EGT2
ENGINEERING TRIPOS PART IIA

Monday 27 April 2026 9.30 to 11.10

Module 3B5

SEMICONDUCTOR ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachment: Sheet of Formulae and Constants (2 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) A semiconductor has a band gap of 1.10 eV, an intrinsic carrier concentration of 10^{16} m^{-3} and an effective density of states in the valence band of 8×10^{24} m^{-3}.

(i) What density of acceptors are required to dope this semiconductor such that at room temperature (298 K) the Fermi energy lies 0.06 eV above the top of the valence band? State any assumptions made. [15%]

(ii) An equal density of dopants, but this time donors, is used to dope another sample of the same semiconductor. Calculate the position of the Fermi energy with respect to the bottom of the conduction band in this sample. [15%]

(b) A p-n junction diode is fabricated using the p-type and n-type materials of part (a) above.

(i) Starting from the Poisson equation, show that the depletion width of the junction w is given by:

$$w = 2\left(\frac{V_o \varepsilon_0 \varepsilon_r}{e N_a}\right)^{0.5}$$

where V_o is the built-in potential and N_a is the doping density of acceptors in the p-type material, which is equal to the doping density N_d in the n-type material. [40%]

(ii) Calculate the depletion width for this p-n junction for $\varepsilon_r = 12$. [30%]

2 (a) Using the *Free Electron Theory*, show that the electron Fermi energy, E_F, in a metal at a temperature of absolute zero is given by:

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{\frac{2}{3}}$$

where m is the electron mass and N/V is the number of free electrons per unit volume. [40%]

(b) Calculate the Fermi energy of magnesium, which is in Group II of the Periodic Table and has an atomic density of $3.92 \times 10^{28}\ \mathrm{m}^{-3}$. [30%]

(c) Calculate the wavelength and velocity of an electron in magnesium whose energy is the Fermi energy of part (b). [30%]

NOTE: The density of states according to the *Free Electron Theory* is given by:

$$g(E)\,dE = \frac{V}{2\pi^2\hbar^3}(2m)^{\frac{3}{2}}E^{\frac{1}{2}}\,dE$$

where V is the volume of the material.

3 (a) The minority carrier concentration in a silicon p-n junction with applied bias is shown below in Fig. 1.

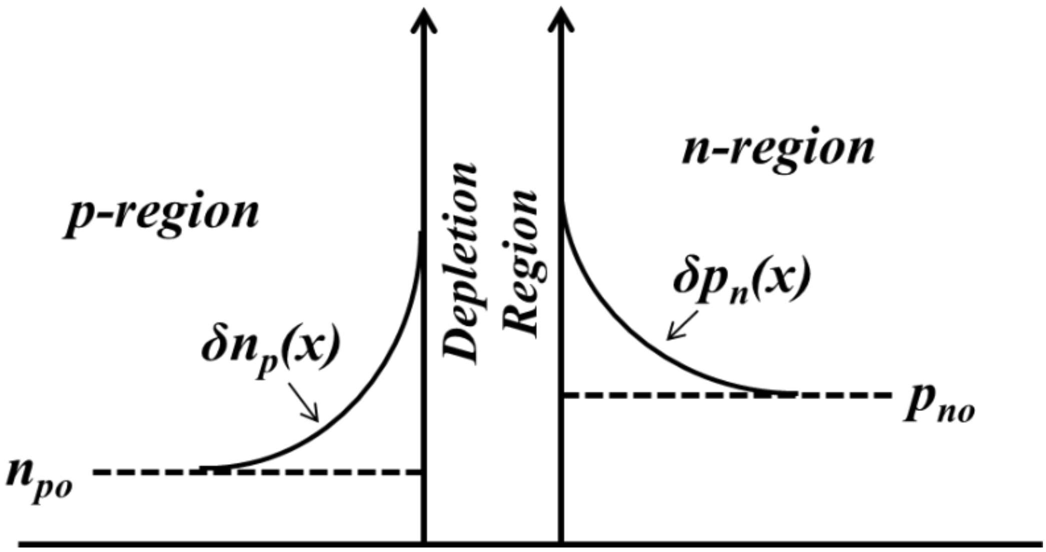

Fig. 1

(i) Is the diode forward biased, reverse biased or in thermal equilibrium (choose one)? Justify your answer. [5%]

(ii) The majority electron concentration on the n-side is larger than, smaller than, or equal to (choose one) the majority hole concentration on the p-side. Justify your answer. [5%]

(iii) If you want to make a p-n junction diode emit light (such as in an LED or a laser), would you forward bias or reverse bias it (choose one)? Justify your answer. [5%]

(iv) Now, consider two ideal p-n junctions at temperature T = 300 K with exactly the same physical parameters (including area A, diffusion coefficients D_n & D_p and diffusion lengths L_n & L_p) and same doping concentrations (N_a & N_d on the p and the n-regions, respectively). The only difference between them is that they have different bandgap energies. The first p-n junction has a bandgap energy of E_{g1} = 0.525 eV and puts out a forward current of 10 mA at V_{a1} = 0.255 V. For the second p-n junction, a forward current of 10 μA was measured at V_{a2} = 0.32 V. What is the bandgap energy E_{g2} of the second p-n junction material? [35%]

 (cont.

