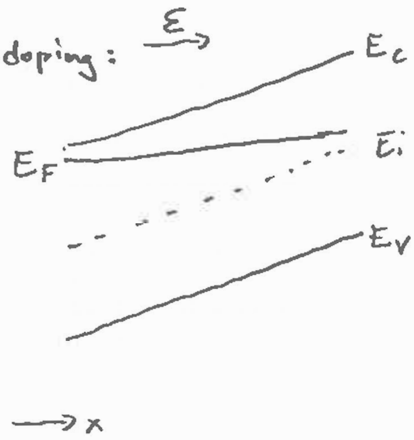
**Q1** (a) $N_D = N_0 \exp(-ax)$

n-type doping:




$$eD\frac{dn}{dx} = n\mu\mathcal{E}e$$

$$\rightarrow \mathcal{E}(x) = \frac{D\frac{dn}{dx}}{n\mu_n}$$

Assume full dopant ionisation, use Einstein relation for $D/\mu_n$

$$= -\frac{kT}{q}\frac{N_0(-a)e^{-ax}}{N_0 e^{-ax}}$$

$$= \frac{kTa}{q}$$

*Hint was meant to help candidates to simple equation of balanced currents, but many candidates started more complex and/or got the derivative wrong.*

(b) $p = 10^{20}\ \mathrm{m^{-3}}$ doped side:

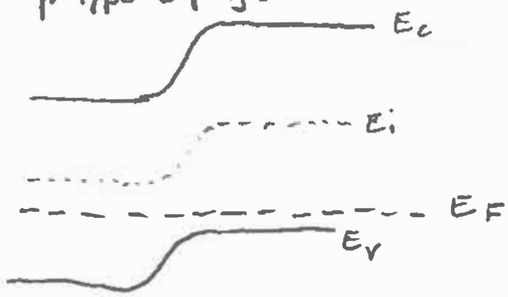
$$p = n_i e^{(E_i - E_F)/kT}$$

$$\rightarrow E_i - E_F = kT \ln\left(\frac{p}{n_i}\right) = 0.24\ \mathrm{eV}$$

$p = 2\cdot 10^{18}\ \mathrm{m^{-3}}$ doped side

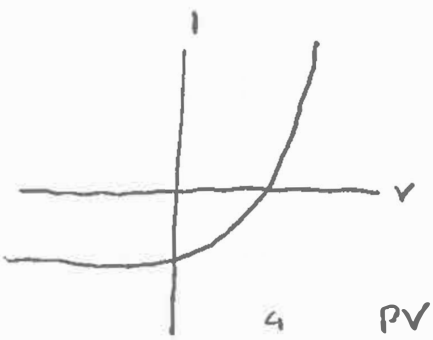
$$E_i - E_F = 0.14\ \mathrm{eV}$$

p-type doping:




*Fermi level calculations and band diagram drawing generally well done.*

(c) photon energy > $E_{gap}$




PV operation, where junction delivers power, is in quadrant 4.

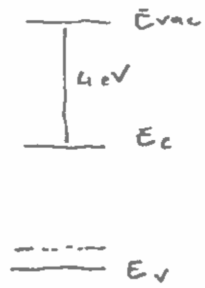
*Usually answered well, but some confused answers.*

Q1 (d) (i) Al is group 3, so Ge is p-doped

$$p = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$E_v - E_F = kT \ln\left(\frac{p}{N_v}\right) = -0.11\,eV$$

Assume full dopant ionisation




Work function of doped Ge

$$\phi(Ge) = 4\,eV + 0.66\,eV - 0.11\,eV = 4.55\,eV$$

For Schottky contact and p-Ge

$$\phi(Ge) > \phi(\text{metal}) \rightarrow \phi(\text{metal}) < 4.55\,eV$$

*Generally answered well, with most candidates also recognising p-type doping situation.*

(ii) Poisson equation

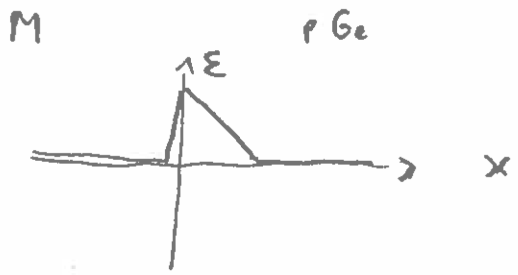
$$\nabla^2 V = \frac{\rho}{\varepsilon_0 \varepsilon_r} = \frac{e N_A}{\varepsilon_0 \varepsilon_r}$$

$$\rightarrow \mathcal{E} = \frac{dV}{dx} = \frac{e N_A (w - x)}{\varepsilon_0 \varepsilon_r}$$

