EGT2 ENGINEERING TRIPOS 2024/25

Module 3C8: MACHINE DESIGN

M. SUTCLIFFE

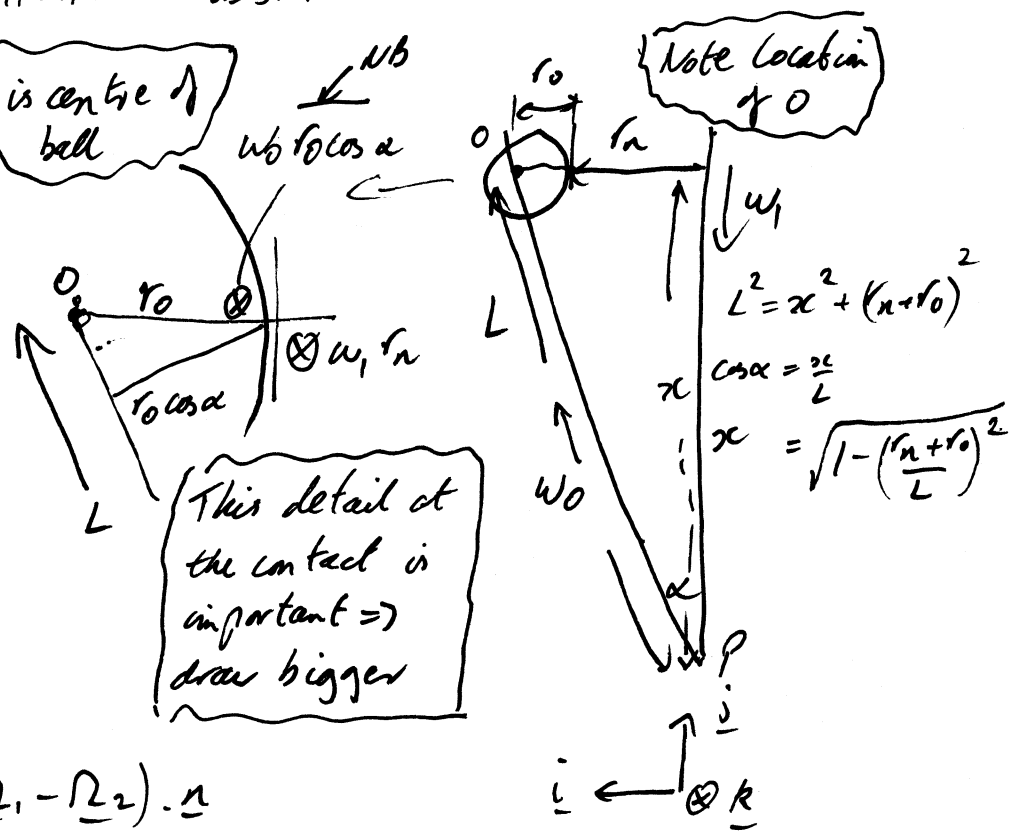
1(a)

No slip at contact

$\Rightarrow \omega_1 r_n = \omega_0 r_0 \cos\alpha$

$$\frac{\omega_0}{\omega_1} = \frac{r_n}{r_0 \cos\alpha}$$

$$\frac{\omega_0}{\omega_1} = \frac{r_n}{r_0} \frac{1}{\sqrt{1-\left(\frac{r_n+r_0}{L}\right)^2}}$$



(b) Spin $\omega_z = (\underline{\Omega}_1 - \underline{\Omega}_2) \cdot \underline{n}$

$\underline{\omega}_1 = -\omega_1 \underline{j}$, $\underline{\omega}_0 = \omega_0(\underline{j}\cos\alpha + \underline{i}\sin\alpha)$, $\underline{n} = \underline{i}$

$\Rightarrow \omega_z = \omega_0 \sin\alpha = \omega_1 \tan\alpha \times \frac{r_n}{r_0}$

(c)

No slip $\Rightarrow \omega_1\left(\frac{r_2+r_3}{2}\right) = \omega_0 r_0 \cos(\beta-\alpha)$

