3C8 2023/24 MACHINE DESIGN CRIB (M SUTCLIFFE)

EGT2: ENGINEERING TRIPOS PART IIA

Q 1(a)
- contact small compared with geometry (small strains)
- linear elastic
- smooth surfaces
- non-conforming
- frictionless

(b) Circular contact: from data sheet $\delta = \frac{1}{2}\left(\frac{9}{2}\frac{P^2}{E^{*2}R}\right)^{\frac{1}{3}}$

For A $E^* = \frac{E}{2(1-\nu^2)}$, $R = r$, $P = F_A$

$$\Rightarrow \delta_A = \frac{1}{2}\left(\frac{18\,F_A^2(1-\nu^2)^2}{E^2 r}\right)^{\frac{1}{3}}$$

(c) Here $\delta_A = \delta_B = \delta_C = \delta_T$ {this critical step sometimes missed}

For B, $R = 2r$, $P = F_B$

C is the same as A.

$$\delta_A = \delta_B \Rightarrow \frac{F_A^2}{r} = \frac{F_B^2}{2r} \Rightarrow F_B = \sqrt{2}\,F_A$$

$$F_T = 2F_A + F_B = (2+\sqrt{2})\,F_A$$

$$\delta_T = \delta_A = \frac{1}{2}\left(18 \cdot \frac{F_T^2}{(2+\sqrt{2})^2}\frac{(1-\nu^2)^2}{E^2 r}\right)^{\frac{1}{3}}$$

{other forms ok}

(d) Stiffness $= \left.\frac{dF_T}{d\delta_T}\right|_{F=F_T}$ {not F_T/δ_T}

$$\frac{d\delta_T}{dF_T} = \frac{1}{2}\cdot\frac{2}{3}\left(\frac{18\cdot(1-\nu^2)^2}{E^2 r\,(2+\sqrt{2})^2}\right)^{\frac{1}{3}} F_T^{-\frac{1}{3}}$$

$$\text{stiffness} = \frac{1}{d\delta_T/dF_T} = 3\,F_T^{\frac{1}{3}}\left(\frac{E^2 r(2+\sqrt{2})^2}{18\,(1-\nu^2)^2}\right)^{\frac{1}{3}}$$

Q 1 (e) $\tau_{max} = 0.310\, p_0$: at first yield $\tau_{max} = \tau_y$

$$p_0 = \frac{1}{\pi}\left(6\,\frac{P E^{*2}}{R^2}\right)^{\frac{1}{3}} \qquad E^* = \frac{210}{1-0.3^2}\,\frac{1}{2} = 115\,\text{GPa}$$

Need to check critical contact : $\dfrac{F_A}{r^2} > \dfrac{F_B}{(2r)^2} = \dfrac{\sqrt{2}\,F_A}{4r^2}$

{this sometimes missed}

so A is critical

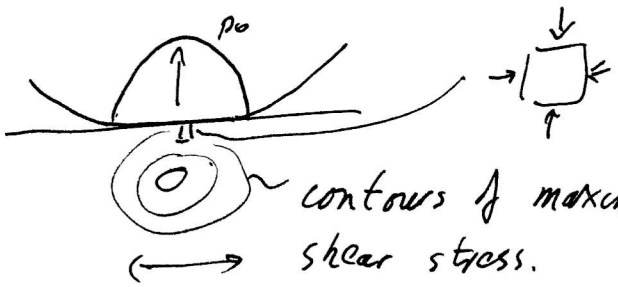
So $$\frac{\tau_y}{0.310} = \frac{1}{\pi}\left(6\,\frac{F_A E^{*2}}{r^2}\right)^{\frac{1}{3}}$$

$$\Rightarrow \left(\frac{\pi \times 500\times 10^{6}}{0.310}\right)^3 = \frac{6 . F_A . (115\times 10^{9})^2}{(10\times 10^{-3})^2} \qquad \text{in SI units}$$

$$\Rightarrow F_A = 164\,N$$

$$F_T = (2+\sqrt{2})F_A = 560\,N$$

(f)



contours of maximum shear stress. Maximum 0.48a below surface.