Assume no fields outside depletion region (width $w$), $\mathcal{E} = 0$ at $x = w$

$\rightarrow$ $\mathcal{E}_{max}$ at $x = 0$

$$\mathcal{E}_{max} = \frac{e N_A}{\varepsilon_0 \varepsilon_r}$$




*Max. field calculation generally done well, some sketches not very clear.*

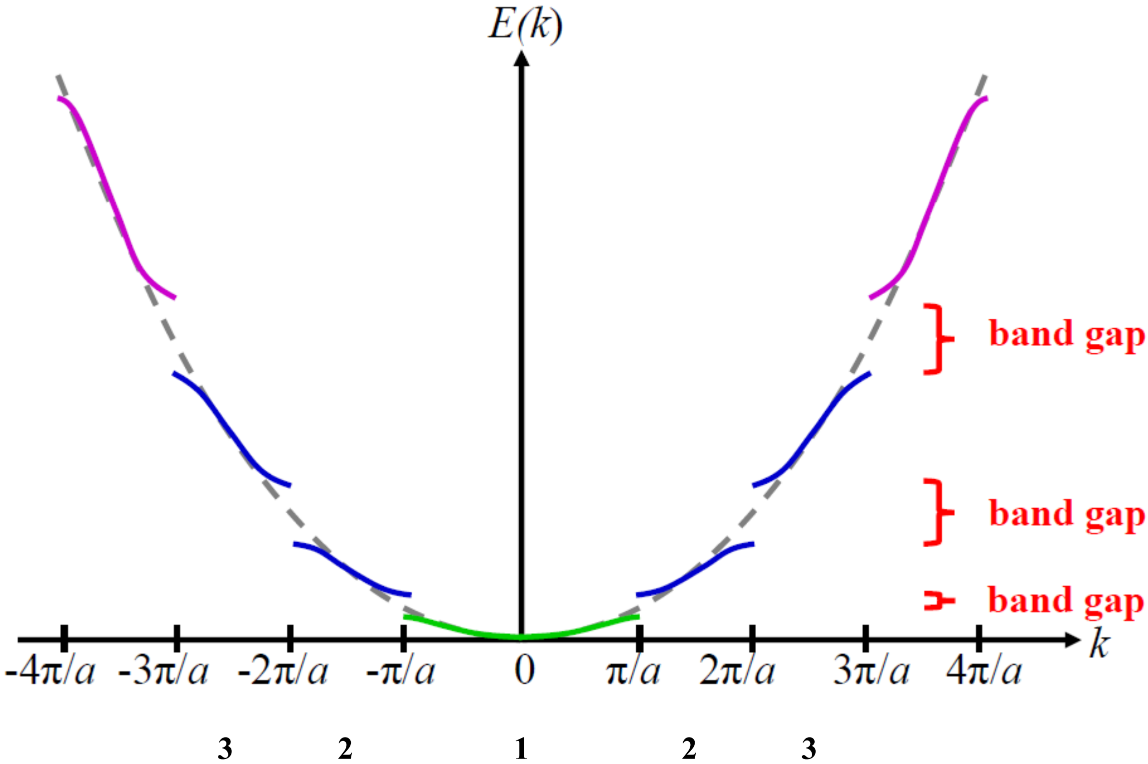
Q1 (d) (iii)


The higher doping density will lead to lower width $w$ of depletion in Ge. The junction capacitance $C \sim \frac{\varepsilon \varepsilon_r}{w}$, hence will go up for higher doping.

Thermionic emission theory does not include eg tunneling contribution. Likelihood of tunneling through barrier increases with $w$ going smaller. That is lower doped junction will show more ideal behaviour.

*Band diagram drawing generally ok (not penalised here for p/n-type mix-up again). Some confusion re ideality and junction C behaviour for higher doped semiconductor.*

**Q2 a)**




When Bragg's condition is verified incident and reflected waves form standing waves for

$k = \pm\frac{n\pi}{a}$ with n an integer (1)

This results in two situations.

1) Low probability of the electron being near the nuclei $\Rightarrow$ higher energy
2) High probability of the electron being near the nuclei $\Rightarrow$ lower energy

This results in an energy gap between the 2 states when (1) is satisfied.

