EGT2 Engineering Tripos IIA, 2026

3C8: Machine Design (M Sutcliffe)

1. (a) Crossed cylinders, use datasheet elliptical formula.

$A = 0.3\,m$, $B = 0.55\,m$, $R = (AB)^{-\frac{1}{2}} = 0.406\,m$ $[R_e = 0.396\,m]$

$b/a = \left(\frac{A}{B}\right)^{2/3} = 0.668$

Many candidates used the formula for parallel cylinders - this didn't incur loss of consequential mark errors

$$P = \frac{98 \times 1000 \times 9.81}{12} = 80.1\,kN$$

Using circular contact formula as per the datasheet:

$$p_0 = \frac{1}{\pi}\left(\frac{6PE^{*2}}{R^2}\right)^{\frac{1}{3}} = \frac{1}{\pi}\left(\frac{6 \times 80.1 \times 10^3 \times (115 \times 10^9)^2}{(0.406)^2}\right)^{\frac{1}{3}} = 1.08\,GPa$$

- the same at all contacts

(b) $$a = \left(\frac{3PR}{4E^*}\right)^{\frac{1}{3}} = \left(\frac{3 \times 80.1 \times 10^3 \times 0.406}{4 \times 115 \times 10^9}\right)^{\frac{1}{3}} = 5.96 \times 10^{-3}\,m$$

$$\delta = a^2/R = (5.96 \times 10^{-3})^2 / 0.406 = 8.75 \times 10^{-5}\,m$$

this normally missed

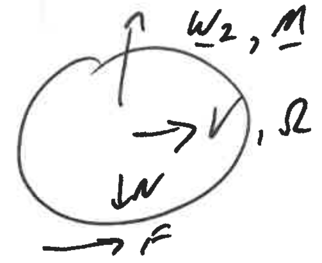
Surface deflections on wheel and rail the same, each equal to $\frac{1}{2}$ the approach of centres $= 43.8\,\mu m$

(c) $\tau_{max} = 0.310\,p_0$ at $z = 0.48a$

$= 335\,MPa$ at a depth of $2.86\,mm$

The same for each wheel-rail contact

1(e)



<u>Kinematics</u>: $\Omega = V/R \quad (R = 0.55\,m)$

$w_2 = (\underline{\Omega}_1 - \underline{\Omega}_2)\cdot\underline{1} \approx \phi\,\Omega$

where $\phi = \frac{2\pi}{180}$ rad

$\Delta v = e\,w_2 = e\phi\,\Omega$

Power transmitted $= 8FV$

Power lost $= \underbrace{8F\Delta v + 8Mw_2}_{\text{driven}} + \underbrace{4Mw_2}_{\text{non-driven}}$

this equation often missed

For non-driven: $e = 0$, $I_F = 0$, $I_m = 0.665$

For driven: $I_F = 0.75 \Rightarrow e = 0.74$, $I_m = 0.33$

$F = I_F\mu N$, $M = I_m\mu N\sqrt{ab}$ where $\sqrt{ab} = 5.96\times10^{-3}\,m$

$$\frac{\text{Power lost}}{\text{power transmitted}} = \frac{8(I_F\cancel{\mu}\cancel{N})\frac{e}{\sqrt{ab}}\phi\cancel{\Omega} + 8(I_m\cancel{\mu}\cancel{N}\frac{\sqrt{ab}}{\sqrt{ab}}\phi\cancel{\Omega}) + 4I_m\cancel{\mu}\cancel{N}\frac{\sqrt{ab}}{\sqrt{ab}}\phi\cancel{\Omega}}{8(I_F\cancel{\mu}\cancel{N})\cancel{\Omega}R/\sqrt{ab}}$$

driven → F Δv I_m driven w_2 non-driven

F

÷ top and bottom by $\sqrt{ab}$

$$= \frac{[8\cdot0.75\cdot0.74 + 8\cdot0.33 + 4\cdot0.665]\cdot\frac{2\pi}{180}}{8\times0.75\times0.55/5.96\times10^{-3}}$$

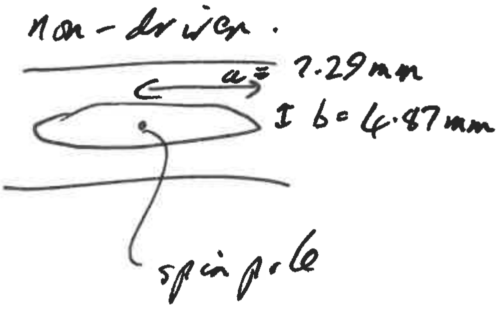
ϕ

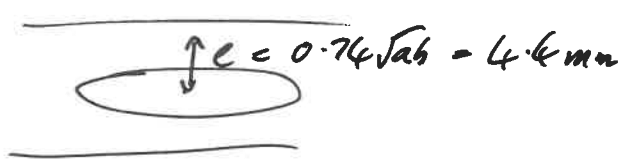
$= 6.1\times10^{-4}$

$\eta = 1 - 6.1\times10^{-4} = \underline{99.939\,\%}$

marks awarded for including the various steps in the calc

(f)