$\underline{n} = \underline{i}\cos\beta - \underline{j}\sin\beta$

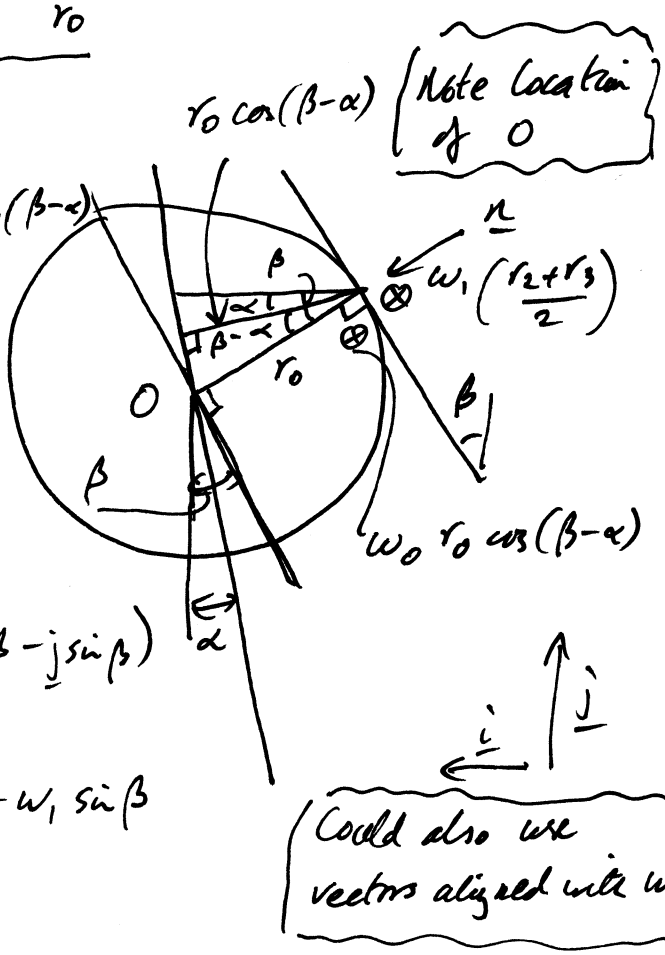
$\omega_2 = (\underline{\omega}_0 - \underline{\omega}_1)\cdot\underline{n}$

$$= \left[\omega_1 \frac{r_2+r_3}{2} \frac{1}{\omega_0 r_0 \cos(\beta-\alpha)} (\underline{j}\cos\alpha + \underline{i}\sin\alpha) + \omega_1 \underline{j}\right] \cdot (\underline{i}\cos\beta - \underline{j}\sin\beta)$$

$$= \frac{\omega_1(r_2+r_3)}{2r_0}\left(-\frac{\cos\alpha\sin\beta + \sin\alpha\cos\beta}{\cos(\beta-\alpha)}\right) - \omega_1\sin\beta$$

$$= -\frac{\omega_1(r_2+r_3)}{2r_0}\tan(\beta-\alpha) - \omega_1\sin\beta$$

or $+ \ldots +$ – note both signs the same.



1(d) Power transmitted $= T_1 \omega_1$

Power lost $= S\Delta v + M\omega_2$

where traction force S given by $Sr = T_1$

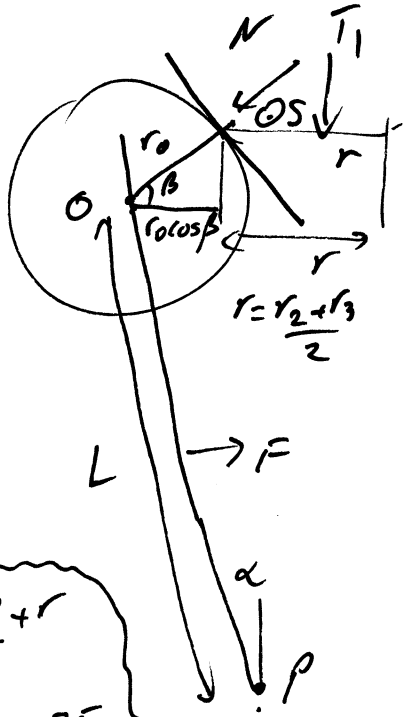
To get normal force N: (This step often wrong)