*Generally well answered.*

**b)** The analysis uses a combination of two approaches:

(i) Wave

(ii) Particle

i) **As a wave**: Let us consider an electron moving through our lattice with some energy, *E*, and wavenumber, *k*, given by the *E*–*k* diagram.
The energy of the particle is partly potential and partly kinetic, so:

$$\omega = \frac{E}{\hbar} = \frac{\frac{1}{2}mv^2 + V(x)}{\hbar}$$

Differentiating gives:

$$\partial\omega = \frac{mv}{\hbar}\partial v$$

From de Broglie's postulate $p = \hbar k,$ so:

$$v = \frac{\hbar k}{m}$$

Differentiating gives us:

$$\partial v = \frac{\hbar}{m}\partial k$$

Substituting this into $\partial\omega = \frac{mv}{\hbar}\partial v$ produces an expression for the velocity of the particle:

$$\partial\omega = \frac{mv}{\hbar}\frac{\hbar}{m}\partial k$$

$$\therefore v = \frac{\partial\omega}{\partial k}$$

We also know the electron wave has energy, given by the Einstein postulate $E = hf = \hbar\omega$. Substituting this into the above gives:

$$v = \frac{1}{\hbar}\frac{\partial E}{\partial k}$$

**(ii) As a particle**: Newtonian mechanics tells us that if we apply a force to a particle of mass $m^*$, it should accelerate and gain energy according to:

$$F = m^*\frac{\mathrm{d}v}{\mathrm{d}t}$$

Substituting in $v = \frac{1}{\hbar}\frac{\partial E}{\partial k}$ from (i)

$$\mathrm{F} = m^*\frac{\mathrm{d}\left(\frac{1}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}k}\right)}{\mathrm{d}t}$$

$$= \frac{m^*}{\hbar}\frac{\mathrm{d}^2E}{\mathrm{d}k^2}\frac{\mathrm{d}k}{\mathrm{d}t}$$

Force is also the rate of change of momentum:

$$F = \frac{\mathrm{d}p}{\mathrm{d}t}$$

Substituting $p = \hbar k$ (de Broglie's postulate) gives:

$$F = \frac{\mathrm{d}}{\mathrm{d}t}(\hbar k) \quad = \hbar\frac{\mathrm{d}k}{\mathrm{d}t}$$

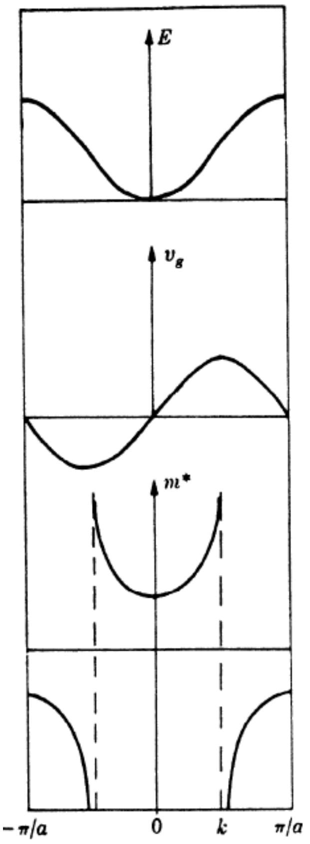
Thus:

$$F = \frac{m^*}{\hbar}\frac{\mathrm{d}^2E}{\mathrm{d}k^2}\frac{\mathrm{d}k}{\mathrm{d}t} = \hbar\frac{dk}{dt}$$

$$\therefore m^* = \hbar^2\left(\frac{\mathrm{d}^2E}{\mathrm{d}k^2}\right)^{-1}$$

*Broadly OK, but a few people struggled with either the particle or the wave approach.*

**c)**




*Generally OK, but a few people did not correctly align the diagrams, or plotted incorrectly the velocity.*

**d) (i)**

For the one dimensional case we have

$E = \frac{\hbar^2 k^2}{2m*}$ = (1.05457×10$^{-34}$ J s)$^2$×(2×10$^9$m$^{-1}$)$^2$/(2×0.5×9.1094×10$^{-31}$kg)=0.30483eV

*Several mistakes in orders of magnitude, and use of J instead of eV.*

**(ii)** $k = \frac{\sqrt{2m*E}}{\hbar}$

with E=$k_B$T= 1.380649×10$^{-23}$ J K$^{-1}$× (273+21)K=4.0591×10$^{-21}$ J=0.02534eV

k= (2×0.5×9.1094×10$^{-31}$kg×4.0591×10$^{-21}$J)$^{0.5}$/1.05457×10$^{-34}$ J s=5.77×10$^8$ m$^{-1}$

At T=0K a LED would emit light with frequency corresponding to the band gap: hf=$E_g$. At T>0K, thermal fluctuations broaden the spectral width if the emitted light. Thus, the frequency uncertainty would be Δf= $k_B$T/h=4.0591×10$^{-21}$J/6.62607×10$^{-34}$ J s=6.13 ×10$^{12}$ Hz.

