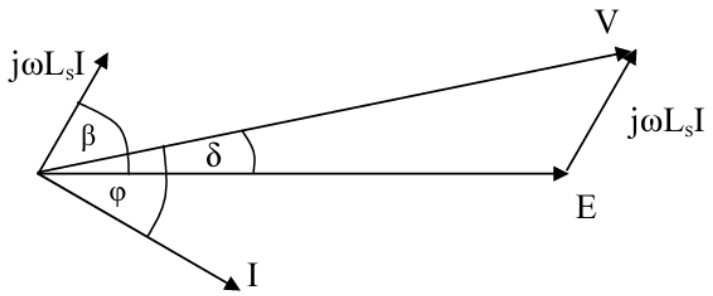
1. (a) Three advantages: higher power and torque density; smooth torque production (minimal reluctance torque); fast speed of response. Three disadvantages: greater sensor requirement (sensorless control is possible for the trapezoidal BLDCM whereas rotor position sensing is required for the sinusoidal BLDCM); stator current has to be resolved into orthogonal components and so greater hardware requirement and hence cost; control strategies are more complex.

General phasor diagram is shown below.



Start from the data book expression $P_{out} = \dfrac{3VE\sin\delta}{\omega L_s}$

By trigonometry: $V\sin\delta = \omega L_s I\sin\beta$

Also: $E = k\omega_s = k\dfrac{\omega}{p}$

Substituting: $P_{out} = T\omega_s = \dfrac{3k\omega_s\omega L_s I\sin\beta}{\omega L_s} = 3k\omega_s I\sin\beta$ giving $T = 3kI\sin\beta$

By operating with a torque angle of 90^0 the maximum torque per stator current is obtained. This means that for a given torque and power output, power losses are minimised and so efficiency is maximised.

The drive system utilises a variable voltage, variable frequency inverter so that the amplitude, phase and frequency of the applied phase voltages can be controlled. By feeding back the instantaneous rotor position and stator currents, the phase, amplitude and frequency of the applied voltages can be calculated by the microprocessor which controls the switching of the inverter transistors using the phasor diagram above but with a torque angle of 90^0. Note that for speeds above rated speed, the torque angle condition would be relaxed (field-weakening), but the controller software will account for this. [30%]

Given data

Number of poles	$P = 4 \Rightarrow p = 2$ pole pairs	Speed at test	$n = 1000$ rpm

Open-circuit **line voltage** at 1000 rpm	$V_{LL} = 20$ V	Phase resistance	$R = 1\ \Omega$
DC supply voltage	$V_{dc} = 40$ V	Rated current	$I_r = 5$ A
Maximum inverter frequency	$f_{\max} = 200$ Hz		

Mechanical angular speed at 1000 rpm: $\omega = \frac{2\pi n}{60} = \frac{2\pi \times 1000}{60} = 104.7$ rad/s

Back-EMF constant (from open-circuit line voltage): $K_e = \frac{V_{LL}}{\omega} = \frac{20}{104.7} = 0.191$ V\cdotps/rad

For a BLDC motor in SI units:$K_t = K_e = 0.191$ N\cdotpm/A

(i) Rated torque and rated speed

$$T_r = K_t I_r = 0.191 \times 5$$
$$\boxed{T_r \approx 0.96 \text{ N\cdotpm}}$$

Maximum fundamental line-to-line voltage from a 40 V DC inverter:

$$V_{LL,\max} \approx \frac{V_{dc}}{\sqrt{3}} = \frac{40}{\sqrt{3}} \approx 23.1 \text{ V}$$