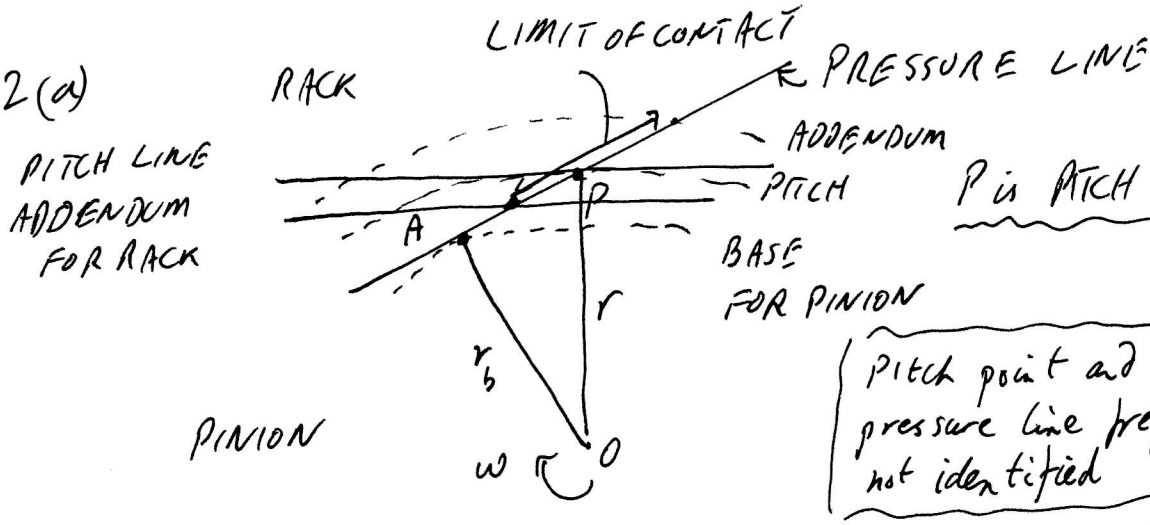
Vertical stress under contact semi-elliptical, Horizontal stress compressive in central region both decay into the contacting solids

For A, $$a = \left(\frac{3PR}{4E^*}\right)^{\frac{1}{3}} = \left(\frac{3 . 164 . 10^{-2}}{4\times 115\times 10^{9}}\right)^{\frac{1}{3}} = 0.22\,\text{mm}$$

so the contact patch is much smaller than the hemisphere radii and it is a good approximation to treat the contacts independently.

{Discussion of stresses not well done}

2(a)

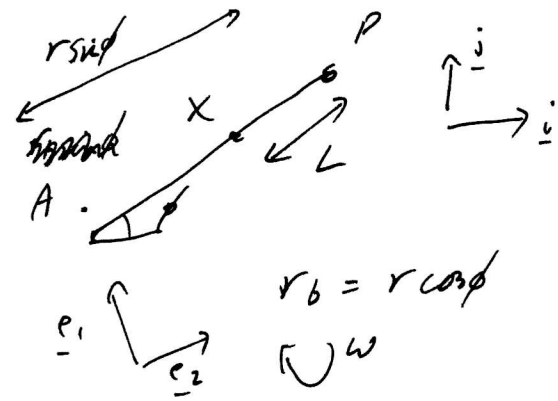


P is PITCH POINT

Pitch point and pressure line frequently not identified

BECAUSE THE INVOLUTE IS AN UNWINDING GEOMETRY FROM THE BASE CIRCLE, THE CENTRE OF CURVATURE OF THE PINION TOOTH ALWAYS LIES AT A. THE RACK PROFILE IS STRAIGHT PERPENDICULAR TO THE PRESSURE LINE. HENCE THE COMMON NORMAL ALWAYS LIES ALONG THE PRESSURE LINE, SO THAT THIS IS THE LOCATION OF THE TOOTH CONTACT.

(b) To maintain contact the component of velocity along the common normal needs to be the same.



On rack:

$$\underline{v}_{xr} = v\underline{i} = v\cos\phi\,\underline{e}_2 - v\sin\phi\,\underline{e}_1$$

On pinion

$$\underline{v}_{xp} = rw\cos\phi\,\underline{e}_2 - \overset{(r\sin\phi - L)}{\cancel{r\sin\phi}}\,w\,\underline{e}_1$$

In $\underline{e}_2$ direction: $v\cos\phi = rw\cos\phi = \frac{v}{w} = r =$ constant

independent of L

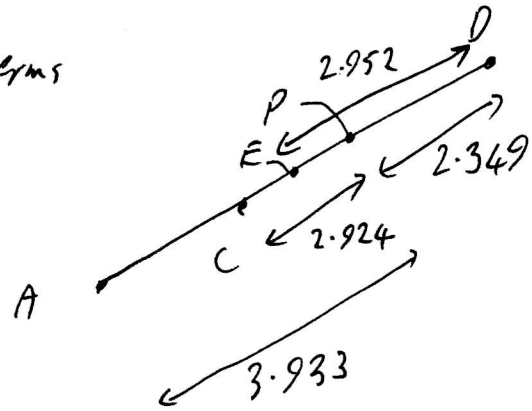
In $\underline{e}_1$ direction $(\underline{v}_{xr} - \underline{v}_{xp})\cdot\underline{e}_1$

$$= -\cancel{v\sin\phi} + \cancel{rw\sin\phi} - wL = -wL \quad \text{since } v = wr$$

Sign depends on convention.