*Several mistakes in orders of magnitude, and use of J instead of eV, very few correct LED answers.*

**Q3 a)**

The first term in the Time-Independent Schrodinger Equation (TISE) represents the kinetic energy of a particle multiplied by the wavefunction, $\psi$, where $\hbar$ is the Plank constant divided by $2\pi$, $m$ is the mass of the particle and $x$ is position. The second term is the potential energy of the particle, $V$, multiplied by the wavefunction and the third term is the total energy of the particle, $E$, multiplied by the wavefunction. Therefore, the TISE is a statement of the conservation of energy: the total energy of a particle is the sum of its kinetic and potential energies.

*Generally well answered.*

**b)** (i) Outside the well, $V = \infty$, and the TISE becomes:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + \infty\Psi = E\Psi$$

This only has two possible solutions:

$\Psi(x) = 0$ or $\Psi(x) = \infty$

The only physically realistic solution is $\Psi(x) = 0$ given that $|\Psi|^2$ is the probability of finding the electron outside the well, of which there is clearly not an infinite probability.

Iside the well, the TISE becomes:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = E\Psi$$

which has the general solution:

$$\Psi = A\cos(kx) + B\sin(kx) \qquad (1)$$

with $A$, $B$, $k$ unknown constants.

We can now apply boundary conditions to determine $A$, $B$, $k$

For the wavefunction to be continuous and single-valued (postulate III), the wavefunctions for inside and outside the boundary must be identical at the boundary, so:

$\Psi = 0$ at $x = 0, L$

Applying the first boundary condition ($\Psi = 0$ at $x = 0$) to (1) we get:

$$A = 0$$
$$\therefore \Psi = B\sin(kx)$$

Applying the other boundary condition ($\Psi = 0$ at $x = L$) gives:

$$0 = B\sin(kL)$$

This means that $k$ can only take certain values given by:

$$k = \frac{\pi n}{L}$$

where $n$ is an integer greater than or equal to 1 (a positive, non-zero integer). Thus:

$$\Psi = B\sin\left(\frac{\pi n}{L}x\right)$$

To solve for $B$ we need one more boundary condition:

- The final condition is that the integral of $|\Psi|^2$ over all space is 1.
- This is because the probability of finding the particle is $|\Psi|^2$, and the total integrated probability over all space must be 1.

$$\int_0^L |\Psi|^2 \, dx = 1$$

$$\int_0^L |\psi|^2 dx = \int_0^L B^2 \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$1 = \int_0^L \frac{B^2}{2}\left\{1 - \cos\left(\frac{2n\pi x}{L}\right)\right\} dx$$

$$1 = \frac{B^2}{2}\left[x - \frac{L}{2n\pi}\sin\left(\frac{2n\pi x}{L}\right)\right]_0^L$$

$$\frac{2}{B^2} = \left[L - \frac{L}{2n\pi}\sin(n\pi)\right] - [0]$$

$$B = \left(\frac{2}{L}\right)^{1/2}$$

Thus:

$$\Psi = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi n}{L}x\right)$$

(ii)

*Broadly ok, but a few people skipped or got wrong some key steps.*

Inside the well, the TISE is:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = E\Psi$$

At the ground state n=1, therefore:

$$\frac{d^2\psi}{dx^2} = \frac{d^2}{dx^2}\left\{\left(\frac{2}{L}\right)^{1/2}\sin\left(\frac{\pi x}{L}\right)\right\}$$

$$= -\left(\frac{2}{L}\right)^{1/2}\left(\frac{\pi}{L}\right)^2\sin\left(\frac{\pi x}{L}\right)$$

$$\frac{d^2\psi}{dx^2} = -\left(\frac{\pi}{L}\right)^2\psi$$

Thus:

$$E = \frac{\hbar^2\pi^2}{2mL^2}$$

*Generally well answered*

(iii)
The Heisenberg uncertainty principle states that we cannot know precisely both the position and momentum of a particle. If we are confining our electron within a well of 2 nm width, then this must be the uncertainly in our knowledge of the electron's position, $\Delta x$, to a first approximation.

