

EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 5 May 2026 9.30 to 11.10

Module 3C9

FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Attachment: 3C9 Fracture Mechanics of Materials and Structures (8 pages)

Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A circular fibre of radius a is made from an elastic solid of Young's modulus E and Poisson ratio equal to zero. The fibre is adhesively bonded into a rigid foundation as shown in Fig. 1, with the top portion of the fibre protruding a height h ($h \gg a$) above the rigid block. Adhesive is not applied around the circumference of the fibre over a length l ($l \gg a$).

(a) On subjecting the top of the fibre to a tensile load P , it displaces upwards by u . Obtain an expression for the compliance $C = u/P$ in terms of the geometry and relevant material parameters. [10%]

(b) Assume no friction between the fibre and the foundation over the debonded zone. Show that the energy release rate G for advance of the debond is given by

$$G = \frac{P^2}{4\pi a} \frac{\partial C}{\partial l}$$

when the load P is held fixed. [30%]

(c) Hence obtain an expression for load P_c to initiate debonding in terms of the toughness G_c of the adhesive, and the debond length l . [30%]

(d) Assuming that the fractured adhesive exerts a constant frictional stress τ_f , calculate the dependence of applied load P on a finite extension Δl of debond. [30%]

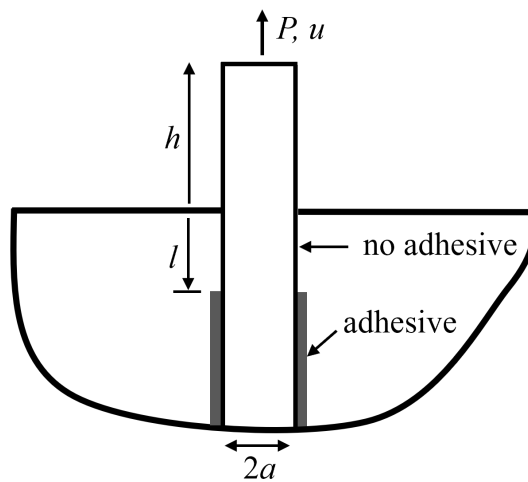


Fig. 1

2 (a) Specimens of grade 2024-T3 aluminium alloy were subjected to fully reversed cyclic loading. Failure occurred after 500 cycles when tested at a cyclic plastic strain range $\Delta\epsilon^p = 0.02$, and after 2000 cycles at $\Delta\epsilon^p = 0.005$. Calculate the constants in the Coffin-Manson fatigue law for this material, making use of the Materials Databook as appropriate. [30%]

(b) The amplitude of cyclic stress-strain curve for the same 2024-T3 aluminium alloy is idealised by the bi-linear relation

$$\frac{\Delta\epsilon}{2} = \frac{1}{E} \frac{\Delta\sigma}{2}, \text{ for } \frac{\Delta\sigma}{2} \leq \sigma_y$$

and

$$\frac{\Delta\epsilon}{2} = \frac{\sigma_y}{E} + \frac{1}{E_T} \left(\frac{\Delta\sigma}{2} - \sigma_y \right), \text{ for } \frac{\Delta\sigma}{2} > \sigma_y$$

in terms of the Young's modulus $E = 70$ GPa, post-yield tangent modulus $E_T = 7$ GPa, and yield strength $\sigma_y = 300$ MPa. A large sheet of the 2024-T3 aluminium alloy contains a circular hole (with elastic stress concentration factor $k_T = 3$) and the plate is subjected to a fully reversed uniaxial cyclic stress of range $\Delta\sigma^\infty = 500$ MPa. Estimate the number of cycles for crack initiation by making use of Neuber's rule, $k_\sigma k_\epsilon = k_T^2$, where k_σ is the stress magnification factor and k_ϵ is the strain magnification factor at the notch root. [50%]

(c) Explain the significance of mean stress relaxation at a notch root when a remote cyclic stress has a compressive mean component. [20%]