These parts (a) and (b) were not well answered.

2 (c) In terms of m



$m/\sin\phi = 2.924\,m$ $\boxed{PC}$

$r = \frac{mN}{2}$, $r_b = r\sin\phi$
$= 3.933\,m$ $\boxed{AP}$

Data book 2(a) 'l' for $N=23$
$\Rightarrow l = 2.349\,m$ $\boxed{PD}$

$p_b = \pi m\cos\phi = 2.952\,m$ $\boxed{DE}$

Contact ratio $= \frac{2.924+2.349}{2.952} = 1.79$

Need to consider single or double contact, find pinion tooth curvature

Double contact, at C : $\frac{R_C}{m} = \frac{AC}{m} = 3.933 - 2.924 = 1.009$

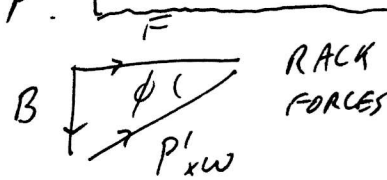
Single contact, at E : $\frac{R_E}{m} = \frac{AE}{m} = 3.933 + 2.349 - 2.952 = 3.33$

Failure : $p_0^2 = \sigma_s^2 = \frac{P'/Q\, E^*}{\pi R}$ (1) where Q is number of contacts assuming force shared

So critical case is double contact since $\frac{R_C \times 2}{R_E} = \frac{1.009\times 2}{3.33} < 1$

$P' = F/w\cos\phi = F/m\cos\phi$

Need to relate rack force to P'. $\boxed{\text{This sometimes missed}}$



Note bearing force B, resultant $P'.w$ is along pressure line.

(1) $\rightarrow (800\times10^6)^2 = \frac{5\times10^3}{Q} \cdot \frac{115\times10^9}{\frac{2m\cos20}{\pi \cdot 1.009m}}$

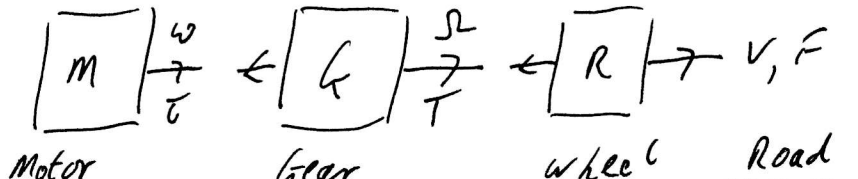
(in SI units)

$\Rightarrow m = 12.2\,mm \rightarrow 13\,mm$ for standard modules

$\boxed{\text{Marks dropped for not choosing standard modules}}$

(d) Rack force doubled : geometry and critical case are unchanged. $p_0 \sim F/m^2$ so $m\times\sqrt{2} = 17.4 \rightarrow 18\,mm$

3(a)



$G = \frac{\omega}{\Omega}$, $F = mgC_R + 0.5\rho C_D A V^2$

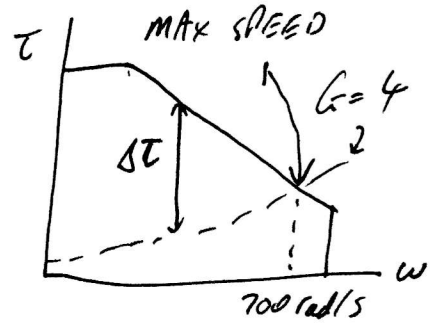
Mostly well done, some calculator errors and omitting the final V calc

Need to convert F, V to ω, τ in load characteristic

$F = \frac{T}{R} = \frac{\tau G}{R}$, $V = \Omega R = \frac{\omega R}{G}$

$\Rightarrow \frac{\tau G}{R} = mgC_R + 0.5\rho C_D A\left(\frac{\omega R}{G}\right)^2$

