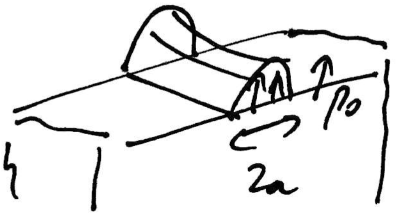
Engineering IIA, 3C8, Machine Design, 2020/21
Crib (Michael Sutcliffe)

1. (a) (i)



- Semielliptical pressure.
- line contact

$p' = P/L$, $a = 2\left(\frac{PR}{L\pi E^*}\right)^{\frac{1}{2}}$

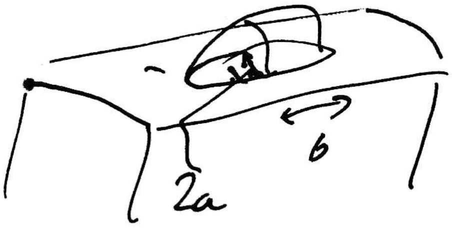
$R = r_w$, $p_0 = \left(\frac{PE^*}{L\pi R}\right)^{\frac{1}{2}}$

(ii)

- Assume contact is in middle ignoring edge effects
- semi-elliptical pressure profile
- circular patch

$R = r_w$, $a = \left(\frac{3PR}{4E^*}\right)^{\frac{1}{3}}$, $p_0 = \frac{1}{\pi}\left(\frac{6PE^{*2}}{R^2}\right)^{\frac{1}{3}}$

(iii)



- Mildly elliptical, elongated along rail

Actually this is a mistake and the patch should be elongated across the rail

$B = \frac{1}{r_w}$, $A = \frac{1}{3r_w}$ $\frac{b}{a} = (3)^{\frac{2}{3}}$ $R = \sqrt{3}\,r_w$

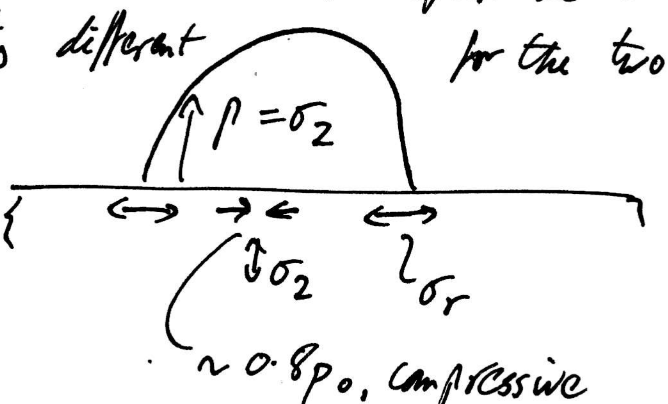
p_0 as for (ii) $ab = \left(\frac{3PR}{4E^*}\right)^{\frac{2}{3}}$

Comment: more detail given above than needed but decent sketches of the requested pressure distribution were often missing.

1 (b)

Circular contact ⇒ circular symmetry

The pressure distribution at the surface on both parts is the same – the semi-elliptical shape discussed in (a). At the surface in-plane compression acts in the middle, tension at the edges. The radial stress ~~or~~ σ_r depends on ν so could be very slightly different for the two steels.



For aluminium rails E^* is smaller so the contact patch increases in size and p_0 falls. Again the normal stresses are semi-elliptical and the same in the two components at the surface. σ_r could be a bit different due to different Poisson's ratios.

Comment: again more detail given than needed. People tended not to comment particularly on the surface stresses as requested.

②

1(c) (i) For the flat there isn't a 'centre' - it is at ∞ - and so the equation blows up with the $\ln\left(\frac{4R}{b}\right)$ term.

(ii) Contact stiffness $= \left(\frac{d\delta}{dP}\right)^{-1}$

this part sometimes missed

$$R = r_w\,,\; b = 2\left(\frac{PR}{\pi L E^*}\right)^{\frac{1}{2}} \Rightarrow \frac{db}{dP} = 2\left(\frac{R}{\pi L E^*}\right)^{\frac{1}{2}} \frac{1}{2} P^{-\frac{1}{2}} = \frac{1}{2}\frac{b}{P}$$

$$\delta = \frac{4P}{\pi L}\left(\frac{2}{E^*}\left(\ln\left(\frac{4R}{b}\right) - \frac{1}{2}\right)\right)$$