3 Account for the following observations:

- (a) It is necessary to qualify the safety of a nuclear reactor pressure vessel made from welded steel using the J -integral approach rather than linear elastic fracture mechanics. [25%]
- (b) Metallic alloys are tougher but weaker than freshly drawn silica glass. [25%]
- (c) Non-propagating fatigue cracks can exist at the root of a small notch but not at the root of a large notch. [25%]
- (d) Severe turbulence on a metallic aircraft wing can extend its fatigue life. [25%]

- 4 (a) Compare the shape of the R-curve in metallic alloys, in ceramic-matrix composites and in elastomers, and give the physical basis for the different characteristics. [30%]
- (b) Consider a large metallic panel containing a centre-crack of initial length $2a_0$ and subjected to a tensile stress σ^∞ . Upon making use of the typical R-curve for a metallic alloy, explain why the K -value associated with the tensile strength of the panel is adequately given by the fracture toughness when the initial crack is short. Why is the K -value associated with the tensile strength increased for the case of a longer initial crack? [30%]
- (c) A cantilever beam, of length ℓ , and uniform square cross-section of side t is subjected to a cyclic end load from zero to a maximum value P . The cantilever is made from a high strength aluminium alloy with fatigue response given by Basquin's law and Goodman's law, as given in the Materials Databook. Obtain an expression for its fatigue life N_f as a function of P . State any assumptions that you make. [40%]

END OF PAPER

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ENGINEERING TRIPOS PART IIA

Module 3C9 – FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

DATASHEET

Crack tip plastic zone sizes

$$\text{diameter, } d_p = \begin{cases} \frac{1}{\pi} \left(\frac{K_I}{\sigma_y} \right)^2 & \text{Plane stress} \\ \frac{1}{3\pi} \left(\frac{K_I}{\sigma_y} \right)^2 & \text{Plane strain} \end{cases}$$

Crack opening displacement

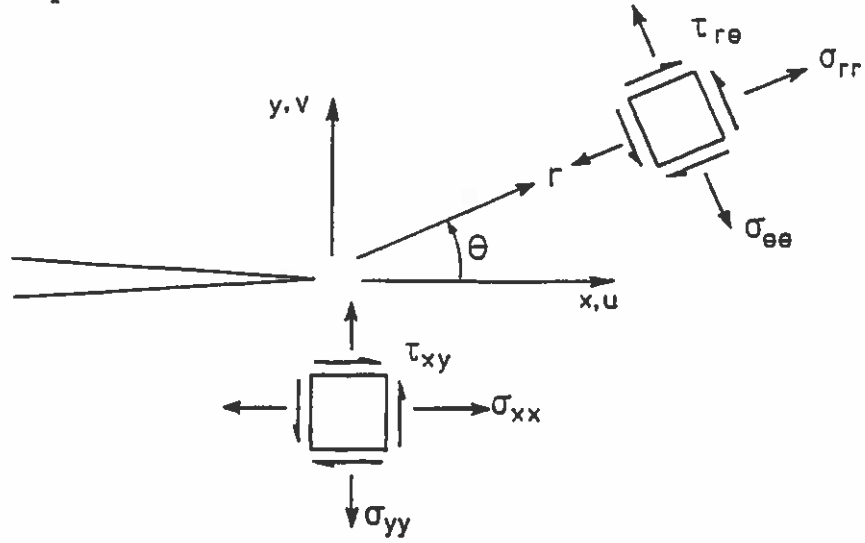
$$\delta = \begin{cases} \frac{K_I^2}{\sigma_y E} & \text{Plane stress} \\ \frac{1}{2} \frac{K_I^2}{\sigma_y E} & \text{Plane strain} \end{cases}$$

Energy release rate

$$G = \begin{cases} \frac{1}{E} K_I^2 & \text{Plane stress} \\ \frac{1-\nu^2}{E} K_I^2 & \text{Plane strain} \end{cases}$$

$$\text{Related to compliance } C: \quad G = \frac{1}{2} \frac{P^2}{B} \frac{dC}{da}$$