$V_R = I_r R = 5 \times 1 = 5$ V	$V_{emf} = 23.1 - 5 = 18.1$ V
$\omega_r = \frac{V_{emf}}{K_e} = \frac{18.1}{0.191} = 94.8$ rad/s	$n_r = \frac{60\omega_r}{2\pi} \approx \boxed{905 \text{ rpm}}$

$$\boxed{T_r \approx 0.96 \text{ N\cdotpm}, n_r \approx 900 \text{ rpm}}$$

(ii) Maximum speed at 25% rated torque

$$I = 0.25 I_r = 1.25 \text{ A}$$

$V_R = 1.25 \times 1 = 1.25$ V	$V_{emf} = 23.1 - 1.25 = 21.85$ V
$\omega = \dfrac{21.85}{0.191} = 114.4$ rad/s	$n = \dfrac{60\omega}{2\pi} \approx \boxed{1090 \text{ rpm}}$

Maximum mechanical speed from inverter frequency: $n_{\max,f} = \frac{60 f_{\max}}{p} = \frac{60\times200}{2} = 6000$ rpm

Voltage limit dominates, not frequency.

$$\boxed{n_{\max} \approx 1100 \text{ rpm at 25\% rated torque}}$$

Final answers (summary)

- Rated torque: ≈ 0.96 N·m
- Rated speed: ≈ 900 rpm
- Max speed at 25% torque: ≈ 1100 rpm

Part c)

Given

Total moment of inertia: $J = 10 \times 10^{-3} = 0.01$ kg\cdotpm^2

Load torque at 800 rpm: $T_L = B\omega = 0.4$ N\cdotpm

Motor current during acceleration: $I = 4$ A

$$K_t = 0.191 \text{ N\textbackslash cdotpm/A}$$

$T_m = K_t I = 0.191 \times 4$ $\boxed{T_m \approx 0.764 \text{ Nm}}$	$\omega_{800} = \dfrac{2\pi \times 800}{60} = 83.8$ rad/s $B = \frac{T_L}{\omega} = \frac{0.4}{83.8} = 4.77 \times 10^{-3}$ Nm/rad/s

Final (steady-state) speed

$T_m = T_L = B\omega_f$ $\omega_f = \dfrac{T_m}{B} = \dfrac{0.764}{4.77 \times 10^{-3}} \approx 160$ rad/s	$n_f = \dfrac{60\omega_f}{2\pi} \approx \boxed{1530 \text{ rpm}}$

$$\boxed{T_m \approx 0.764\ \text{Nm}, n_f \approx 1530\ \text{rpm}}$$

(ii) Drive speed as a function of time

$J\dfrac{d\omega}{dt} = T_m - B\omega$	$\dfrac{d\omega}{dt} + \dfrac{B}{J}\omega = \dfrac{T_m}{J}$

This is a first-order linear system.

Time constant $\tau = \frac{J}{B} = \frac{0.01}{4.77\times10^{-3}} \approx 2.10$ s

Speed response (starting from rest)

$\omega(t) = \omega_f\left(1 - e^{-\frac{t}{\tau}}\right) = 160\left(1 - e^{-t/2.10}\right)$ rad/s	$\boxed{n(t) = 1530\left(1 - e^{-t/2.10}\right)\ \text{rpm}}$

$$\boxed{\omega(t) = \omega_f\left(1 - e^{-t/\tau}\right), \tau \approx 2.10\ \text{s}}$$

(iii) Time to reach 50% of final speed

$\omega = 0.5\,\omega_f$ $0.5 = 1 - e^{-t/\tau}$ $e^{-t/\tau} = 0.5$ $t = \tau \ln\ 2$	$t_{50\%} = 2.10 \times 0.693 \approx \boxed{1.46\ \text{s}}$

Final Answers (summary)

(i)

- Motor torque: ≈ **0.764 N·m**
- Final speed: ≈ **1530 rpm**

(ii)

$$\omega(t) = 160\left(1 - e^{-t/2.10}\right)\ \text{rad/s}$$

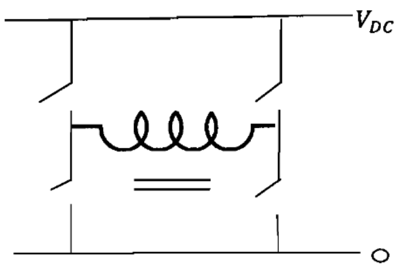
(iii)