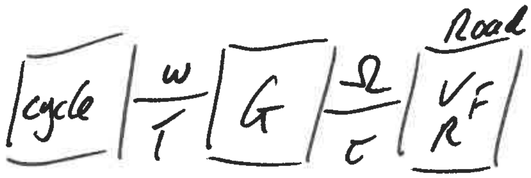


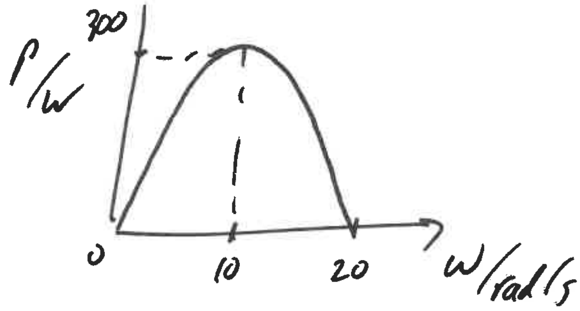


$\sqrt{ab} = 5.96\times10^{-3}\,m \quad \frac{b}{a} = 0.668 \Rightarrow b = 4.87\,mm,\ a = 7.29\,mm$

Same for both contacts.

2. (a)





$T = 60 - 3\omega$,

$\text{Power} = T\omega = 60\omega - 3\omega^2$

Max when $0 = 60 - 6\omega$

$\Rightarrow \omega = 10 \text{ rad/s}, \; P = 300 \text{ W}$

(b) Min time $\Rightarrow$ max V $\Rightarrow$ max Power

So $\omega = 10$ rad/s, $P = 300$ W

$\alpha = 0$ $\quad P = \frac{1}{2}\rho C_D A V^2 \times V \Rightarrow 300 = \frac{1}{2} \cdot 1.2 \cdot 0.8 \cdot 0.5 V^3$

$\Rightarrow V = 10.8$ m/s

$G = \frac{\Omega}{\omega} = \frac{V}{R\omega} = \frac{10.8}{0.35 \times 10} = 3.08$

$\alpha = 0.05$ $\quad P = (mg\alpha + \frac{1}{2}\rho C_D A V^2) \cdot V \Rightarrow 300 = 50V + 0.24V^3$

$\Rightarrow V = 5.29$ m/s

$G = \frac{5.29}{3.5} = 1.51$ $\qquad$ Generally these parts were well done

$\text{Minimum time} = \sum \frac{\text{Distance}}{\text{speed}} = \frac{5000}{10.8} + \frac{5000}{5.29} = 1408 \text{ s}$

(c) The rated torque is the torque that can be continuously operated without overheating. This rated torque can be exceeded for a short while, prolonged use requires higher current, overheating, damage to the permanent magnets, insulation and other parts, so reducing the life or causing permanent damage.

2 (d)

Maximum acceleration when the extra torque above the load is maximum.

due to load

This occurs at zero speed, when $F=0$, $T = 80\,\mathrm{Nm}$

This often missed, 20 from motor 60 from cyclist

$$\Delta F V = \omega T$$

$$\Rightarrow \Delta F = \frac{\omega T}{V} = \frac{\omega T}{\Omega R} = \frac{T}{GR} = \frac{80}{1.1 \times 0.35}\,\mathrm{N}$$

$$a = \frac{\Delta F}{m} = \frac{80}{1.1 \times 0.35 \times 100} = 2.08\ \mathrm{ms^{-2}}$$

(e) $$\frac{dv}{dt} = \frac{F}{m} = \frac{\frac{1}{2}\rho C_D A v^2}{m} = 0.0024 v^2$$

