\,\mu H.$$

The minimum L_m is $500\,\mu H$.

(b) (i) $f_r = \dfrac{1}{2\pi\sqrt{L_r C_r}}$, $\omega_r = \dfrac{1}{\sqrt{L_r C_r}}$

(ii) $L_m \gg L_r$, LLC is approximately LC.



$$R_T = \left(\frac{N_2}{N_1}\right)^2 \frac{8R}{\pi^2}.$$

$$V_o = \left|\frac{R_T}{R_T + Z_r}\right| \cdot V_{ac} = \frac{1}{2}\left|\frac{R_T}{R_T + Z_r}\right| V_i$$

$$Z_r = \frac{j\omega L_r \cdot \frac{1}{j\omega C_r}}{j\omega L_r + \frac{1}{j\omega C_r}} = \frac{\frac{L_r}{C_r}}{\frac{1-\omega^2 L_r C_r}{j\omega C_r}} = \frac{j\omega L_r}{1-\omega^2 L_r C_r} = \frac{j\omega L_r}{1-\omega^2/\omega_r^2}$$

$$\left|\frac{R_T}{R_T + Z_r}\right| = \left|\frac{R_T}{R_T + \frac{j\omega L_r}{1-\left(\omega/\omega_r\right)^2}}\right|$$

$$= \left|\frac{1}{1 + \frac{j\omega L_r / R_T}{1-\left(\frac{\omega}{\omega_r}\right)^2}}\right| \quad \text{if } \frac{\omega L_r}{R_T} = Q$$

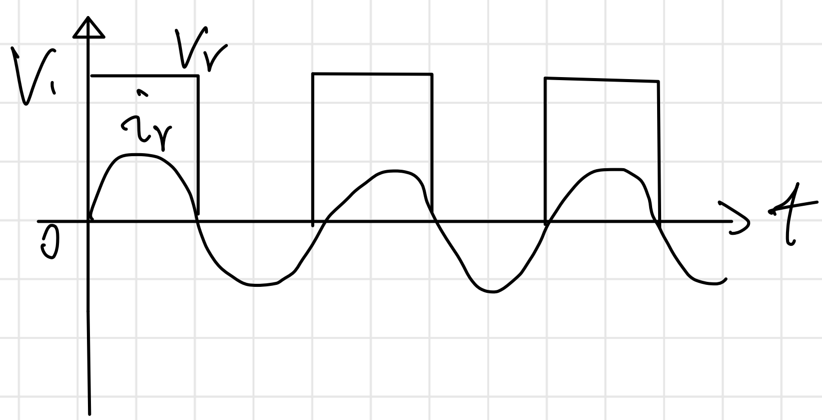
$$= \left|\frac{1}{1 + \frac{jQ}{1-\left(\frac{\omega}{\omega_r}\right)^2}}\right| = \frac{1}{\sqrt{1 + \frac{Q^2}{\frac{(1-\omega^2)^2}{\omega_r^2}}}}$$

$$\text{if } \frac{\omega}{\omega_r} = \omega_n$$

$$= \frac{1}{\sqrt{1 + Q^2\left(\frac{1}{\omega_n} - \omega_n\right)^2}} . \qquad \text{Gain} = \frac{1}{\Sigma}\frac{1}{\sqrt{1 + Q^2\left(\frac{1}{\omega_n} - \omega_n\right)^2}}$$

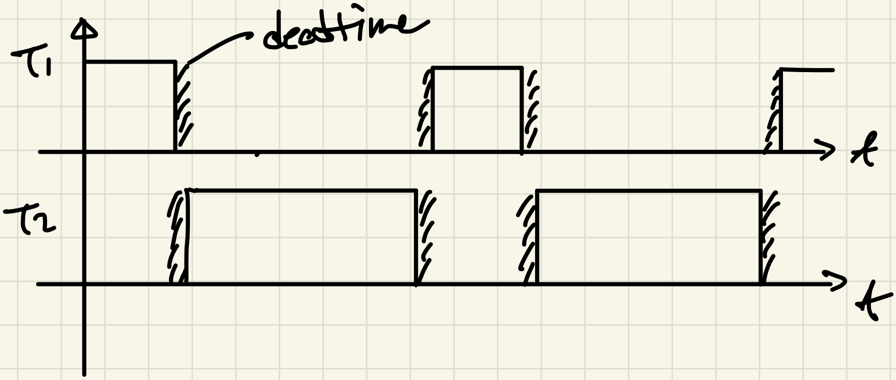
$$Q = \frac{\omega L_r}{R_T}, \qquad R_T = \frac{N_2^2}{N_1^2}\frac{8R}{\pi^2}$$

(iii)



Q4.

a)



Deadtime is used to have both high side (T_1) and low side (T_2) MOSFETs at the off state. This deadtime acts like a time buffer to avoid incident short through when T_1 and T_2 have states being switched.

b) $$I_{Lo} = \frac{P_{in}}{V_{in}} = \frac{250}{V_o \cdot (1-D)} = \frac{250}{100 \times 0.25} = 10A$$