$\ln\frac{4R}{b} = -\ln\frac{b}{4R}$

$$\frac{d\delta}{dP} = \frac{4}{\pi L}\left(\frac{2}{E^*}\left(\ln\left(\frac{4R}{b}\right) - \frac{1}{2}\right)\right) + \frac{4P}{\pi L}\cdot\frac{2}{E^*}\left(-\frac{4R}{b}\cdot\frac{1}{4R}\cdot\frac{1}{2}\frac{b}{P}\right)$$

$$= \frac{8}{\pi L E^*}\left(\ln\left(\frac{2\cancel{4} r_w}{\cancel{2}\left(\frac{P\cdot r_w}{\pi L E^*}\right)^{\frac{1}{2}}}\right) - 1\right)$$

- answer is inverse of this

(iii) $P = 100 \times 1000 \times 9{\cdot}81/12 = 8{\cdot}175 \times 10^4\ N$

$L = 0{\cdot}1\,m$

$r_w = 0{\cdot}5\,m$

$E^* = 115 \times 10^9\ Nm^{-2}$

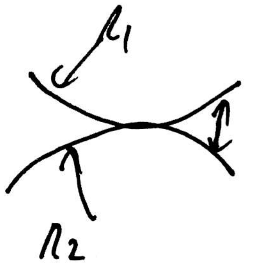
$$\frac{d\delta}{dP} = \frac{1}{115 \times 10^9}\cdot\frac{8}{\pi \times 0{\cdot}1}\left(\ln\left(\frac{\overset{2}{\cancel{4}} \times 0.5}{\left(\frac{8{\cdot}175 \times 10^4 \times 0{\cdot}5}{\pi \times 0{\cdot}1 \times 115 \times 10^9}\right)^{\frac{1}{2}}}\right) - 1\right) = 1{\cdot}29 \times 10^{-9}\ m/N$$

Stiffness $\frac{dP}{d\delta} = \cancel{773}\ 7.73 \times 10^8\ N/m$

(iv) More force on outside rail, less on inner rail, so increased stiffness on outside rail as contact area rises.

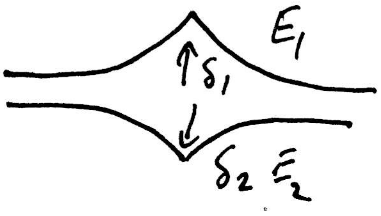
Comment: less popular question, not a very high average with quite a few parts, each with their pitfalls.

2(a) (i)



Contact defined by gap

$h \sim \frac{1}{R} \Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$



Effective δ given by sum of effects of E_1 and E_2

$\delta \sim \frac{F}{E} = \frac{1}{E^*} = \frac{1}{E_1} + \frac{1}{E_2}$

(ii) Data sheet

$\bar{p} = \frac{2}{3} p_0 = \frac{2}{3\pi}\left(\frac{6PE^{*2}}{R^2}\right)^{\frac{1}{3}}$