$$\int_{v_0}^{v} \frac{dv'}{v'^2} = \int_0^t 0.0024\, dt'$$

$$\left[-\frac{1}{v'}\right]_{v_0}^{v} = 0.0024 t$$

$$\frac{1}{v} - \frac{1}{v_0} = 0.0024 t$$

$v_0 = 10\,\mathrm{ms^{-1}}$

$$v = \frac{ds}{dt} = \left(0.0024 t + \frac{1}{v_0}\right)^{-1}$$

Boundary condition

$v = 3 \Rightarrow t = 97.2\,\mathrm{s}$

$$s = \int v\,dt = \int_0^{97.2} \left(0.0024 t + \frac{1}{v_0}\right)^{-1} dt$$

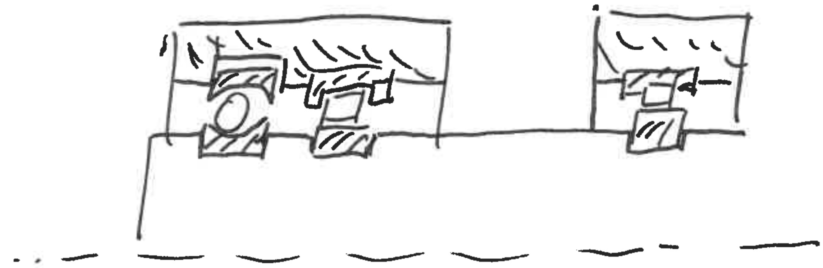
$$= \frac{\left[\ln\left(0.0024 t + \frac{1}{v_0}\right)\right]_0^{97.2}}{0.0024}$$

$$= 959 - \cancel{478}\ 458 \approx 500\,\mathrm{m}$$

$$= \cancel{481\,\mathrm{m}}\ \cancel{481\,\mathrm{m}}$$

This integration wasn't done so well. Several students used numerical integration successfully.

3. (a) (i)



Points to note:

- inner races all positively located on the shaft
- outer races positively located on housing
- rollers only located on outer races, can move axially relative to inner races
- gap provided radially on deep groove bearing

(ii) Design of cylindrical roller bearings allows axial movement so can accommodate shaft thermal expansion.

The deep groove bearing isn't designed to accommodate shaft thermal expansion, but an axial clearance between the outer race and housing could be provided to allow movement of the bearing relative to the housing.

(iii) $L = a_1 a_{23} \left(\frac{C}{P}\right)^p$ where $p = 10/3$, $a_1 = 0.21$, $a_{23} = 1$, $L = 0.02$

C_a for NU 310 EC = 110 kN $\Rightarrow$ $P = 223$ kN

Static load rating $C_0 = 112$ kN. So need to keep the load below 112 kN, not 223 kN, to avoid brinelling.

(iv) Pros - higher axial load capacity
- improved axial stiffness

Cons - higher friction
- increased complexity

3 (b) (i) The assembly rotates as a rigid body

$\Omega_0 = \Omega_c$

All torques are the same, independent of speed.
Return to this at end.

This point normally missed.

(ii) Use tabular approach

	Ω_c	Ω_s	Ω_0
First put $\Omega_c = 0$	0	X	$+X\lambda$
Superimpose rotn Y	Y	$Y+X$	$Y+\lambda X$

$\lambda = \frac{A}{B}\cdot\frac{D}{E}$

With $\Omega_s = 0 \Rightarrow Y = -X \Rightarrow \dfrac{\Omega_0}{\Omega_c} = \dfrac{Y - \lambda Y}{Y} = 1 - \dfrac{AD}{BE}$

[Check $\Omega_s = \Omega_c \Rightarrow X = 0 \Rightarrow \Omega_0 = \Omega_c$]

(iii) $\Omega_s = -\Omega_c/2 \Rightarrow Y + X = -\frac{Y}{2} = \frac{3Y}{2} = -X$

$$\frac{\Omega_0}{\Omega_c} = \frac{Y - \frac{3\lambda}{2}Y}{Y} = 1 - \frac{3}{2}\lambda = 1 - \frac{3}{2}\frac{AD}{BE}$$

Now consider torques: Virtual work: $\Omega_c' T_c + \Omega_0' T_0 + \Omega_s' T_s = 0$

$$\Rightarrow \frac{T_0}{T_c} = -\left(\frac{\Omega_c'}{\Omega_0'}\right)_{\Omega_s'=0} = \frac{1}{1 - \frac{AD}{BE}} = \frac{BE}{BE - AD}$$

from (ii)

Not such a well answered question.
Need to set up table carefully.

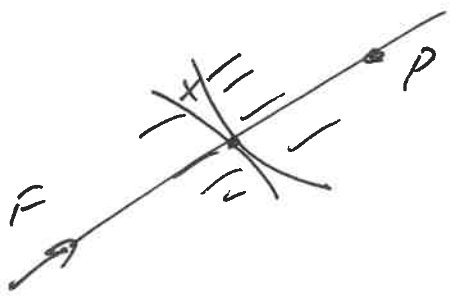
4 (a) - lubricant acts to separate surfaces, aiming for a film thicker than the gear roughness

- reduces friction and wear

- also acts as a coolant

- may be difficult ~~to use~~ for eg. polymer gears or space applications

(b)



