

EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 6 May 2025 9.30 to 11.10

Module 3C9

FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Attachment: 3C9 Fracture Mechanics of Materials and Structures (8 pages)

Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Explain why the residual stress state in a fusion weld is commonly approximated by the tensile yield strength. [25%]

(b) What is the significance of this residual stress state upon fatigue crack growth and fracture from a pre-existing flaw in the weld? [25%]

(c) The design life of a welded steel bridge is typically 120 years. Explain why most of the fatigue life is consumed while the cracks are short, and consequently the threshold stress intensity range is of more concern in bridge design than the fracture toughness of the material. [25%]

(d) What steps can be taken to maximise the transition flaw size of a steel for monotonic loading? [25%]

2 (a) Explain why the transition flaw size of a structural steel for cyclic loading is much less than that for monotonic loading, and comment on the implications. [30%]

(b) A high strength titanium alloy has a fatigue limit σ_0 , defined as the stress amplitude of fully reversed loading that corresponds to a fatigue life of 10^7 cycles. The material fails after 10^4 cycles at a stress amplitude of $1.5\sigma_0$.

(i) If a circular cylindrical bar of this alloy is subjected to rotating bending such that 70% of its life is spent at an amplitude of bending stress of $1.1\sigma_0$ and the remaining fraction at $1.2\sigma_0$, determine its fatigue life. Use may be made of the fatigue formulae in the Materials Data Book, as appropriate. [40%]

(ii) Assume that an identical circular cylindrical bar is subjected to an axial tension of 0.2 times the tensile strength in addition to the cyclic loading as detailed in part (i) above. Obtain an expression for its fatigue life, again by making suitable use of the Materials Data Book, as appropriate. [30%]

3 (a) A double cantilever beam, of depth b into the page and of in-plane geometry shown in Fig. 1, is used for performing toughness tests on an adhesive layer. The height h of each beam varies linearly with distance x from the left-hand end of the beam according to $h = h_0 + \alpha x$, where $h = h_0$ is the beam height at the left hand-end and α is the slope of beam height. Both beams are made from a high strength steel of Young's modulus E , and the beams are adhered by an epoxy adhesive of toughness $\mathcal{G}_c$ and negligible thickness. The double cantilever beam contain an edge crack of length a and each beam is loaded by a transverse force P , as shown in the figure.

- (i) Determine the potential energy of the double cantilever beam under the prescribed end load P . Hint: it may be helpful to make use of the standard formula:

$$\int \frac{x}{(A + Bx)^3} dx = \frac{A}{2B^2}(A + Bx)^{-2} - \frac{1}{B^2}(A + Bx)^{-1} + C,$$

where C is a constant of integration.

[50%]

- (ii) Hence, or otherwise, determine the energy release rate $\mathcal{G}$ as a function of crack length a .

[25%]

(b) Explain why the toughness of a structural alloy can increase with crack extension whereas the toughness of an elastic, brittle solid such as a rubber or epoxy is usually independent of crack extension.

[25%]

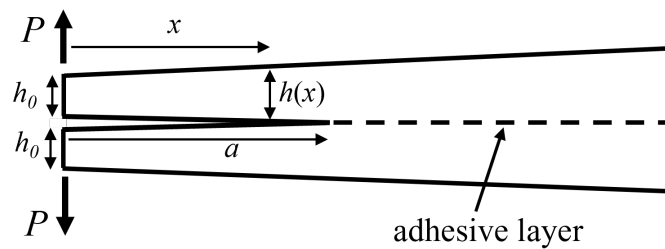


Fig. 1

4 A semi-infinite crack is loaded by a remote mode I stress intensity factor K^∞ and by a pair of forces P that act on opposing faces of the crack at a distance x_0 behind the crack tip, as shown at the top of page 5 of the 3C9 datasheet.

(a) Write down the total stress intensity factor at the tip of this semi-infinite crack and explain why linear superposition holds. [10%]

(b) Using the Dugdale model for a semi-infinite crack and the K calibration in part (a), obtain an expression for the plastic zone size d_y at the crack tip due to the remote mode I stress intensity factor K^∞ , for a solid of yield strength σ_0 . [30%]

(c) Now consider a load cycle that involves loading the semi-infinite crack by a remote mode I stress intensity factor K^∞ , followed by remote unloading to zero. Obtain an expression for the cyclic plastic zone size d_{cy} at the crack tip after this load cycle, again by use of the Dugdale model. [30%]

(d) Explain the practical limitation of the small scale yield assumption in a specimen of finite crack length a . [30%]