For $G = 4 \Rightarrow \tau = 15 + 1.22\times10^{-4}\omega^2$



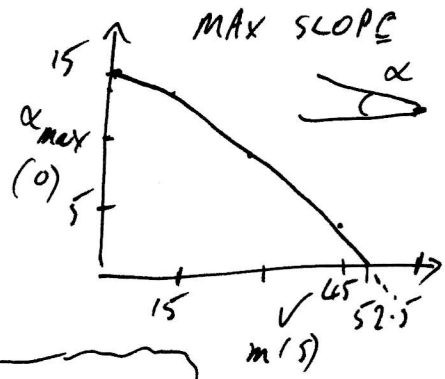
Maximum speed, 700 rad/s $= \frac{700\times R}{G} = 52.5$ m/s

(b) Hill climbing or acceleration due to extra torque ΔT (see fig)

Need to convert to $\Delta F = \frac{\Delta T}{R} = \frac{\Delta\tau G}{R}$. $\Delta F = mg\sin\alpha$ or $\Delta F = ma$

Construct a small table. 108 km/h = 30 m/s

V (m/s)	0	15	30	45	52.5
ω (rad/s)	0	200	400	600	700
ΔT (Nm)	185	180	~~150~~ 115	40	0
ΔF (N)	2470	2400	1530	533	0
α (°)	14.3	13.9	8.8	3	0
a (m s⁻²)	~~0.247~~ 2.47	~~0.24~~ 2.40	~~0.15~~ 1.50	~~0.05~~ 0.50	0



$a = \frac{\Delta v}{\Delta t} \Rightarrow \Delta t = \Delta v / a$ ← This step important

Simple estimate. 0 → 15 m/s, $\bar{a} \approx$ ~~0.24~~ 2.4 m s⁻², Δv = 15 m/s, Δt = ~~3.65~~ 6.3 s

15 → 30 m/s, $\bar{a} \approx$ ~~0.20~~ 2.0 m s⁻², Δv = 15 m/s, Δt = ~~3~~ 7.5 s

Total time 0 → 108 km/h = 13.8 ≈ 14 s

Could improve the estimate with more data points or integrating 1/a with respect to v.

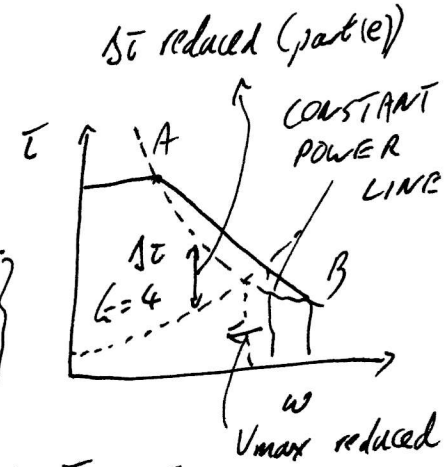
Not necessary to have a dense table to get full marks

(5)

3(c) Power at A and B = 40 kW

Maximum power at intermediate point.

Either draw up a table or analytical:



$P = \tau\omega = (250 - 0.25\omega)\omega$

$\frac{dP}{d\omega} = 250 - 0.5\omega$

Need to justify location of max power

Max when $\omega = 500$ rad/s, $P_{max} = 62.5$ kW, $\tau = 125$ Nm

From load characteristic, Power $= FV = 200V + 0.29V^3$

Putting $V = 56.1$ m/s (202 km/h): $= 200 \cdot 56.1 + 0.29 \times (56.1)^3$

$= 62.6$ kW

So close to P_{MAX} of 62.5 kW. Max speed $= 56.1$ m/s $= 202$ km/h

$G = \frac{\omega R}{V} = \frac{500 \times 0.3}{56.1} = 2.67$

(d) Points A and B both have the same power so the speed is the same.

At B the motor is at the maximum speed so no possibility of going faster on downhill sections.

(e) The constant power line, sketched in the figure above, comes inside the original characteristic.

So for $G=4$ the maximum possible speed is reduced.

Also $\Delta\tau$ and hence acceleration performance are reduced (see above figure).