- Force acts along pressure line → transmitted power

- local sliding speed is proportional to the distance from the pitch point

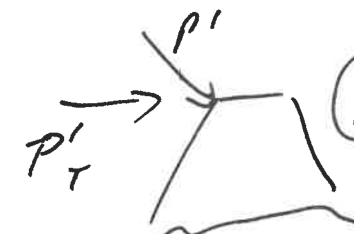
- local friction force depends on normal force and local friction coefficient

- Frictional power given by product of force and velocity

- need to average over meshing ~~ob~~ cycle

- as N increases, this contact effective friction coefficient becomes a small fraction of the local friction coefficient μ, see the data sheet formula ~~[illegible]~~ $\mu\pi\left(\frac{1}{N_1}+\frac{1}{N_2}\right)$

4 (c) (i)



$P'_T = P'\cos\phi$

$r = \frac{mN}{2}$

$P' w r \cos\phi = Q$

$$\sigma_b = \frac{P'_T}{Jm} = \frac{Q/wr}{Jm} = \frac{Q\ 2}{Jw\,m^2 N}$$

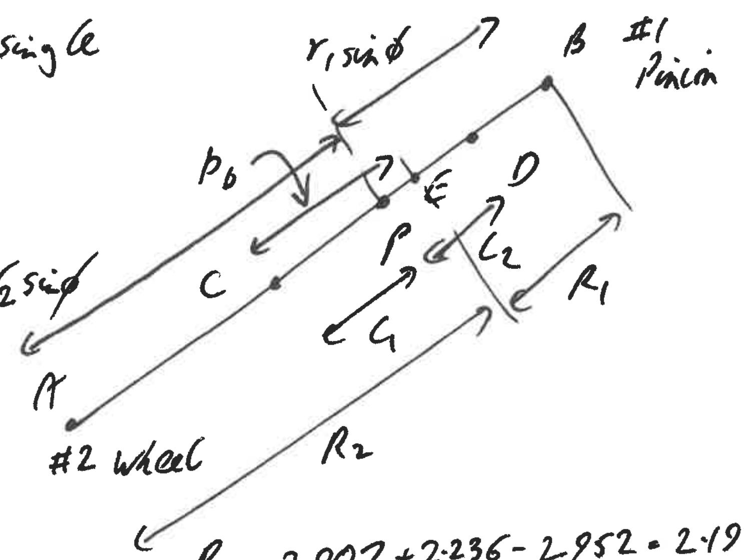
For the pinion, $N = 17$, $J = 0.225$ (no load sharing)

$$\Rightarrow Q_p = \sigma_b \cdot J \cdot 4m^3 N_p / 2 = 300\times10^6 \cdot 0.225 \cdot 4 \cdot (4\times10^{-3})^3 \cdot 17/2$$

$w = 4m$

$$= \underline{147\ \text{Nm}}$$

(ii) E is critical condition for single pair contact.



$R_1 = r_1\sin\phi + L_1 - p_b$

$R_2 = r_1\sin\phi + r_2\sin\phi - R_1$

$\frac{r_1\sin\phi}{m} = \frac{\cancel{m}N_1}{2}\frac{\sin\phi}{\cancel{m}} = 2.907$

$\frac{r_2\sin\phi}{m} = 10.632$

$p_b/m = \pi\cos\phi = 2.952$

$\frac{L_1}{m} = \sqrt{0.02924 \cdot 17^2 + 17 + 1} - 0.1710 \cdot 17 = 2.236$

$\frac{L_2}{m} = \quad " \ \{G1\} = 2.638$

$\frac{R_1}{m} = 2.907 + 2.236 - 2.952 = 2.191$

$\frac{R_2}{m} = 2.907 + 10.632 - 2.191 = 11.148$

$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \cancel{0.566}\,m = \cancel{2.78}$ $\quad 1.831 \quad 7.324$ mm

(iii) Now critical point is D - no load sharing assumed.

$R_1 = r_1\sin\phi - L_2 = 0.269m$, $R_2 = r_1\sin\phi + r_2\sin\phi - R_1 = 13.07m$, $R = 1.054$ mm

Line contact $\quad p_0 = \sigma_s = \left(\frac{P'E^*}{\pi R}\right)^{\frac{1}{2}} = \left(\frac{Q\,E^*}{\pi R\,4m \cdot mN/2\,\cos\phi}\right)^{\frac{1}{2}}$

$$\Rightarrow Q = (800\times10^6)^2 \cdot \pi \cdot \underset{1.054}{\cancel{7.324}}\times10^{-3} \cdot 2 \cdot (4\times10^{-3})^2 \cdot 17 \cdot \cos 20 / 115\times10^9 = \underline{9.42\ \text{Nm}}$$

$R \sim m \quad \sim m^2$

(iv) All dimensions including r and R scale with m. $\quad \underline{R\times 2}$

The allowed torques in (i) and (ii) scale with m^3 $\quad \underline{Q\times 8}$