END OF PAPER

ENGINEERING TRIPOS PART IIA

Module 3C9 – FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

DATASHEET

Crack tip plastic zone sizes

$$\text{diameter, } d_p = \begin{cases} \frac{1}{\pi} \left(\frac{K_I}{\sigma_y} \right)^2 & \text{Plane stress} \\ \frac{1}{3\pi} \left(\frac{K_I}{\sigma_y} \right)^2 & \text{Plane strain} \end{cases}$$

Crack opening displacement

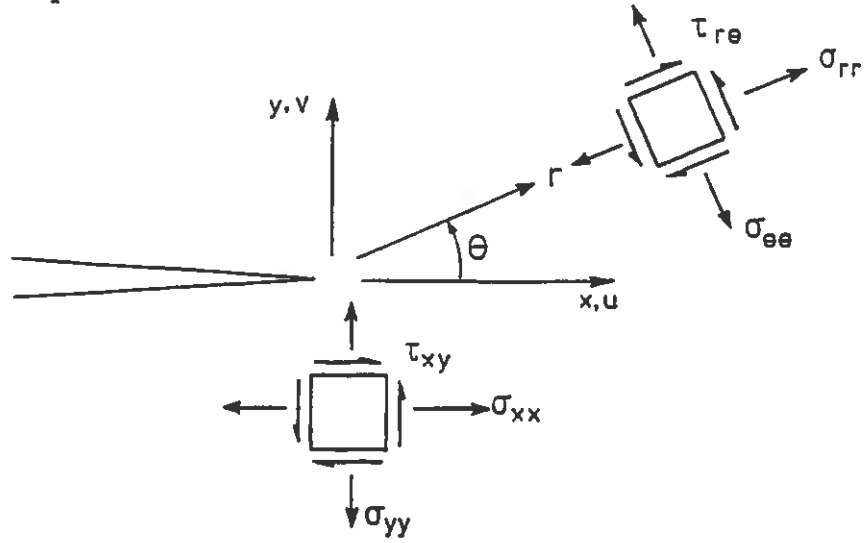
$$\delta = \begin{cases} \frac{K_I^2}{\sigma_y E} & \text{Plane stress} \\ \frac{1}{2} \frac{K_I^2}{\sigma_y E} & \text{Plane strain} \end{cases}$$

Energy release rate

$$G = \begin{cases} \frac{1}{E} K_I^2 & \text{Plane stress} \\ \frac{1-\nu^2}{E} K_I^2 & \text{Plane strain} \end{cases}$$

$$\text{Related to compliance } C: G = \frac{1}{2} \frac{P^2}{B} \frac{dC}{da}$$

Asymptotic crack tip fields in a linear elastic solid



Mode I

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{5}{4} \cos \frac{\theta}{2} - \frac{1}{4} \cos \frac{3\theta}{2} \right)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{3}{4} \cos \frac{\theta}{2} + \frac{1}{4} \cos \frac{3\theta}{2} \right)$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{1}{4} \sin \frac{\theta}{2} + \frac{1}{4} \sin \frac{3\theta}{2} \right)$$

$$u = \begin{cases} \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{1-\nu}{1+\nu} + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(1 - 2\nu + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{2}{1+\nu} - \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(2 - 2\nu - \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Crack tip stress fields (cont'd)

Mode II

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left(2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{rr} = \frac{K_{II}}{\sqrt{2\pi r}} \left(-\frac{5}{4} \sin \frac{\theta}{2} + \frac{3}{4} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{\theta\theta} = -\frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{3}{4} \sin \frac{\theta}{2} + \frac{3}{4} \sin \frac{3\theta}{2} \right)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{1}{4} \cos \frac{\theta}{2} + \frac{3}{4} \cos \frac{3\theta}{2} \right)$$

$$u = \begin{cases} \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{2}{1+\nu} + \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(2 - 2\nu + \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{\nu-1}{1+\nu} + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(-1 + 2\nu + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Mode III

$$\tau_{zx} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin \frac{\theta}{2}$$

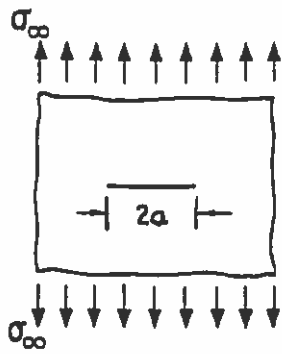
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = 0$$