Asymptotic crack tip fields in a linear elastic solid



Mode I

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{5}{4} \cos \frac{\theta}{2} - \frac{1}{4} \cos \frac{3\theta}{2} \right)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{3}{4} \cos \frac{\theta}{2} + \frac{1}{4} \cos \frac{3\theta}{2} \right)$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{1}{4} \sin \frac{\theta}{2} + \frac{1}{4} \sin \frac{3\theta}{2} \right)$$

$$u = \begin{cases} \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{1-\nu}{1+\nu} + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(1 - 2\nu + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{2}{1+\nu} - \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(2 - 2\nu - \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Crack tip stress fields (cont'd)

Mode II

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left(2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{rr} = \frac{K_{II}}{\sqrt{2\pi r}} \left(-\frac{5}{4} \sin \frac{\theta}{2} + \frac{3}{4} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{\theta\theta} = -\frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{3}{4} \sin \frac{\theta}{2} + \frac{3}{4} \sin \frac{3\theta}{2} \right)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{1}{4} \cos \frac{\theta}{2} + \frac{3}{4} \cos \frac{3\theta}{2} \right)$$

$$u = \begin{cases} \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{2}{1+\nu} + \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(2 - 2\nu + \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{\nu-1}{1+\nu} + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(-1 + 2\nu + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Mode III

$$\tau_{zx} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin \frac{\theta}{2}$$

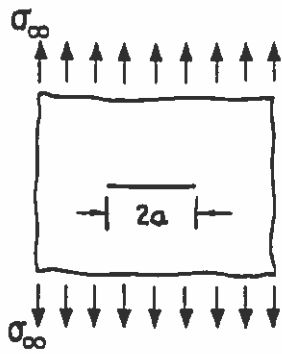
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = 0$$

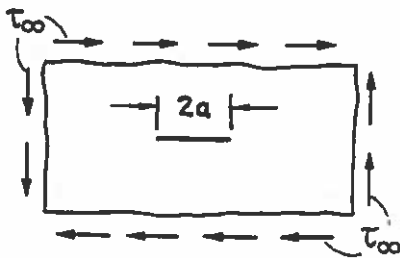
$$w = \frac{K_{III}}{G} \sqrt{\frac{2r}{\pi}} \sin \frac{\theta}{2}$$

$$u = v = 0$$

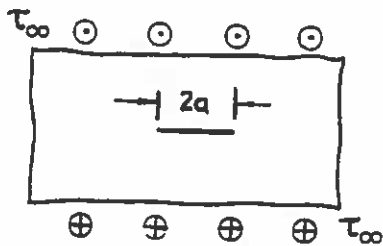
Tables of stress intensity factors



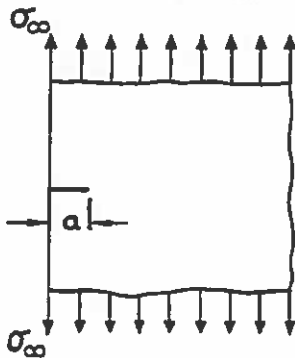
$$K_I = \sigma_{\infty} \sqrt{\pi a}$$



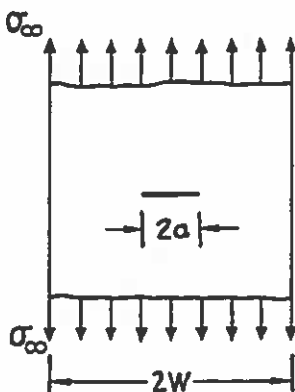
$$K_{II} = \tau_{\infty} \sqrt{\pi a}$$



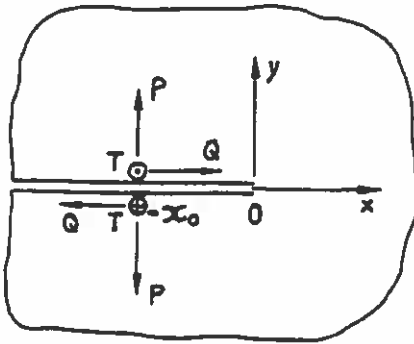
$$K_{III} = \tau_{\infty} \sqrt{\pi a}$$