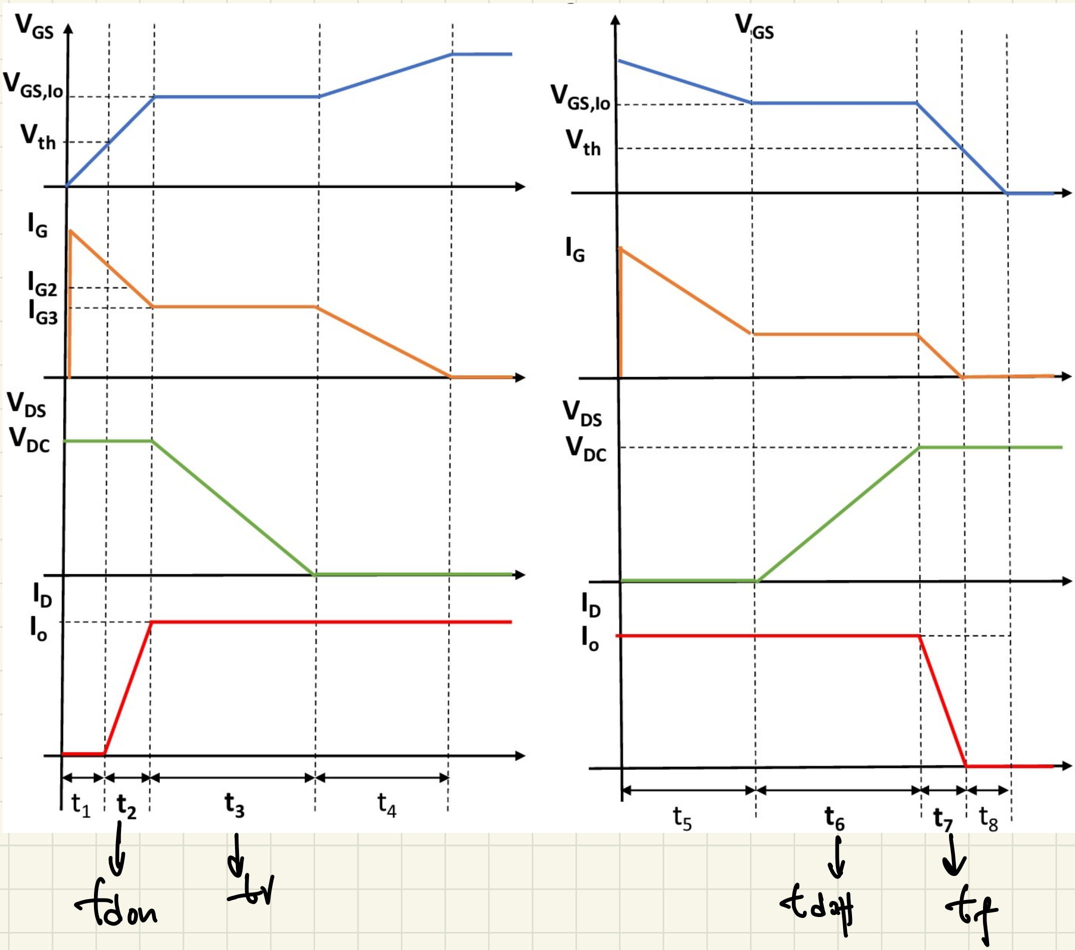
$$\Delta I_L = 10 \times 40\% = 4$$

$$P_{cond\,T_2} = R_{don} \cdot \left(\sqrt{I_{Lo}^2 D + \frac{\Delta I_L^2 D}{12}}\right)^2$$

$$= 50 \times 10^{-3} \times \left(10^2 \times 0.75 + \frac{4^2 \times 0.75}{12}\right)$$

$$= 3.8W$$

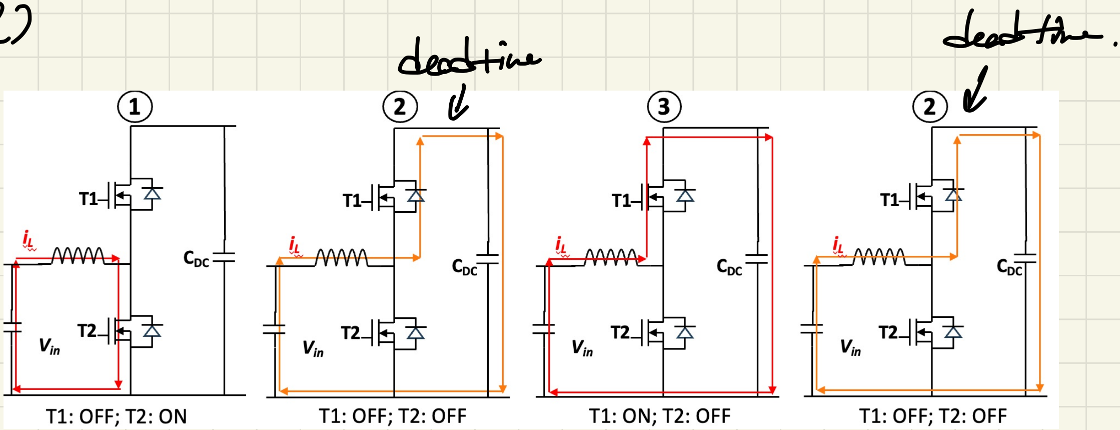
c)



d) $$P_{sw T_2} = \left(\frac{V_o \cdot I_L}{2}\right) \cdot \left(\frac{t_{don} + t_r}{T}\right) \cdot 2$$

$$= \left(\frac{100 \times 10}{2}\right) \times \frac{(14+16) \times 10^{-9}}{1/(100 \times 10^3)} \times 2 = 3W$$

e)



During the deadtime, the inductor current needs to commutate before T_1 is switched on. This commutating current is conducted by the body diode of T_1. This means that T_1 has nearly zero voltage before being switched on, as the body diode forward bias voltage is much smaller than V_0. This gives much smaller switching loss of T_1 than T_2.