(b) Consider a Si (bandgap $E_g = 1.12$ eV) and a GaAs (bandgap $E_g = 1.42$ eV) p-n junction diode that are connected in series in an electronic circuit as shown below in Fig. 2. Both diodes have an area of $A = 100 \times 100\ \mu\text{m}^2$, an acceptor concentration of $N_a = 5 \times 10^{15}$ cm^{-3} and a donor concentration of $N_d = 1 \times 10^{18}$ cm^{-3}. The circuit carries a current of 1 mA.

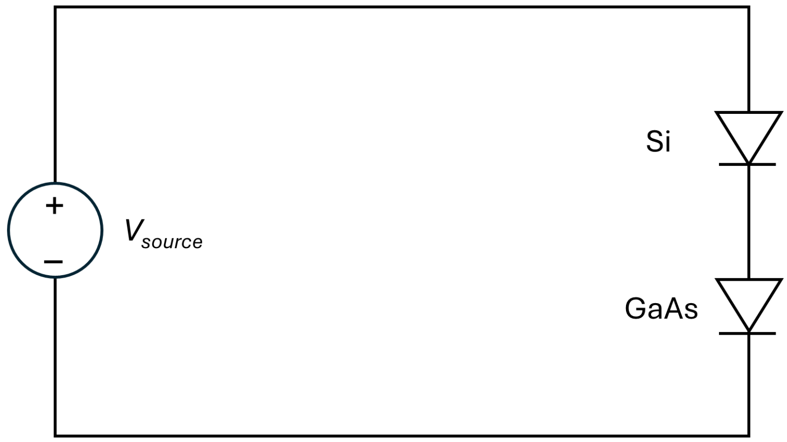

Fig. 2

For Silicon: $D_n = 35$ cm^2/sec, $D_p = 12$ cm^2/sec, $\tau_p = \tau_n = 10^{-6}$ s and $n_i = 1.0 \times 10^{10}$ cm^{-3}
For GaAs: $D_n = 220$ cm^2/sec, $D_p = 10$ cm^2/sec, $\tau_p = \tau_n = 10^{-8}$ s and $n_i = 2.0 \times 10^{6}$ cm^{-3}

(i) Calculate the forward-bias voltages (V_a) of the two diodes, as well as the source voltage, V_{source}. [40%]

(ii) The forward-bias voltages for the two diodes calculated in (i) above are different for the Si and the GaAs diode. What is the physical origin of this difference? [5%]

(iii) Ge has a bandgap energy of $E_g = 0.66$ eV. Assume that a Ge diode has the same geometrical dimensions and doping concentrations as the two other diodes. Qualitatively estimate the forward voltage of the Ge diode for a current of 1 mA (*i.e.*, is it smaller, the same, or larger than the Si and/or GaAs, and why)? [5%]

4 (a) Consider a CMOS device shown below in Fig. 3 fabricated on Silicon doped with 5×10^{16} cm^{-3} atoms. The gate material is Aluminum (Al) which has a work function $e\phi_m = 4.1$ eV. Assume the electron affinity of silicon to be $e\chi_s = 4.01$ eV and its bandgap to be $E_g = 1.12$ eV. The oxide thickness is 50 nm. Assume oxide to be charge free, and the permittivity of silicon and oxide to be $\varepsilon_{Si} = 11.7\varepsilon_0$ and $\varepsilon_{ox} = 3.9\varepsilon_0$, respectively.

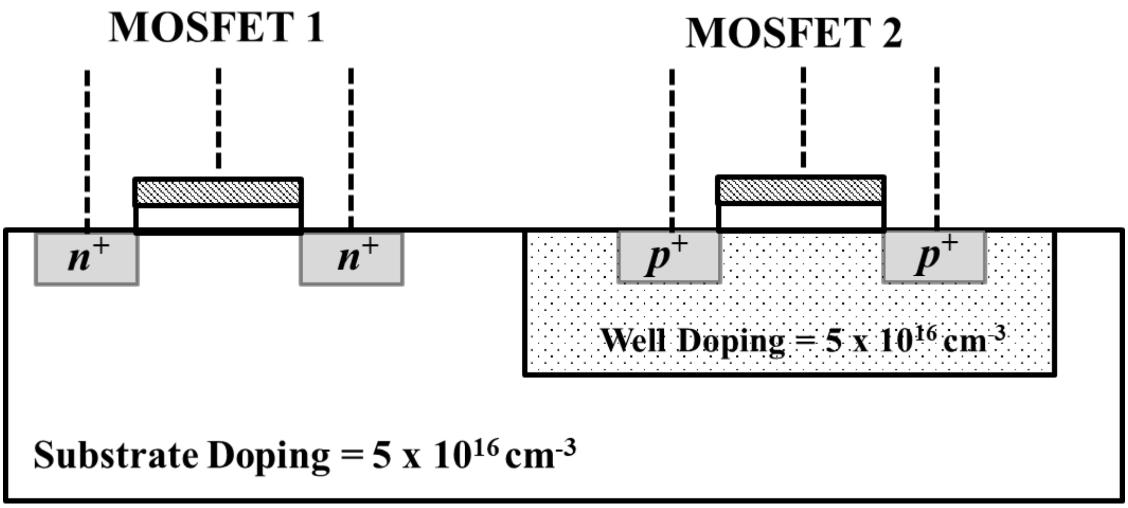

Fig. 3

(i) First identify each device: MOSFET 1 is (choose one: NMOS or PMOS) and MOSFET 2 is (choose one: NMOS or PMOS). [5%]