$$K_I = 1.12 \sigma_{\infty} \sqrt{\pi a}$$



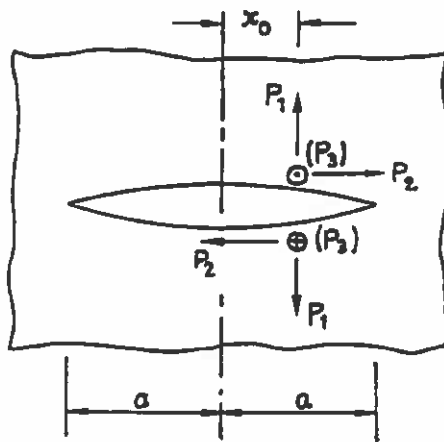
$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{1 - a/2W + 0.326a^2/W^2}{\sqrt{1 - a/W}} \right)$$



$$K_I = \frac{2P}{\sqrt{2\pi x_0}}$$

$$K_{II} = \frac{2Q}{\sqrt{2\pi x_0}}$$

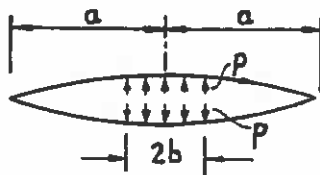
$$K_{III} = \frac{2T}{\sqrt{2\pi x_0}}$$



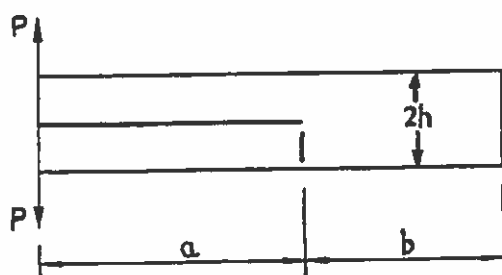
$$K_I = \frac{P_1}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$

$$K_{II} = \frac{P_2}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$

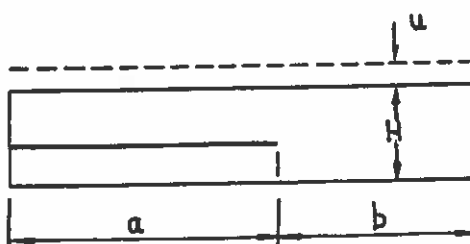
$$K_{III} = \frac{P_3}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$



$$K_I = \frac{2pb}{\sqrt{\pi a}} \frac{a}{b} \arcsin \frac{b}{a}$$

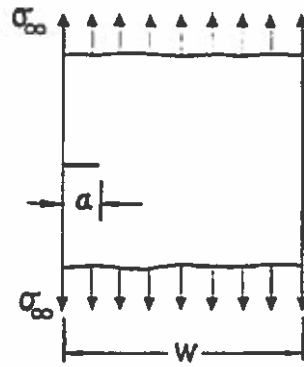


$$K_I = \frac{2\sqrt{3}}{h\sqrt{h}} \frac{Pa}{B} \quad h \ll a \text{ and } h \ll b$$



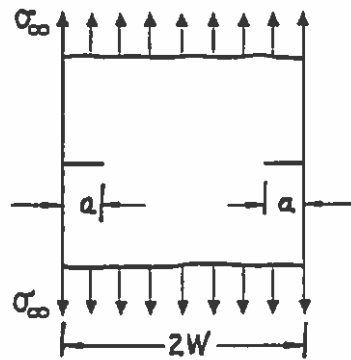
$$K_I = \sqrt{\frac{1}{2\alpha H}} Eu \quad H \ll a \text{ and } H \ll b$$

$$\alpha = \begin{cases} 1 - \nu^2 & \text{Plane stress} \\ 1 - 3\nu^2 - 2\nu^3 & \text{Plane strain} \end{cases}$$