Moments $\circlearrowleft$ $F \times \frac{L}{2}\cos\alpha = \underbrace{N\cos(\beta-\alpha)}_{\text{component } \perp \text{ to output shaft}} \times L$

$$\Rightarrow N = \frac{FL\cos\alpha}{2L\cos(\beta-\alpha)}$$

$$= \frac{2000\cos 17.5}{2\times\cos 27.5} = 1075\,N$$

$$S = \frac{T_1}{r} = \frac{100}{0.175} = 572\,N$$



$$\sin\alpha = \frac{r_0\cos\beta + r}{L}$$

$$= \frac{0.05/\sqrt{2} + 0.175}{0.7}$$

$$= 0.301$$

$$\alpha = 17.5°$$

This element missed or incorrect

Need to find hertz contact radius a

$$a = \left(\frac{3PR}{4E^*}\right)^{\frac{1}{3}} \text{ for circular contact} = \left(\frac{3\times 1075\times 0.05}{4\times 115\times 10^9}\right)^{\frac{1}{3}} = 705\,\mu m$$

To get Δv, first use $I_F = \frac{F}{\mu N} = \frac{572}{1\times 1075} = 0.53$

From graph in data sheet, for a circular contact, $\frac{e}{a} \simeq 0.68$, $I_m = 0.68$

Also $\omega_2 = \omega_1\left(\frac{0.175}{2\times 0.05}\tan 27.5 - \sin 45\right) = 0.204\,\omega_1$

$$\Delta v = e\omega_2 = 0.68\times 7.05\times 10^{-4}\times 0.204\,\omega_1\,(m) = 6.9\times 10^{-5}\,\omega_1\,(m)$$

Frictional moment $M = I_m \mu N a = 0.68\times 1\times 1075\times 7.05\times 10^{-6} = 0.36\,Nm$

Loss $= S\Delta v + M\omega_2 = (572\times 6.9\times 10^{-5} + 0.36\times 0.204)\omega_1 = 0.11\,\omega_1$

Power transmitted $= T_1\omega_1 = 100\,\omega_1$. $\eta = 1 - \frac{0.11}{100} = \underline{99.89\%}$

Marks were lost for errors in geometry and angles. Big diagrams help here. For the final part the many elements make getting the answer hard so need to explain the method.

2(a) $F = mgC_r + 0.5\rho C_D A v^2$

$= m\ 10 \times 0.005 + 0.5 \times 1.2 \times 0.5 \times 6\, v^2$

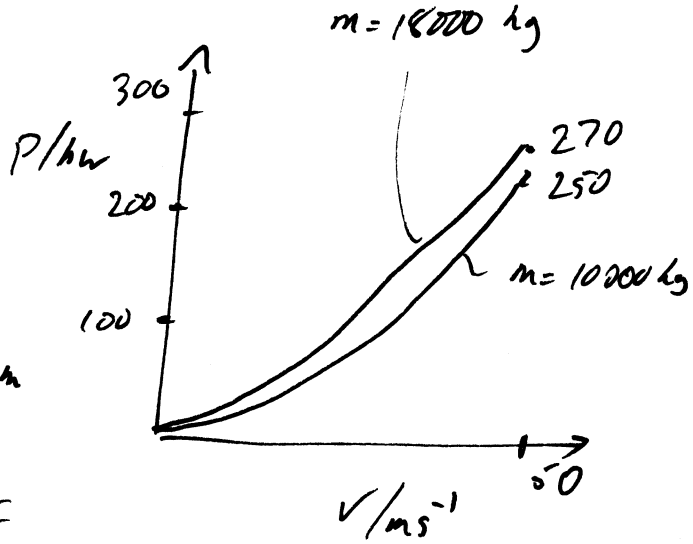
$= \begin{matrix} 500 + 1.8v^2 \\ 900 + 1.8v^2 \end{matrix}$ $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ $\begin{matrix} m = 10000\ kg \\ = 15000\ kg \end{matrix}$