(ii) The substrate is (choose one: n-type, p-type or intrinsic), and the well of MOSFET 2 is (choose one: n-type, p-type or intrinsic). [5%]

(iii) Determine the metal-semiconductor work-function difference ϕ_{ms} and the flat-band voltage V_{FB} of MOSFET 1? [15%]

(iv) Determine the metal-semiconductor work-function difference ϕ_{ms} and the flat-band voltage V_{FB} of MOSFET 2? [15%]

(v) Determine the threshold voltages (V_T) of MOSFET 1 and MOSFET 2. [20%]

(b) A Schottky diode and a p-n junction diode have the same cross-sectional area of $A = 7 \times 10^{-4}$ cm^2. The reverse-saturation current densities at temperature $T = 300$ K of the Schottky diode and p-n junction are 4×10^{-8} A/cm^2 and 3×10^{-12} A/cm^2, respectively.

 (cont.

A forward-bias current of 0.8 mA is required in each diode.

(i) Determine the forward-bias voltage (V_a) required across each diode. [20%]

(ii) If the voltage from part (i) is maintained across each diode, determine the current in each diode if the temperature is increased to $T = 400$ K. Assume $E_g = 1.12$ eV for the p-n junction diode and $\phi_B = 0.82$ V for the Schottky diode. [20%]

END OF PAPER

THIS PAGE IS BLANK

3B5 Semiconductor Engineering: Sheet of Formulae and Constants

Planck's constant	$h = 6.626 \times 10^{-34}$ Js
Reduced Planck's constant	$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ Js
Speed of light in a vacuum	$c = 2.998 \times 10^{8}$ m/s
Mass of electron	$m_{\mathrm{e}} = 9.109 \times 10^{-31}$ kg
Charge of electron	$-e = -1.602 \times 10^{-19}$ C
Permittivity of free space	$\epsilon_0 = 8.854 \times 10^{-12}$ C^2/(Jm)
Boltzmann constant	$k_{\mathrm{B}} = 1.381 \times 10^{-23}$ J/K

de Broglie relation
$$p = \frac{h}{\lambda}$$

Planck's equation
$$E = \hbar\omega$$

Bragg's law
$$n\lambda = 2d\sin\theta$$

Heisenberg's uncertainty principle
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Time-dependent Schrödinger equation
$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V\psi(\mathbf{r},t) = j\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t)$$

Time-independent Schrödinger equation
$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r}) + V\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$$

Laplacian in Cartesian coordinates
$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Laplacian in spherical polar coordinates
$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

Density of states in three-dimensional free electron model
$$g(E)\mathrm{d}E = \frac{V}{2\pi^2\hbar^3}(2m^*)^{\frac{3}{2}}E^{\frac{1}{2}}\mathrm{d}E$$

Fermi–Dirac function
$$f(E) = \frac{1}{\exp\left(\frac{E-E_F}{k_{\mathrm{B}}T}\right) + 1}$$

Law of mass action
$$np = n_i^2$$

Density of electrons in conduction band	$n = N_C \exp\left(-\frac{E_C - E_F}{k_\mathrm{B}T}\right)$
Density of holes in valence band	$p = N_V \exp\left(\frac{E_V - E_F}{k_\mathrm{B}T}\right)$
Current density	$J = \sigma\varepsilon$
Conductivity	$\sigma = ne\mu_\mathrm{e} + pe\mu_\mathrm{h}$
Mobility	$\mu = \frac{q\tau_\mathrm{scatt}}{m^*}$
Continuity equation for electrons	$\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau_\mathrm{e}} + \mu_\mathrm{e}\varepsilon\frac{\partial(\Delta n)}{\partial x} + D_\mathrm{e}\frac{\partial^2(\Delta n)}{\partial x^2}$
Continuity equation for holes	$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau_\mathrm{h}} - \mu_\mathrm{h}\varepsilon\frac{\partial(\Delta p)}{\partial x} + D_\mathrm{h}\frac{\partial^2(\Delta p)}{\partial x^2}$
Poisson equation	$\nabla^2 V = -\frac{\rho}{\epsilon_0\epsilon_r}$
Einstein's relation	$D = \mu\frac{k_\mathrm{B}T}{e}$
Diffusion length	$L = \sqrt{D\tau}$
Debye screening length	$L_\mathrm{D} = \sqrt{\frac{\epsilon_0\epsilon_r k_\mathrm{B}T}{e^2 n_0}}$