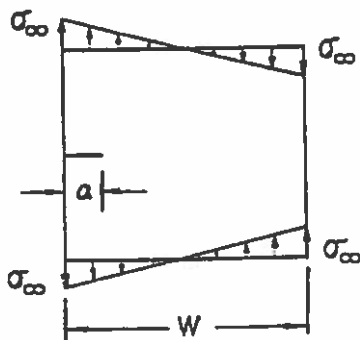


$$a/W < 0.7$$

$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(1.12 - 0.23 \frac{a}{W} + 10.6 \frac{a^2}{W^2} - 21.7 \frac{a^3}{W^3} + 30.4 \frac{a^4}{W^4} \right)$$

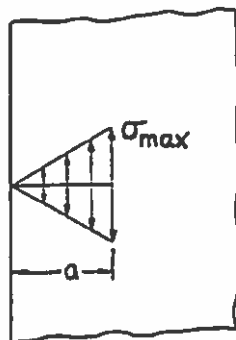


$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{1.12 - 0.61a/W + 0.13a^3/W^3}{\sqrt{1 - a/W}} \right)$$

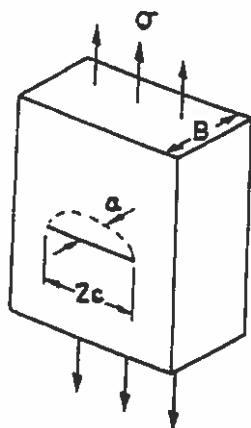


$$a/W < 0.7$$

$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(1.12 - 1.39 \frac{a}{W} + 7.3 \frac{a^2}{W^2} - 13 \frac{a^3}{W^3} + 14 \frac{a^4}{W^4} \right)$$

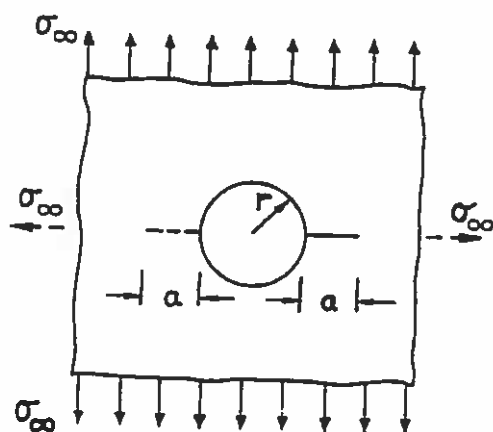
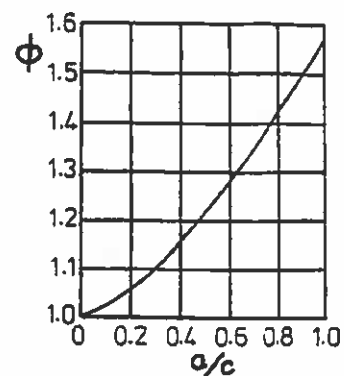


$$K_I = 0.683 \sigma_{\max} \sqrt{\pi a}$$



$$K_I = \frac{1.12}{\Phi} \sigma \sqrt{\pi a}$$

$$\Phi = \int_0^{\pi/2} \left(1 - \frac{c^2 - a^2}{c^2} \sin^2 \theta \right)^{\frac{1}{2}} d\theta$$