$P = Fv = \begin{matrix} 500v + 1.8v^3 \\ 900v + 1.8v^3 \end{matrix}$ $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ $\begin{matrix} m = 10{,}000\ kg \\ 15{,}000\ kg \end{matrix}$

Need to add some numbers to the sketch

(b)(i) Max speed $\Rightarrow$ max power

$= 225\ kW$



$\Rightarrow \Omega$ in range $4000 \leq \Omega \leq 7000$ rpm

Motor	$\frac{\Omega}{\omega}$	G	$\frac{w}{v}$	R	$\frac{F}{V}$

$G = \frac{\Omega}{w}$ $\qquad v = wR$

Power $= F \cdot v \Rightarrow 900v + 1.8v^3 = 225 \times 10^3$

$v \approx 46.5\ m/s$ (by iteration or use the calculator)

Gear ratio $= \frac{\Omega}{w} = \frac{\left\{\begin{matrix} 4000 \\ 7000 \end{matrix}\right\} \times \frac{2\pi}{60}}{46.5/0.5} = 4.5 \rightarrow 7.9$

Mistakes included errors in going from rpm to rad/s and not identifying a range

2(b)(ii)

$F = 900 + 1.8V^2 + 1.8\times10^5 \sin 0.18$

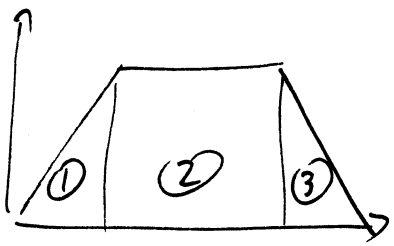
$= 33.1\ kW$

Power $= FV = 99.6\ kW$ $\quad 1770 = 4000 \times \frac{99.4}{225}$ from graph

Need motor speed > 1770 rpm approx to get this power

$\Rightarrow G \geq 31$ {need to include inequality}

(b)(iii)



③ Braking – no power used

② Constant speed @ 20 m/s

$F = 900 + 1.8V^2 = 1620\ N$

Power $= FV$. Work done = Power × time $= 1620 \times 20 \times 600 = 13.0\ MJ$

① Acceleration $v = t/5$, $a = 0.2\ ms^{-2}$ $\quad \frac{dx}{dt} = v = t/5$

$F = 900 + 1.8v^2 + ma = 900 + 0.072V^2 + 3600 = 4500 + 0.072V^2$

$$W.D. = \int F\,dx = \int_{t=0}^{t=100} \left[4500 + 0.072\left(\frac{t}{5}\right)^2\right] \cdot \frac{t}{5}\,dt = \left[\frac{4500}{5}\frac{t^2}{2} + \frac{0.072}{5^2}\frac{t^4}{4}\right]_0^{100}$$

$= 4.51\ MJ$

Total work $= 13.0 + 4.5 = 17.5\ MJ$

Required power $= \frac{17.5}{0.95} = 18.6\ MJ$

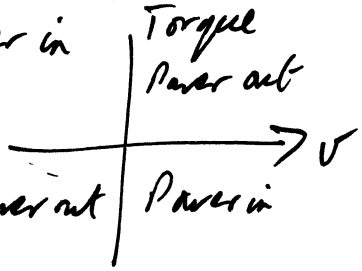
{Some did numerical integration – here an analytical solution is easier}

(c) v. high speed – magnetic hysteresis and eddy current losses

v. low speed – when torque is high, current is high giving high heating losses. when torque is low bearing losses become significant

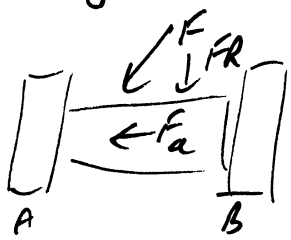
(d) Using convention +ve torque is torque applied by motor

Quadrants 2 & 4 with power in allow regenerative braking.