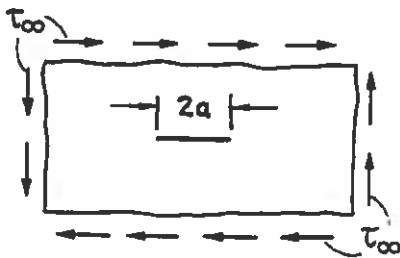
$$w = \frac{K_{III}}{G} \sqrt{\frac{2r}{\pi}} \sin \frac{\theta}{2}$$

$$u = v = 0$$

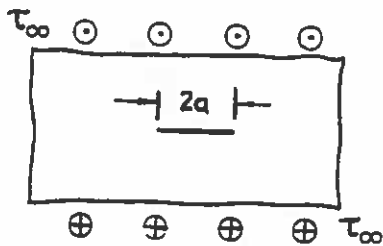
Tables of stress intensity factors



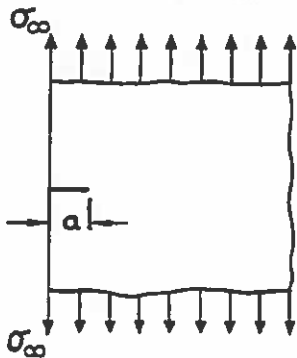
$$K_I = \sigma_{\infty} \sqrt{\pi a}$$



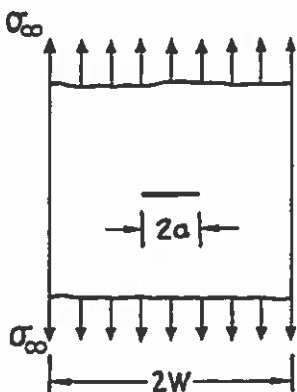
$$K_{II} = \tau_{\infty} \sqrt{\pi a}$$



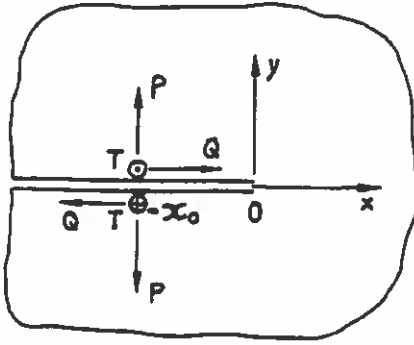
$$K_{III} = \tau_{\infty} \sqrt{\pi a}$$



$$K_I = 1.12 \sigma_{\infty} \sqrt{\pi a}$$



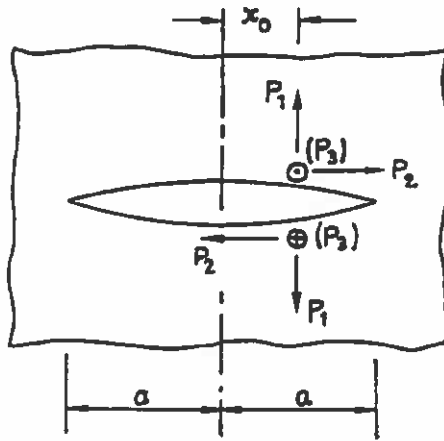
$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{1 - a/2W + 0.326a^2/W^2}{\sqrt{1 - a/W}} \right)$$



$$K_I = \frac{2P}{\sqrt{2\pi x_0}}$$

$$K_{II} = \frac{2Q}{\sqrt{2\pi x_0}}$$

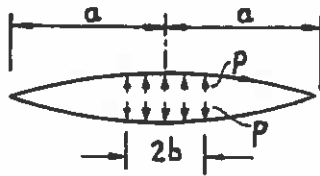
$$K_{III} = \frac{2T}{\sqrt{2\pi x_0}}$$



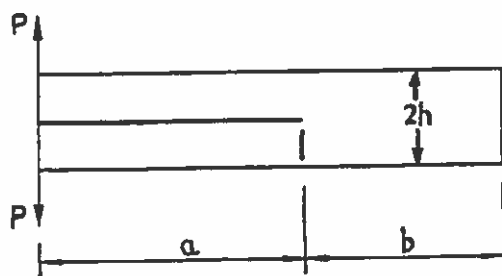
$$K_I = \frac{P_1}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$

$$K_{II} = \frac{P_2}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$

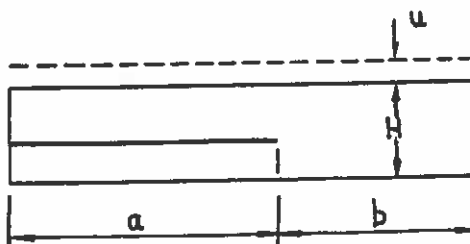
$$K_{III} = \frac{P_3}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$