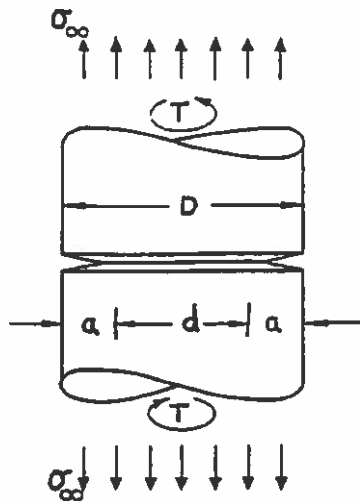


$$K_I = \sigma_{\infty} \sqrt{\pi a} F\left(\frac{a}{r}\right)$$

value of $F(a/r)^{\dagger}$

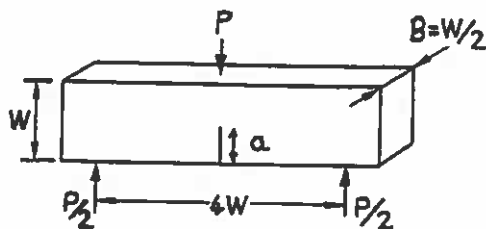
	One crack		Two cracks	
	U	B	U	B
$\frac{a}{r}$				
0.00	3.36	2.24	3.36	2.24
0.10	2.73	1.98	2.73	1.98
0.20	2.30	1.82	2.41	1.83
0.30	2.04	1.67	2.15	1.70
0.40	1.86	1.58	1.96	1.61
0.50	1.73	1.49	1.83	1.57
0.60	1.64	1.42	1.71	1.52
0.80	1.47	1.32	1.58	1.43
1.0	1.37	1.22	1.45	1.38
1.5	1.18	1.06	1.29	1.26
2.0	1.06	1.01	1.21	1.20
3.0	0.94	0.93	1.14	1.13
5.0	0.81	0.81	1.07	1.06
10.0	0.75	0.75	1.03	1.03
∞	0.707	0.707	1.00	1.00

$\dagger U = \text{uniaxial } \sigma_{\infty} \quad B = \text{biaxial } \sigma_{\infty}.$

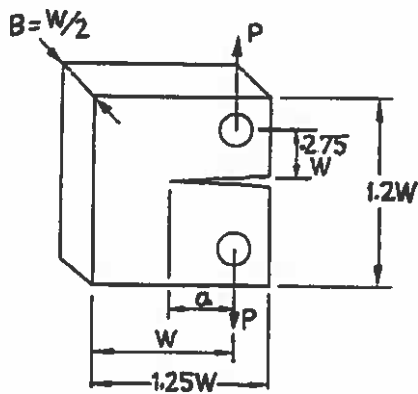


$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{D}{d} + \frac{1}{2} + \frac{3}{8} \frac{d}{D} - 0.36 \frac{d^2}{D^2} + 0.73 \frac{d^3}{D^3} \right) \frac{1}{2} \sqrt{\frac{D}{d}}$$

$$K_{III} = \frac{16T}{\pi D^3} \sqrt{\pi a} \left(\frac{D^2}{d^2} + \frac{1}{2} \frac{D}{d} + \frac{3}{8} + \frac{5}{16} \frac{d}{D} + \frac{35}{128} \frac{d^2}{D^2} + 0.21 \frac{d^3}{D^3} \right) \frac{3}{8} \sqrt{\frac{D}{d}}$$



$$K_I = \frac{4P}{B} \sqrt{\frac{\pi}{W}} \left\{ 1.6 \left(\frac{a}{W} \right)^{1/2} - 2.6 \left(\frac{a}{W} \right)^{3/2} + 12.3 \left(\frac{a}{W} \right)^{5/2} - 21.2 \left(\frac{a}{W} \right)^{7/2} + 21.8 \left(\frac{a}{W} \right)^{9/2} \right\}$$



$$K_I = \frac{P}{B} \sqrt{\frac{\pi}{W}} \left\{ 16.7 \left(\frac{a}{W} \right)^{1/2} - 104.7 \left(\frac{a}{W} \right)^{3/2} + 369.9 \left(\frac{a}{W} \right)^{5/2} - 573.8 \left(\frac{a}{W} \right)^{7/2} + 360.5 \left(\frac{a}{W} \right)^{9/2} \right\}$$