3 (a)(i) Advantage – can take axial load
Disadvantages – requires careful fitting and problematic with thermal expansion

(ii)



A closes, B opens $\Rightarrow F_{aB} = 0$

$F_{aA} = F\sin\alpha$

$F_R = F\cos\alpha$, $\Rightarrow F_{rA} = F_{rB} = \frac{F\cos\alpha}{2}$

(iii) $F_a = 5.16\,kN$, $F_r = 9.66\,kN$ for bearing A which is critical

$P = 1.0 P_r + 0.3 P_a = 11.2\,kN$

$L = 6000 \times 60 \times 1000/10^6 = 240$

$p = 10/3$, $a_{23} = 1$, $a_1 = 0.44$

$L = a_1 a_{23} (C/P)^p \Rightarrow C = (546)^{3/10} \times 11.2 = 74.2\,kN$

Mostly well answered

Minimum outer diameter taper roller bearing with inner diameter from datasheet is 33110 bearing ($D = 95\,mm$, $C = 85.8\,kN$)

(b)(i) Although the speed ratio is lower, the planet carrier is stationary make the planet bearings, which can be heavily loaded, more easy to lubricate. The difference in direction may also be useful.

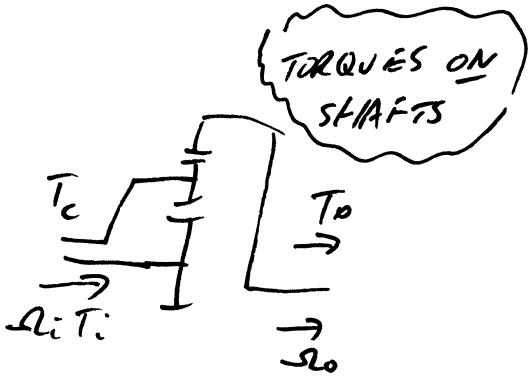
(ii)
Epicyclic speed eqn:

$\omega_S = (1+R)\omega_C - R\omega_A$ where $R = \frac{A}{S}$

$\frac{A}{S} = \frac{r_A}{r_S} = \frac{\frac{3}{2}N}{\frac{1}{2}N} = 3$

$\omega_C = 0 \Rightarrow \frac{\Omega_o}{\Omega_i} = \frac{\omega_A}{\omega_S} = -\frac{1}{R} = -\frac{1}{3}$

3 (b) (iii)



$\eta = 0.98 \Rightarrow \left|\frac{T_o \Omega_o}{T_i \Omega_i}\right| = 0.98$

(no power through stationary carrier)

Here $T_o \Omega_o < 0$ as power out of output.

Speed ratio is unchanged $\Rightarrow$ $-\frac{T_o \Omega_o}{T_i \Omega_i} = 0.98$, $T_o = -0.98 \frac{\Omega_i}{\Omega_o} = 2.94\, T_i$

Equilibrium $T_c + T_i + T_o = 0 \Rightarrow T_c = -T_i - T_o = -3.94\, T_i$

{Care needed to get signs right} {Cannot use virtual power due to losses}

(iv) $\Omega_c = \frac{\Omega_i}{2}$. Using epicyclic speed equation again

$$\Omega_i = 4\Omega_c - 3\Omega_o$$
$$= 2\Omega_i - 3\Omega_o$$
$$\Rightarrow \Omega_o = \Omega_i / 3$$

No losses, now we can use virtual power:

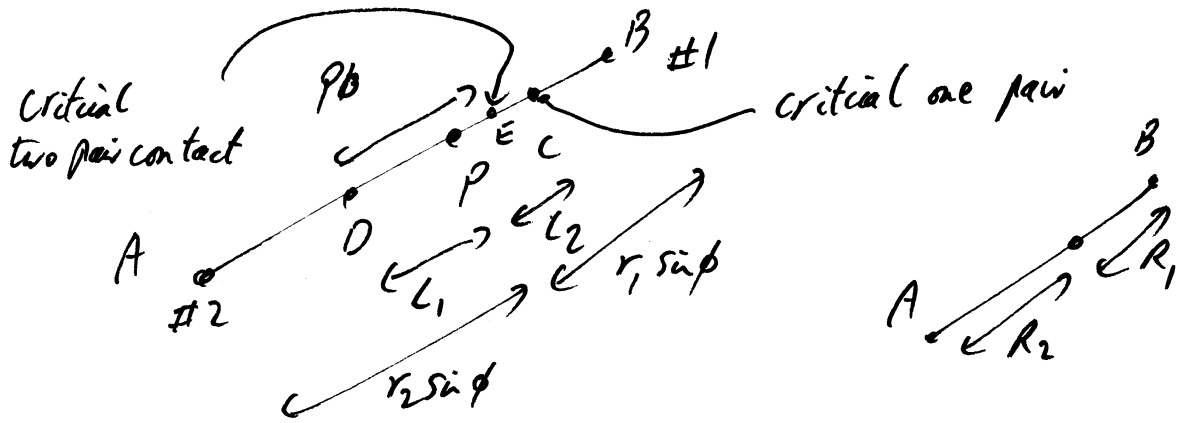
$$T_i \Omega_i' + T_c \Omega_c' + T_o \Omega_o' = 0$$

Put $\Omega_c' = 0 \Rightarrow \frac{T_o}{T_i} = -\left(\frac{\Omega_i'}{\Omega_o'}\right)_{\Omega_c' = 0} = 3$

$$T_o = 3 T_i$$

{Virtual power makes the algebra easier}

4 (a)



Using standard teeth with $a = m$ and $\phi = 20°$

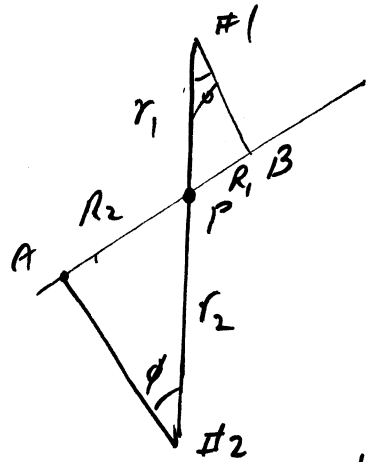
All the relevant dimensions depend only on N, α and m.

Specifically $r_1 = \frac{mN}{2}$, $r_2 = \frac{m\alpha N}{2}$, $\frac{L_1}{m}$ and $\frac{L_2}{m}$ are functions only of N and αN from data sheet

Finally $p_b = \pi m \cos\phi$

With R_1 and $R_2 = f_n(N, \alpha, m)$ and $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$, $R = f_n(N, \alpha, m)$

(b)



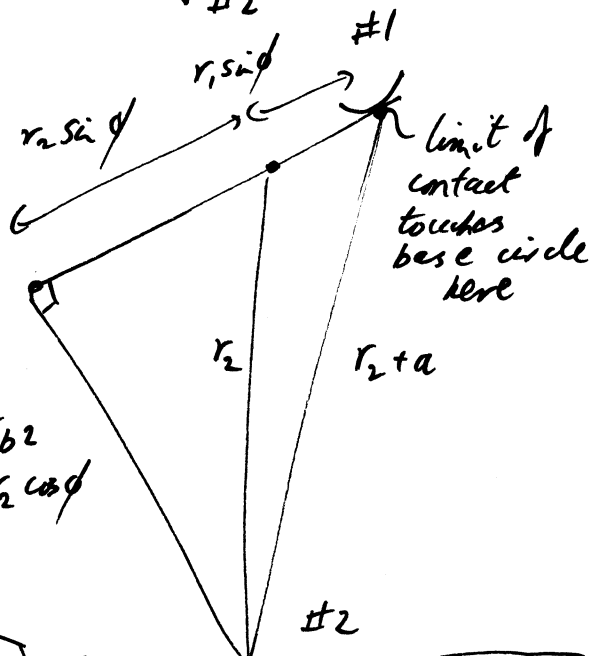
$$R_1 = r_1 \sin\phi = \frac{mN}{2}\sin\phi$$

$$R_2 = r_2 \sin\phi = \frac{\alpha m N}{2}\sin\phi$$

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2} = \underline{\frac{\alpha m N \sin\phi}{2(\alpha+1)}}$$

(c) As contact approaches a base circle R_1 or $R_2 \to 0$ so $R = \frac{R_1 R_2}{R_1 + R_2} \to 0$