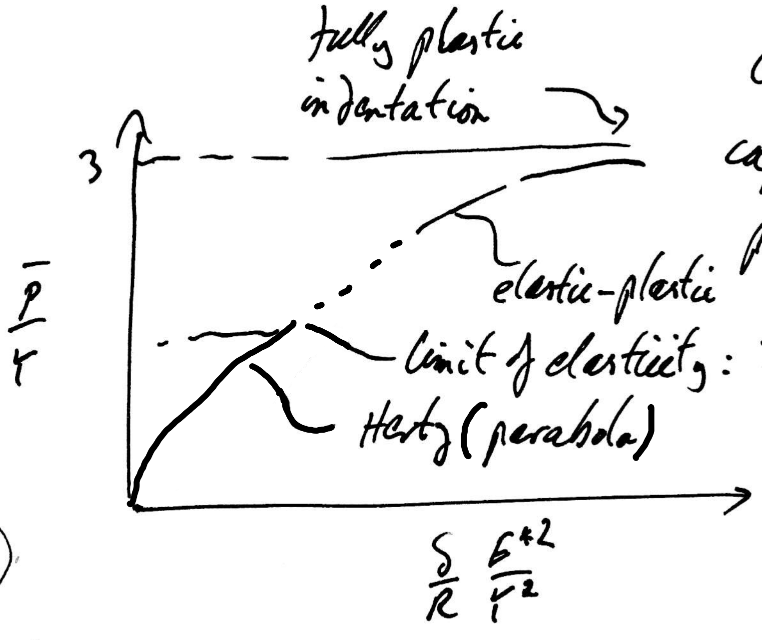
this section ~~answered~~ answered well

$\delta = \frac{1}{2}\left(\frac{9}{2}\frac{P^2}{E^{*2}R}\right)^{\frac{1}{3}}$

$\Rightarrow \frac{\bar{p}^6}{\delta^3} = \frac{\left(\frac{2}{3\pi}\right)^6\left(\frac{6PE^{*2}}{R^2}\right)^2}{\left(\frac{1}{2}\right)^3\frac{9}{2}\frac{P^2}{E^{*2}R}} \Rightarrow$ P's cancel out

$\delta = \bar{p}^{-2}\,\frac{9\pi^2}{16}\,\frac{R}{E^{*2}}$ – parabolic with constant as given.

(iii)



Choose groups which capture elastic and plastic response

: $\tau_y = Y/2 = 0.31\,p_0$

$= 0.31 . \frac{3}{2}\bar{p}$

$\Rightarrow \frac{\bar{p}}{Y} \approx 1.08$

not used commonly

This part not answered well.

2(b)(i) Line contact $p_0 = \sqrt{\dfrac{P' E^*}{\pi R}}$ where $P' = \dfrac{\overset{\text{torque}}{T_j}}{\underset{\text{base circle radius}}{r_b}\;\underset{\text{width}}{w}}$

$$1 = \left(\frac{p_{o1}}{p_{o2}}\right)^2 = \frac{P'_1}{P'_2}\,\frac{R_2}{R_1} = \frac{\overset{\text{ratio}=2}{T_{j1}}}{T_{j2}}\;\underbrace{\frac{R_2 r_{b2} w_2}{R_1 r_{b1} w_1}}_{\text{all those scale with size}}$$

$\Rightarrow$ scaling factor $= \sqrt[3]{2} = 1.26$

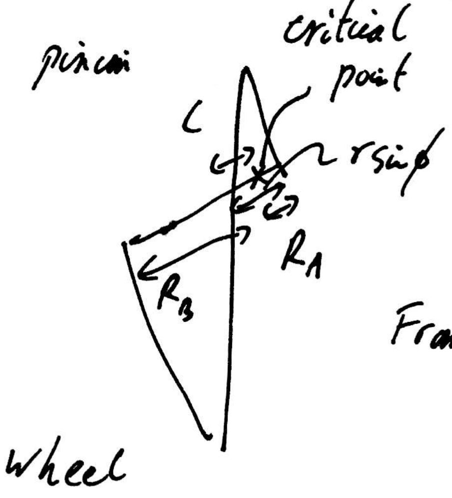
(ii) Since dimensions scale except m and a the larger gear set will have more teeth – 19 and 57.

not spotted by many

$$\frac{T_{j2}}{T_{j1}} = \frac{R_2}{R_1}\;\frac{\overset{\left(\frac{19}{17}\right)^2}{r_{b2}\,w_2}}{r_{b1}\,w_1} \quad \text{as before}$$

Comment: this part not well completed

No load sharing – critical point is at end of pressure line near pinion. ← Important to spot



$$\frac{1}{R} = \frac{1}{r_1 \sin\phi - l} + \frac{1}{r_2 \sin\phi + l}$$

where $r_1 = \dfrac{m N_p}{2}$ $r_2 = \dfrac{m N_w}{2}$

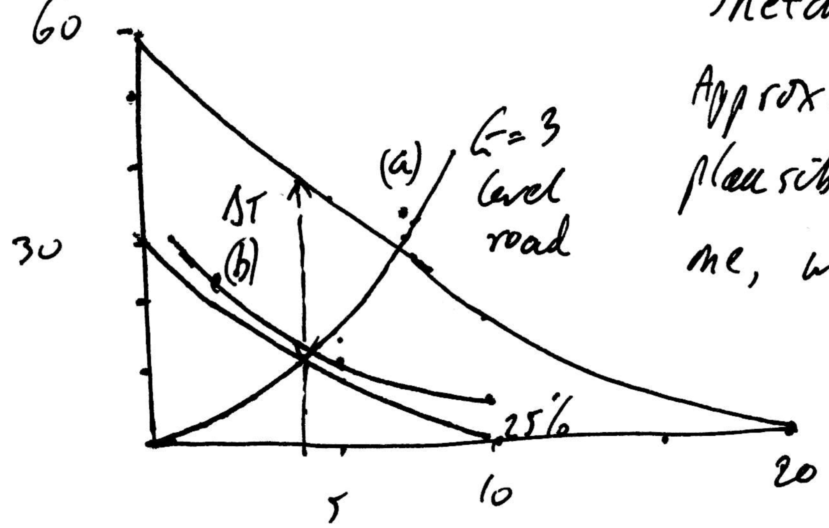
From data sheet $\dfrac{l}{m} = (0.02924N^2 + N + 1)^{\frac{1}{2}} - 0.1710N$