Hence:

$\Delta p \Delta x \geq \hbar/2$

$\Delta p \geq \hbar/2L$

The minimum kinetic energy E must be:

$$E \geq \frac{\hbar^2}{2m4L^2}$$

E≥ $(1.05457\times10^{-34}\text{ J s})^2/(8\times9.1094\times10^{-31}\text{kg}\times4\times10^{-18}\text{ m}^2)$=2.38meV

The ratio of E derived in (ii) and (iii) is

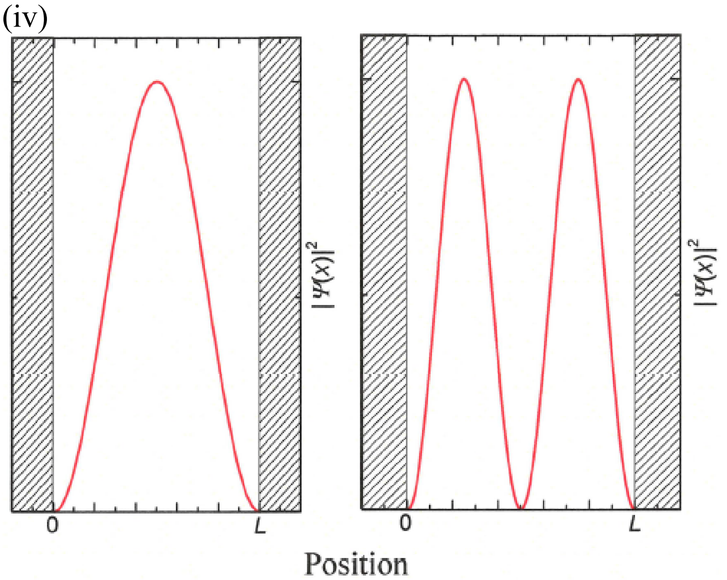
$\frac{\hbar^2\pi^2}{2mL^2} : \frac{\hbar^2}{2m4L^2} = 4\pi^2 = 39.5$

Hence the minimum E in (ii) is 93.95meV

This is entirely compatible with the uncertainty principle as the momentum of the electron will always be above the minimum required by Heisenberg.

*Several mistakes in orders of magnitude, calculations or units*

(iv)




$n = 1$ $\qquad$ $n = 2$

This is radically different to the classical equivalent in several ways. First, the minimum energy that the particle could possess classically is zero. At this energy, the particle would not be moving. The particle could also possess any energy above this minimum. In both cases, there would be an equal probability of finding the particle at any position inside the potential well. It would certainly not have 'favoured locations' as suggested by the quantum model

*A few perfect answers, but many with mistakes.*

Q4 (a) (i) $C_{max}$ corresponds to the capacitance due to the oxide layer. With an applied bias (in reverse bias) a depletion region forms in the Si, which presents another capacitane. Hence total capacitance decreases.

Once inversion has occurred, the depletion region does not expand anymore but gets filled with inversion carriers. Hence overall capacitance goes back up again to $C_{max}$.

The thickness of the insulating layer is

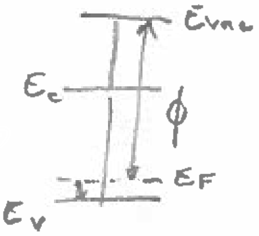
*Usually explained well. Dielectric constant of Si oxide to be taken from CUED data book (quartz), but full marks awarded for any ε assumed.*

$$C_{max} = \frac{\varepsilon_0 \varepsilon_r}{d} \rightarrow d = \frac{\varepsilon_0 \varepsilon_r}{C_{max}}$$

for numerical value, need $\varepsilon_r$ of oxide. Electrical databook lists $\varepsilon_r$ (Quartz) $= 3.8$ $\rightarrow$ $d = 48$ nm (full points awarded to answers without exact numerical value)

(ii) At equilibrium there is no band bending, so $\phi_M = \phi_{SC}$

$\phi_{Si} = 5.12\,eV$




$$\rightarrow E_F - E_V = \chi_{Si} + \text{band gap}_{Si} - \phi_{Si}$$
$$= 0.05\,eV$$

$$\rightarrow p = N_V \exp\left(\frac{E_V - E_F}{kT}\right) = 1.5 \cdot 10^{24}\,m^{-3}$$

*Generally answered well.*

(iii) Max. width of depletion at point of strong inversion, where $C_{min}$ is reached. (0.8 V as in Fig. 1)