- Time to reach 50% of final speed: ≈ **1.46 s**

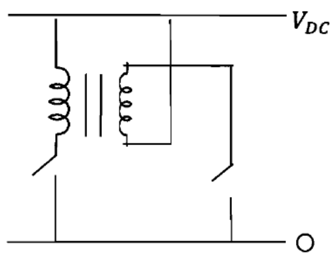
CRIB

4 (a) The brushless dc motor (BLDC) will have overall the better performance in terms of torque density and dynamic performance. However, its control is more complex: commutation requires position sensors and it is likely that knowledge of position will be needed; this implies a position encoder. In contrast, a stepper motor's position will always be determined by the number of steps executed, provided it does not pull out.

(b) The current in the windings in a normally configured hybrid stepper motor has to be reversed during operation, requiring relatively complicated circuitry. Bifilar, that is two independent wires, windings allow current to be directed in either sense with fewer switches, as shown below.



Normal



Bifilar

(c) (i) A stepper motor exhibits **natural resonance** because, near a step position, the motor torque provides a **restoring stiffness**, and together with the rotor–load inertia it behaves like a **spring–mass system**.

Torque–position characteristic

$T(\theta) = \hat{T}\sin(N_s\theta)$	$\sin(N_s\theta) \approx N_s\theta$
$T(\theta) \approx \hat{T}N_s\theta$	$k = \hat{T}N_s$

This is a **linear restoring torque**, equivalent to a torsional spring with stiffness:

$I\dfrac{d^2\theta}{dt^2} + k\theta = 0$	$I\dfrac{d^2\theta}{dt^2} + \hat{T}N_s\theta = 0$

This is the equation of a **simple harmonic oscillator**.

Natural angular frequency	Natural resonant frequency
$\omega_n = \sqrt{\dfrac{\hat{T}N_s}{I}}$	$\boxed{f_c = \dfrac{1}{2\pi}\sqrt{\dfrac{\hat{T}N_s}{I}}}$

- Increasing **torque** or **step resolution** increases stiffness → higher resonant frequency
- Increasing **inertia** lowers resonant frequency
- This resonance causes **oscillation, noise, and step loss** at certain stepping rates
-

ii) Reduction of oscillations using microstepping

Cause of oscillations in full-step operation:

- The rotor is commanded to jump abruptly between discrete equilibrium positions
- This sudden torque step excites the motor's natural resonance
- Oscillations can build up, especially near the resonant stepping frequency

How microstepping reduces oscillations

1. Smaller torque steps

    - Reduces excitation of the natural resonance
2. Smoother torque–angle characteristic
    - Rotor moves gradually toward the new equilibrium
3. Reduced overshoot
    - Less kinetic energy injected at each step
4. Increased effective damping
    - Current control absorbs oscillatory energy
5. Lower vibration and acoustic noise
    - Especially effective at low and mid speeds

(d) Consider a standard 2 phase, 2 stack hybrid stepper motor with 50 teeth on each of the two rotor wheels. The peak static torque at the rated current of 2 A is 400 mN m, and the moment of inertia of the rotor is 0.02 kg m2.

The stepper motor has a phase resistance of 1.5 Ω, phase inductance of 2 mH and an emf constant of 2.5 V s rad–1. The motor is driven at a speed of 100 rpm and is controlled so that it operates at an rms current of 2 A such that the phase current lags the excitation emf by 15°.

Draw a phasor diagram of the stepper motor.