$\dfrac{l}{m} = 2.595$ for 51 teeth, 2.622 for 57 teeth

$$\frac{R_2}{R_1} = \frac{\dfrac{1}{\frac{17\sin 20}{2} - 2.595} + \dfrac{1}{\frac{51\sin 20}{2} + 2.595}}{\dfrac{1}{\frac{19\sin 20}{2} - 2.622} + \dfrac{1}{\frac{57\sin 20}{2} + 2.622}} = 1.965$$

$$\frac{T_{j2}}{T_{j1}} = \left(\frac{19}{17}\right)^2 \cdot 1.965 = \underline{2.46}$$

3.



Sketches required with covid. Approximate answers with plausible sketches, as this one, were acceptable.

(a) $F=0.5v^2$ $\quad G=\frac{\Omega}{\omega}$

Well answered

$v=\Omega R$

Power: $T\omega = Fv = F\Omega R$

$T = F\frac{\Omega}{\omega}R = RGR = 0.5GRv^2 = 0.5GR(\Omega R)^2 = 0.5G^3R^3\omega^2$

For $G=3$ $R=0.35\text{m}$ ~~$T=0.59\omega^2$~~ $T=0.58\omega^2$ (SI units)

ω	5	10	7.5
T	15	59	33

Intersects 100% effort line at $\omega \approx 7$ rad/s $\Rightarrow V = G\omega R \approx 7.4$ m/s

(b) Speed = 5 m/s $\Rightarrow \omega = \frac{v}{GR} = 4.8$ rad/s

Extra torque $\Delta T \simeq 38 - 13 = 15$ Nm

Extra $F = \frac{\Delta T}{GR} = mg\sin\alpha$

Again well answered

$\Rightarrow \sin\alpha = \frac{15}{3\times 0.35}\ \frac{1}{100\times 10} =$ [illegible] 0.014

$\alpha = 0.8°$ (rather flat!)

3 (c)

Need to recognise that G can be changed.

Easiest to work with powers: $Fv = 0.5v^3 = 62.5\,W$

Plot this power line on characteristic to identify optimum G and minimum effort

ω	2.5	5	10
T	25	12.5	6.25

Not a very sharp minimum - $G = 3$ is close to minimum with ≈ 30% effort assuming linear interpolation

(d) Need to find max power output of system with human and motor torques added.

this point often missed

ω	5	10	15	
T	65	48	22	← torques added
P	325	480	330	

↑ maximum (at change in slope)

$$v = \sqrt[3]{2P} = 9.9\,m/s, \quad G = \frac{v}{\omega R} = \frac{9.9}{10 \times 3.5} = 2.8$$

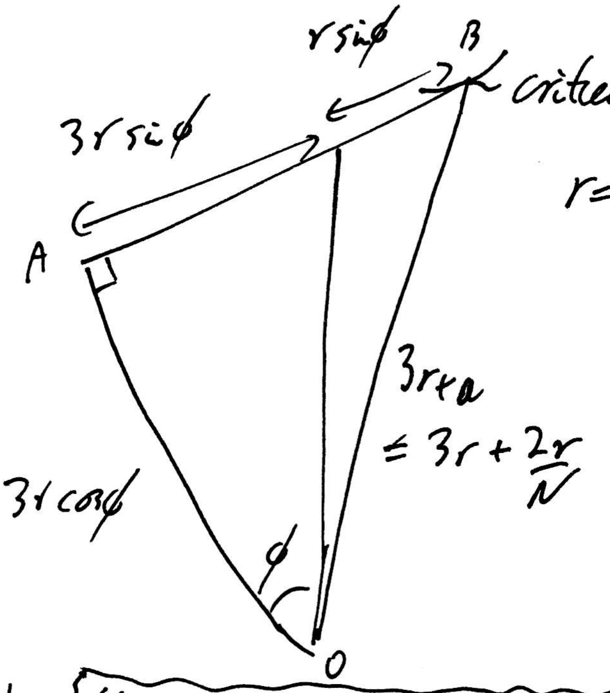
(e) - could couple directly to front or rear wheels but then the motor is often not running at a useful operating point

- couple through epicyclic at crank - maybe could be used to manage power matching with torque variation independent of speed.