$$K_I = \frac{2pb}{\sqrt{\pi a}} \frac{a}{b} \arcsin \frac{b}{a}$$

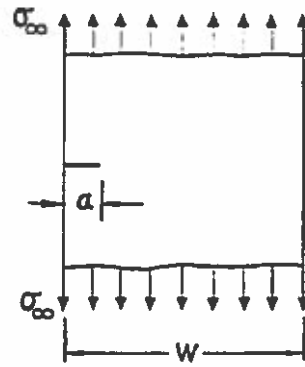


$$K_I = \frac{2\sqrt{3}}{h\sqrt{h}} \frac{Pa}{B} \quad h \ll a \text{ and } h \ll b$$



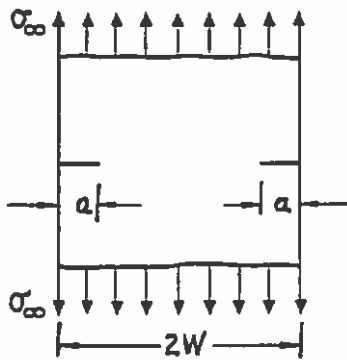
$$K_I = \sqrt{\frac{1}{2\alpha H}} Eu \quad H \ll a \text{ and } H \ll b$$

$$\alpha = \begin{cases} 1 - \nu^2 & \text{Plane stress} \\ 1 - 3\nu^2 - 2\nu^3 & \text{Plane strain} \end{cases}$$

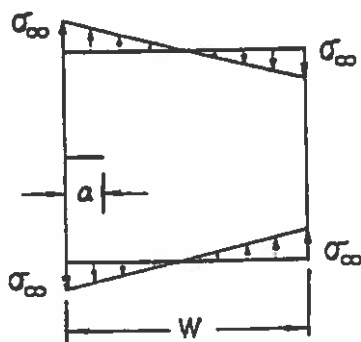


$$a/W < 0.7$$

$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(1.12 - 0.23 \frac{a}{W} + 10.6 \frac{a^2}{W^2} - 21.7 \frac{a^3}{W^3} + 30.4 \frac{a^4}{W^4} \right)$$

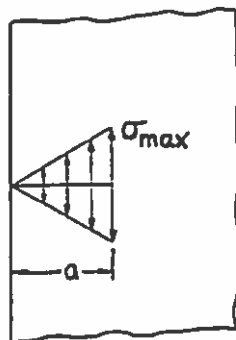


$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{1.12 - 0.61a/W + 0.13a^3/W^3}{\sqrt{1 - a/W}} \right)$$

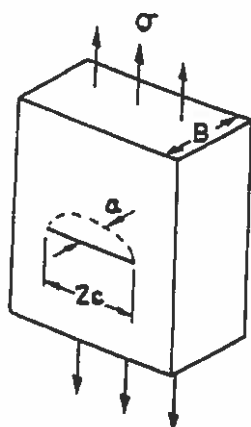


$$a/W < 0.7$$

$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(1.12 - 1.39 \frac{a}{W} + 7.3 \frac{a^2}{W^2} - 13 \frac{a^3}{W^3} + 14 \frac{a^4}{W^4} \right)$$

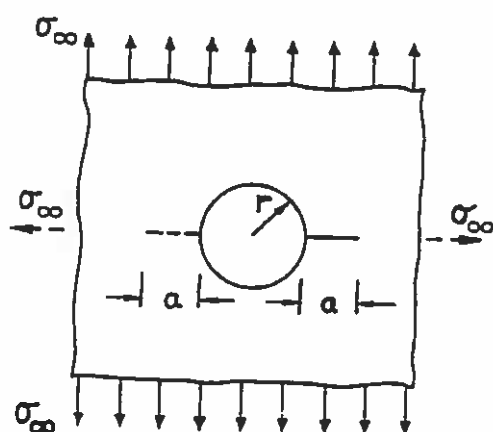
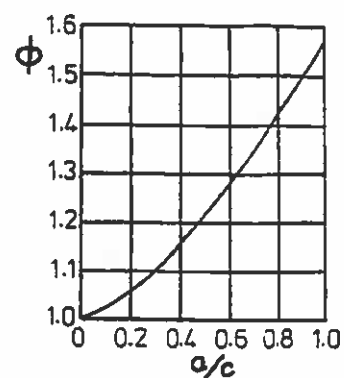


$$K_I = 0.683 \sigma_{\max} \sqrt{\pi a}$$



$$K_I = \frac{1.12}{\Phi} \sigma \sqrt{\pi a}$$

$$\Phi = \int_0^{\pi/2} \left(1 - \frac{c^2 - a^2}{c^2} \sin^2 \theta \right)^{\frac{1}{2}} d\theta$$