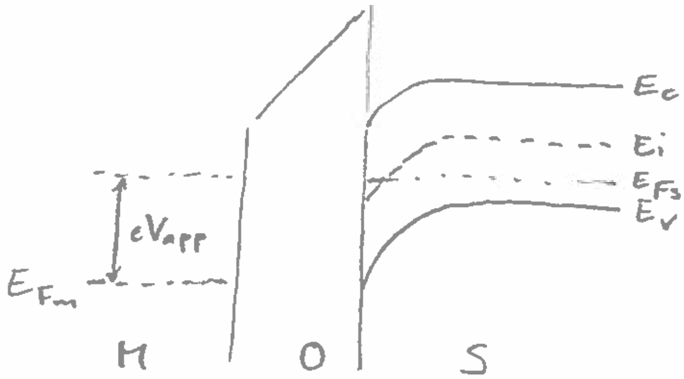
$$C_{min} = \frac{C_{max}\, C_{dep}}{C_{max} + C_{dep}} \rightarrow C_{dep}^{-1} = \frac{C_{max} - C_{min}}{C_{max}\, C_{min}} \Rightarrow$$

$C_{dep} = 2.8\,Fm^{-2}$

$$\rightarrow w_{max} = \frac{\varepsilon_0 \varepsilon_r}{C_{dep}} = 3.8 \cdot 10^{-7}\,m$$

with $\varepsilon_r = 11.9$ as given for Si

Q4 (a)(iii) Band diagram at strong inversion (0.8 V as of Fig 1)



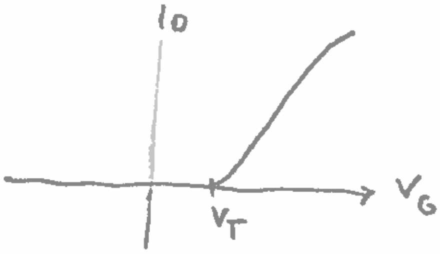

*Band diagram of strong inversion generally drawn well, some candidates did not see that max. depletion links to min. C*

(a)(iv) If the bias is varied so rapidly that charge in inversion layer can not change in response, and depletion region maintains extended. This is called deep depletion and reflected in curve 2 in Fig 1.

*Well answered.*

(b)(i) Most efficient is normally off mode, ie enhancement n-channel MOSFET (p-type Si body as of (a)).

Transfer characteristics:




*Some candidates drew output characteristics or other diagrams. Some nice answers re lowest supply V, but also many confused comments.*

Subthreshold conduction is due to (weak) inversion and thus occurs before $V_T$ (strong inversion). Lowest possible value for subthreshold swing

$$S = \frac{dV_{gs}}{d(\ln I_{DS})} \cdot \ln 10$$

is 60 mV / dec at room temperature.

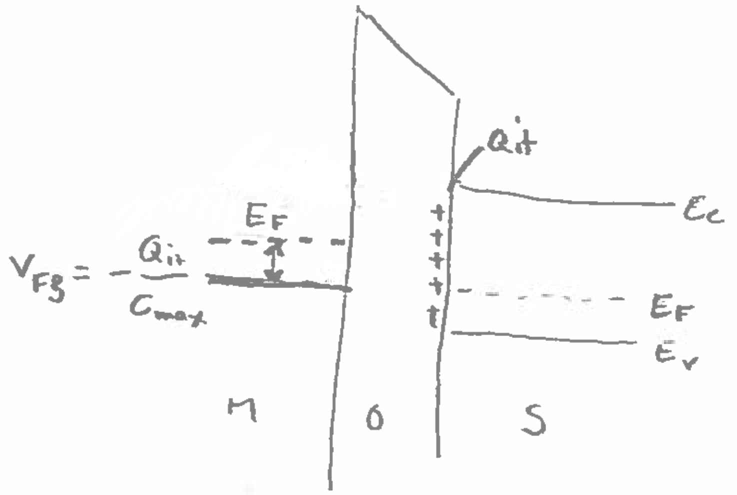
So for on-off ratio of 4 decades min. 240 mV required

Q4 (b) (ii) $Q_i = 10^{11} \frac{1}{cm^2} \cdot e = 1.6 \; 10^{-8} \; C/cm^2 = 1.6 \; 10^{-4} \; C/m^2$

shift in $V_T$ $\quad \Delta V_T = -\frac{Q_i}{C_i} = -\frac{Q_i}{C_{max}} = -0.23 \; V$

*If answered, then generally done well.*

(b) (iii) Band diagram for positive $Q_{it}$ at so called flat band condition




*Not many candidates realised flat band situation.*