4 (a)

	Speed Capability	Load Capability	Shaft Locating Accuracy
Hydrostatic	**HIGH**	**HIGH**	**HIGH**
Hydrodynamic	**HIGH**	**HIGH**	**LOW**
Rolling Element	**MEDIUM**	**MEDIUM**	**HIGH**

(b)

	Accommodating Shaft Expansion	Eliminating Bearing Clearance
Fixed-floating	**YES**	**NO**
Cross-section	**NO**	**YES**

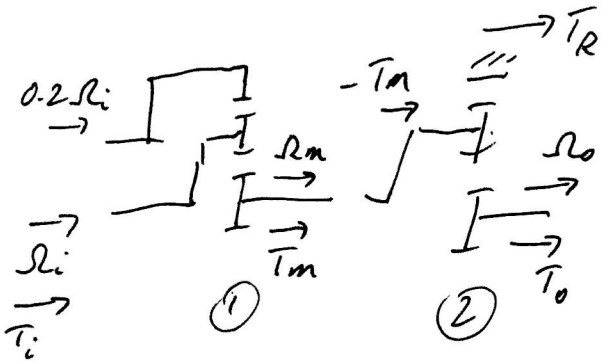
(c) (i) The fixed bearing in a fixed-floating arrangement normally carries axial load but can also carry some radial load. The floating bearing carries radial road only. The floating bearing is normally a deep grove ball bearing, a cylindrical bearing, or a needle bearing. If a roller bearing is required – in comparison to a ball bearing, a roller bearing can carry greater radial load due to line contact – then <u>either cylindrical bearing or needle bearing should be deployed as the floating bearing to carry the main radial load.</u>

(c)(ii) Using the adjusted rating life equation $L_n = a_1 a_{23} (C/P)^p$ (1).

- ✓ Roller bearing → $p = 10/3$
- ✓ Failure probability $n = 1$ → $a_1 = 0.21$
- ✓ Bearing properly lubricated → $a_{23} = 1$
- ✓ 1000 hours, 4000 rpm → L_1 = 1000 hours x 60 min/hour x 4000 rpm = 240 x 10^6 revs = 240 m revs
- ✓ Dynamic load P = 2.5 kN = 2500 N

So, Dynamic load rating C, as per (1) is: <u>$C = (L_1 / a_1 / a_{23})^{1/p}$ x P = 20669.86 N = 20.67 kN.</u>

4 (d) (i)



Torques on shafts

Need to define signs

Epicyclic speed equation: $\omega_S = (1+R)\omega_C - R\omega_A$

① $\Omega_m = (1+R)\Omega_i - 0.2\,\Omega_i$ ② $\Omega_o = (1+R)\Omega_m - R\cancel{\Omega_A}^{\,0}$

Combine: $\Omega_o = (1+R)\left(1+\frac{4}{5}R\right)\Omega_i$. $R=5 \Rightarrow \frac{\Omega_o}{\Omega_i} = 6\times 5 = 30$

(ii) Use virtual power. We can choose any virtual speed set – here put everything except the input and restraint stationary.

This simple step not often seen, though virtual power was used.

Power in $= 0 = \Omega_i' T_i + \Omega_{A2}' T_R$. All other $\Omega' = 0$

① $\Rightarrow \Omega_m' = (1+R)\Omega_i' - R\cancel{\Omega_{A1}'}^{\,0} = 6\Omega_i'$

② $\Rightarrow \cancel{\Omega_o'}^{\,0} = (1+R)\Omega_m' - R\Omega_{A2}' \Rightarrow \Omega_m' = \Omega_{A2}' \times 5/6$

Combine: $\frac{T_R}{T_i} = \left.\frac{-\Omega_i'}{\Omega_{A2}'}\right|_{\Omega'=0} = \frac{-\Omega_m'/6}{\Omega_m' \times \frac{6}{5}} = -5/36$

(iii) Power out $= \eta$ power in. Speeds unchanged.

For epicyclic ①. Input torques unchanged. $\eta = 1 \Rightarrow \frac{T_m}{T_i} = -\left.\frac{\Omega_i}{\Omega_m}\right|_{\Omega_A'=0} = -\frac{1}{6}$

For $\eta \neq 1$, $\frac{T_m}{T_i} = -\eta/6$ (since speeds unchanged and power $= \Omega T$)

For epicyclic ②: Power out $= \eta$ Power in $\Rightarrow -T_o\Omega_o = -T_m\Omega_m\eta$

$\Rightarrow \frac{T_m}{T_o} = 1/\eta\,\frac{\Omega_o}{\Omega_m} = 6/\eta$ (using $\Omega_{A2}=0$)

Equilibrium for ②: $T_R = T_m - T_o \Rightarrow \frac{T_R}{T_i} = \frac{T_m}{T_i} - \frac{T_o}{T_i} = -\frac{\eta}{6} + \left(\frac{\eta}{6}\right)^2$ $\frac{T_o}{T_m}\cdot\frac{T_m}{T_i}$

A tricky part. Marks awarded for understanding how to use η

Check
$\eta = 1$
$\frac{T_R}{T_i} = -\frac{1}{6} + \frac{1}{36} = -5/36$