Worst case will be considering the limit of contact on the smaller gear.



$$(r_2\cos\phi)^2 + ((r_1 + r_2)\sin\phi)^2 = (r_2 + a)^2 \quad \left(r = \frac{mN}{2}\right)$$

$$(\alpha\cos\phi)^2 + (1+\alpha)^2\sin^2\phi = \left(\alpha + \frac{2}{N}\right)^2$$

With $\alpha = 2$, $\phi = 20 \Rightarrow N = 14.2$

Minimum case $\underline{N = 15}$

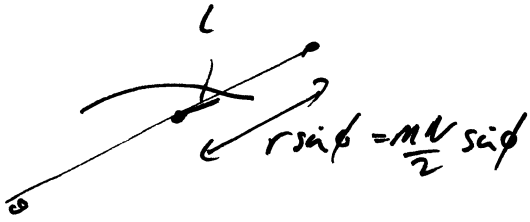
Could also use L formula - need to use the right gear to check

These parts (a) (b) and (c) not done as well as expected

(d) For small N the situation of part (c) occurs and $R \to 0$ for $N = 16.2$

For large N the length L is small compared with $r\sin\phi$ and the situation of part (b) occurs with

$$R \to \frac{\alpha m N \sin\phi}{2\alpha+1} = 0.11\,N\,m$$



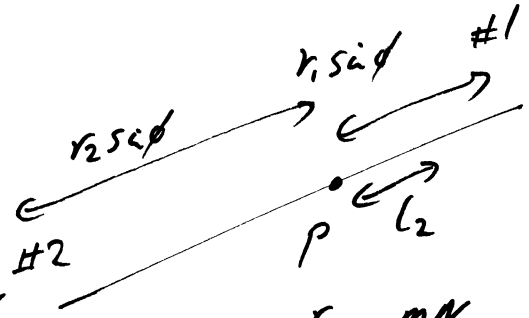
For intermediate values:

$$\frac{L_2}{m} = \sqrt{0.02926\,(\alpha N)^2 + \alpha N + 1} - 0.1710\,\alpha N$$

$$R_1 = r_1\sin\phi - L_2$$

$$R_2 = r_2\sin\phi + L_2$$

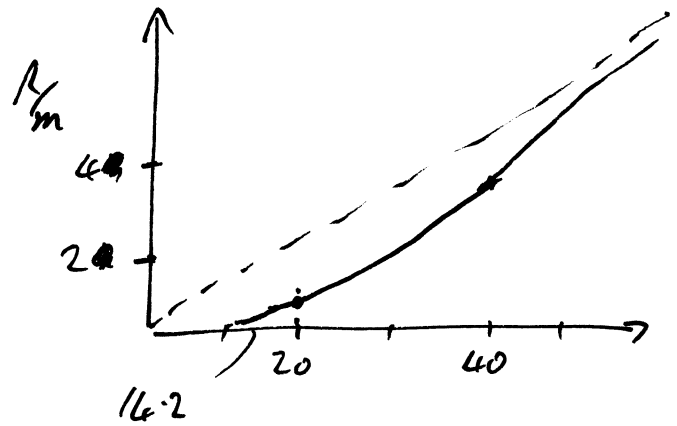
$$R = \frac{R_1 R_2}{R_1 + R_2}$$



$$r_1 = \frac{mN}{2}$$

$$r_2 = \alpha\frac{mN}{2}$$

N	16.2	20	40
L_2/m	[illegible]	2.52	2.69
R_1/m	[illegible]	0.90	4.15
R_2/m		9.36	16.4
R/m	0	0.81	3.31



Not well answered. Good marks were awarded when students used parts (b) and (c) to get the rough form

(e) For small N contact is near a base circle, To decide if single or double pair contact is important compare $\frac{P}{R_1}$ with $\frac{P/2}{R_2}$
↑ single ↑ double

As N increases, R for both single and double contact will tend to the pitch point values.

So the double contact case will be better, with a larger value of R than the corresponding single point case. At the point where the two cases give the same result, there will be a jump by a factor ×2 in R.