Phasor diagram

In **phasor form**, for a stepper motor:

$$\underline{V}_{\text{phase}} = \underline{I}R + j\omega L\underline{I} + \underline{E}_{\text{emf}}$$

Where:

- $\underline{I}$is the phase current phasor
- $\underline{E}_{\text{emf}}$is the induced EMF (phasor)
- $\omega = 2\pi f_{\text{excitation}}$
- $j\omega LI$is the voltage drop across the phase inductance

Phasor relationships

- Draw $\underline{E}_{\text{emf}}$along the horizontal axis (reference)
- Current $\underline{I}$lags EMF by $\phi = 15^\circ$
- Voltage drops:
    - Resistive drop: RIalong current phasor
    - Inductive drop: $X_L I = \omega LI$perpendicular to current phasor (leading $\underline{I}$by 90°)
- Phase voltage phasor is the vector sum:

$$\underline{V}_{\text{phase}} = \underline{E}_{\text{emf}} + R\underline{I} + j\omega L\underline{I}$$

Exam sketch tips:

1. Horizontal axis: EMF (E_{emf})
2. Draw current phasor I lagging by 15°
3. Draw RI along I
4. Draw $X_L I$ perpendicular to I (leading by 90°)
5. Vector sum gives total phase voltage

This is **worth 15% marks**; just a clearly labeled phasor is sufficient.

Q3

1. (a) The voltage of a three-phase motor is limited by the airgap flux density (saturation, iron losses) and the ability of the winding insulation to withstand the peak electric field. The phase current of a three-phase motor is limited by the heat produced due to I^2R losses in the winding (overheating, reduced efficiency). Hence it is the volt-amps of the motor that is limited, and the volt-amp rating is given by the product of the maximum rms phase current and the maximum rms phase voltage and the number of phases.

Starting from $S = 3VI$, taking V and E as equal and substituting for I in terms of specific electric loading, J, and V in terms of specific magnetic loading, B, gives the result.

Note that the expression for E involves B_{rms} and so B_{rms} must be expressed in terms of B using $B_{rms}=\pi B/(2\sqrt{2})$.

d: airgap diameter; l: axial length; ω: angular supply frequency; p: pole-pairs; B: specific magnetic loading; J: specific electric loading.

(b) (i) Rated output power = 100 kW and full-load efficiency is 92%, so input power at full load is 100/0.92 = 108.7 kW. The input power factor is 0.8 and so the volt-amp rating of the motor is 108.7/0.8 = 136 kVA = S.

$B=0.5$ T, $J=20$ kAm^{-1}, $\omega/p=2\pi\times50/5 = 62.8$ rads^{-1} and $d=4l$. Substituting into the expression for S:

$136000=\pi^2\times4l^3\times62.8\times0.5\times20000/\sqrt{2}$ giving $l^3 = 0.00776$ and so $l=0.198$ m and $d=4l=0.792$ m.

(ii) $k_w = \cos(p\alpha/2)\times\sin(mnp\beta/2)/(m\sin p\beta/2)$, number of slots $N_S = 60$ and so $\beta=360/60 = 6^0$. Single layer winding so it must be fully-pitched and so $\alpha=0$. 60 slots means 30 coils (for a single layer winding) and so 10 coils per phase, so since $p=5$, $m=2$ giving $mnp\beta/2=30^0$ and $p\beta/2=15^0$ giving:

$k_w=\cos(0)\times\sin30/(2\times\sin(15)=0.966$.

$E=V=6.6kV/\sqrt{3}=3.81$ kV $=l\times(\omega/p)\times d\times N_{ph}\times k_w\times B_{rms}$ with $B_{rms}=\pi B/(2\sqrt{2})=0.555$ T. Putting in the numbers and solving for N_{ph} gives $N_{ph} = 722.5$ and hence 72.3 turns per coil. This should be rounded up to the nearest integer so that the specific magnetic loading is slightly under the desired value, so $N_{coil} = 73$ and $N_{ph} = 730$.

(iii) Assuming that all the flux crossing a slot pitch enters the tooth then:

$B_t w_t=(w_s+w_t)B\pi/2$ where w_s and w_t are the slot width and tooth width respectively. Rearranging:

$B_t=(1+w_s/w_t)\ B\pi/2$ and solving for w_s/w_t with $B_t=1.57$ gives $w_s/w_t=1$ and so $w_s=w_t$.