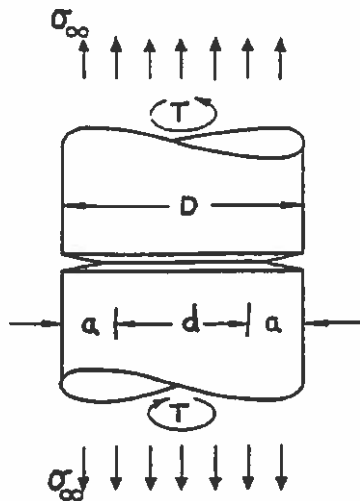


$$K_I = \sigma_{\infty} \sqrt{\pi a} F\left(\frac{a}{r}\right)$$

value of $F(a/r)^{\dagger}$

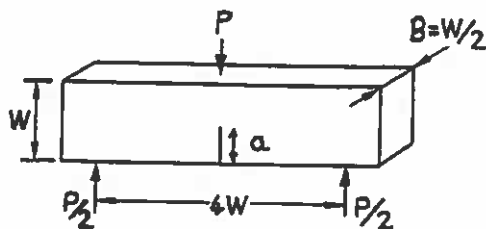
	One crack		Two cracks	
	U	B	U	B
$\frac{a}{r}$				
0.00	3.36	2.24	3.36	2.24
0.10	2.73	1.98	2.73	1.98
0.20	2.30	1.82	2.41	1.83
0.30	2.04	1.67	2.15	1.70
0.40	1.86	1.58	1.96	1.61
0.50	1.73	1.49	1.83	1.57
0.60	1.64	1.42	1.71	1.52
0.80	1.47	1.32	1.58	1.43
1.0	1.37	1.22	1.45	1.38
1.5	1.18	1.06	1.29	1.26
2.0	1.06	1.01	1.21	1.20
3.0	0.94	0.93	1.14	1.13
5.0	0.81	0.81	1.07	1.06
10.0	0.75	0.75	1.03	1.03
∞	0.707	0.707	1.00	1.00

$\dagger U = \text{uniaxial } \sigma_{\infty} \quad B = \text{biaxial } \sigma_{\infty}.$

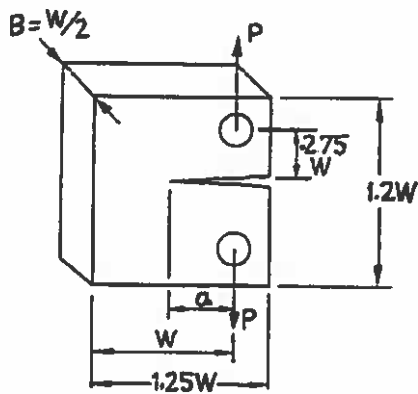


$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{D}{d} + \frac{1}{2} + \frac{3}{8} \frac{d}{D} - 0.36 \frac{d^2}{D^2} + 0.73 \frac{d^3}{D^3} \right) \frac{1}{2} \sqrt{\frac{D}{d}}$$

$$K_{III} = \frac{16T}{\pi D^3} \sqrt{\pi a} \left(\frac{D^2}{d^2} + \frac{1}{2} \frac{D}{d} + \frac{3}{8} + \frac{5}{16} \frac{d}{D} + \frac{35}{128} \frac{d^2}{D^2} + 0.21 \frac{d^3}{D^3} \right) \frac{3}{8} \sqrt{\frac{D}{d}}$$



$$K_I = \frac{4P}{B} \sqrt{\frac{\pi}{W}} \left\{ 1.6 \left(\frac{a}{W} \right)^{1/2} - 2.6 \left(\frac{a}{W} \right)^{3/2} + 12.3 \left(\frac{a}{W} \right)^{5/2} - 21.2 \left(\frac{a}{W} \right)^{7/2} + 21.8 \left(\frac{a}{W} \right)^{9/2} \right\}$$



$$K_I = \frac{P}{B} \sqrt{\frac{\pi}{W}} \left\{ 16.7 \left(\frac{a}{W} \right)^{1/2} - 104.7 \left(\frac{a}{W} \right)^{3/2} + 369.9 \left(\frac{a}{W} \right)^{5/2} - 573.8 \left(\frac{a}{W} \right)^{7/2} + 360.5 \left(\frac{a}{W} \right)^{9/2} \right\}$$