Comment: generally well-answered question

4 (a)(i) When the tooth goes inside the base circle then undercutting occurs and we lose the involute geometry

(ii) Critical point when both contact is on the base circle.



$r = \frac{mN}{2} = \frac{aN}{2} \Rightarrow a = 2r/N$

$OAB: (4r\sin\phi)^2 + (3r\cos\phi)^2 = r^2\left(3+\frac{2}{N}\right)^2$

$3.13 = 3 + \frac{2}{N}$

$N = 14.98 \rightarrow 15$

$N \geq 15$

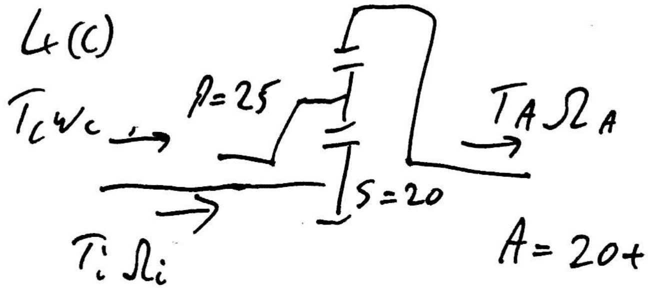
~~(iii)~~ (b) – These questions based on demo clips. Although not all the detail was expected, answers tended not to pay attention to the basics of speed ratio, changes in direction and materials

(i) Rotating to linear motion ⇒ rack & pinion
May be beneficial to include step down stage from motor.
Corrosion / lubrication will be important.

(ii) Large step down in speed – series spur gears. Could use polymer for high speed stage. Corrosion important → brass ?

(iii) Again high speed 'waterwheel' action needs stepping down to low speed oscillation – multi stage drive. Nylon gears to avoid lubrication / corrosion issues.

4(c)



Forces on shafts

$A = 20+25+25 = 70$ this step sometimes missed

$R = \frac{A}{S} = \frac{7}{2}$

(i) Annulus fixed : $\Omega_A = 0$

Epicyclic speed eqn: $\omega_S = (1+R)\omega_C - R\omega_A$

Power $T_i \Omega_i + T_C \Omega_C + T_A \Omega_A = 0$ Mostly well done

Put $\Omega_c' = 0 \Rightarrow$ $\left.\frac{T_A}{T_i} = -\frac{\Omega_i'}{\Omega_A'}\right)_{\Omega_c'=0} = \underline{\underline{R = \frac{7}{2}}}$

Equilibrium : $T_i + T_C + T_A = 0 \Rightarrow T_c = -(T_i + T_A) = \underline{\underline{-\frac{9}{2} T_i}}$

(ii) $\Omega_A = -0.2 T_i$, $T_A = \frac{7}{2} T_i$ as before.

As signs of Ω_A and T_A are different $\Rightarrow$ power out of A this not always noted

Need to find Ω_c : $\Omega_i = (1+R)\Omega_c + R(0.2\Omega_i)$

$$\Rightarrow \frac{\Omega_i}{\Omega_c} = \frac{1+R}{1-0.2R} = 15$$

assuming all negative powers rotating modules here

$$\text{Efficiency} = \frac{\text{Power out}}{\text{Power in}} = \eta = 0.95 = \frac{-(T_C \Omega_c + T_A \Omega_A)}{T_i \Omega_i}$$

$$0.95 = -\frac{T_c}{T_i}\frac{1}{15} + \overset{T_A/T_i}{\frac{7}{2}} \times \underbrace{0.2}_{-\Omega_A/\Omega_i}$$

$$\frac{T_c}{T_i} = -\frac{15}{4}$$

Comment : This last part not so well answered. We can't use virtual work as power isn't conserved. Need to pay attention to signs and power flows.