Also, $w_s+w_t=\pi d/60 = 41.4$ mm and so $w_s=w_t=20.7$ mm

For the stator core thickness, the principle is that all of the pole flux must split in half, travel round the core and then back through the teeth one pole-pitch later. Thus, the peak core flux, φ_c, is given by:

$B_c yl=0.5\times Bl\pi d/(2p)$ where y is the core thickness and B_c is the peak core flux density which is to be 1.4 T. Solving for y gives 44.4 mm.

(iv) The stator slot must be deep enough so that its area is big enough to accommodate the required total conductor area. First find the rated phase current:

$S=3V_{ph}I_{ph}$ gives $136000=\sqrt{3}\times 6600\times I_{ph}$ resulting in $I_{ph}=11.9$ A.

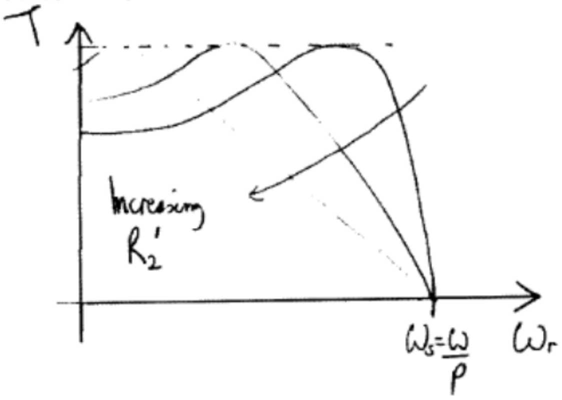
With the rms current density limited to 6Amm^{-2} the conductor cross-sectional area is $11.9/6=1.98\ \text{mm}^2$ and so the total conductor area is $N_{coil}\times 1.98=144.5\ \text{mm}^2$. The slot area required to accommodate this is 144.5/0.7 because of the 70% slot fill factor giving a total slot area of 206 mm^2.

Assuming rectangular stator slots, this area is given by $w_s d_s$ where d_s is the slot depth and w_s was found earlier to be 20.7 mm. Thus $d_s = 9.98$ mm.

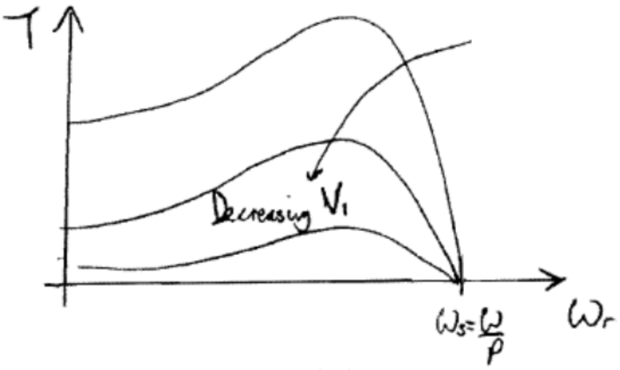
[redacted]

Q4

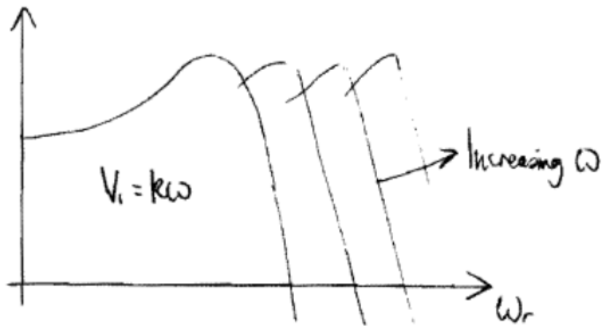
2(a) (i) Torque-speed characteristics for varying R_2.



(ii) Torque-speed characteristics for varying V_1.



(iii) Torque-speed characteristics VVVF speed control.



Advantages of VVVF control: the motor is always operating on the steep part of the torque-speed characteristic, so efficient; wide speed range possible at high efficiency; maximum available torque is unaffected unlike variable voltage control; motor always fully-fluxed which means it is operating efficiently and making good use of the magnetic materials.

(b) Start from the databook expression for torque:

$T = 3I_2^2R_2/s\omega_s$

$I_2^2 = V_1^2/((R_2/s)^2 + \omega^2L_2^2) = s^2V_1^2/(R_2^2 + s^2\omega^2L_2^2)$ assuming that the voltage across the rotor branch, E, is equal to the stator applied voltage, V_1. For small slip, $s\omega L_2 << R_2$ and may be ignored (this corresponds to the steep part of the torque-speed characteristic as required) giving $I_2 = sV_1/R_2$

Substituting into the torque equation gives:

$T = 3V_1^2 s/(\omega_s R_2)$ as required.

The slope of the steep part of the torque-speed characteristic is found by differentiating the above torque expression wrt ω_r. The easiest way to do this is to use the chain rule, so:
$dT/d\omega_r = dT/ds \times ds/d\omega_r$ where $s = (\omega_s - \omega_r)/\omega_s$ and so $ds/d\omega_r = -1/\omega_s$

$dT/ds = 3V_1^2/(\omega_s R_2)$ and so $dT/d\omega_r = -3V_1^2/(\omega_s^2 R_2)$

Substituting in $\omega_s = \omega/p$ gives $dT/d\omega_r = -3p^2 V_1^2/(\omega_s^2 R_2)$

For VVVF control with $V1 = k\omega$ this slope becomes $-3k^2p^2/R_2$ and is therefore fixed.

(c) (i) The unloaded speed at 50 Hz of an 8 pole motor is $60f/p = 3000/4 = 750$ rpm. Therefore the maximum unloaded speed of the drive corresponds to the inverter operating at its maximum frequency of 150 Hz, giving $N_{max} = 750 \times 150/50 = 2250$ rpm.

To find the rated torque, ignore $R_1 + jX_1$ and jX_2. This is justified because the motor will be operating at a small value of slip.

$I_1 = 415\sqrt{3}/jX_m + I_2 = -j2.40 + I_2$

Since $R_2/s >> X_2$, I_2 is approximately in phase with V_1 and so at the stator current limit of 15 A:

$I_2 = \sqrt{(15^2 - 2.4^2)} = 14.8$ A (which shows that $I_1 = I_2$ is a reasonable approximation for an induction motor under full-load conditions)

$k = V_b/\omega_b = 415/\sqrt{3}/(2\pi \times 50) = 0.764$

$I_2 = V_1/(R_2/s) = V_1 s\omega/(\omega R_2) = s\omega k/R_2 = 14.8$ so $s\omega = 14.8 \times 1.2/0.764 = 23.3$ and this is fixed for rated torque and rated rotor current.

$T_{rated} = 3pk^2 s\omega/R_2 = 3 \times 4 \times 0.764^2 \times 23.3/1.2 = 136$ Nm

V_1 is at its maximum when $f = 50$ Hz and delivering rated torque. The slip at this frequency is given by $2\pi \times 50s = 23.3$ so $s = 0.0742$.

Maximum speed at which rated torque can be delivered is $(1 - s) \times 60f/p = (1 - 0.0742) \times 750 =$ 694 rpm.

(ii) $f = 1$ Hz so voltage across magnetizing reactance for rated flux is $(1/50) \times 240 = 4.8$ V. At rated magnetizing current torque is proportional to I_2, and so for 50% of rated torque, $I_2 = 14.8/2 = 7.4$ A.

Thus, total stator current is $(7.4 - j2.4)$ A and this will produce a voltage drop across R_1 (no need to include X_1 since its value will be negligible compared to R_1 at $f = 1$ Hz) of:

$V_{R1} = 1.5 \times (7.4 - j2.4) = 11.1 - j3.6$

and so $V_1 = 4.8 + 11.1 - j3.6 = 15.9 - j3.6$ and the magnitude of V_1 is 16.3 V. Voltage boost is the difference between V_1 and the voltage required to produce rated flux ie the voltage across the magnetizing reactance, which is 4.8 V. Therefore $V_{boost} = 16.3 - 4.8 = 11.5$ V.