Soln.

3D1 - April 2026

Q1 a) Soil particles are formed from weathering of parent rocks. Cracks in rocks are filled with water and subsequent freeze, thaw cycles break the rocks forming smaller fragments. This process continues until smaller soil particles are formed that can be carried by water or air. Soil particles then get deposited as the flow velocity reduces leading to formation of a soil deposit.

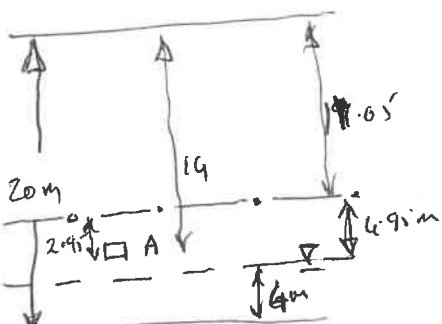
[10%]

b) Soil particles are transported in flowing rivers. As the flow velocity of the water reduces, particles of larger size get deposited first. Smaller soil particles are carried down the river, with finest fraction getting deposited as the rivers confluence with oceans, forming deltas.

Varved deposits contain alternating layers of very fine particles followed by coarser, larger particles. During summer months river water flows reduce and fine particles start to settle down to form a layer. During winter months, as the rainfall increases, the river flow velocity increases carrying larger soil fractions further down the river that settle to form a layer of coarser material. This process continues year after year leading to formation of varved deposits

[15%]

c)



$G_s = 2.61$; $e = 0.68$

$D_{10} = 0.006$ mm

$\gamma_w = 10$ kN/m^3

$T = 7.42 \times 10^{-2}$ N/m

Consider a soil element 'A' at 14 m depth

Height of capillary rise : $h_c = \dfrac{4T}{\gamma_w D_{10}}$

$$\therefore h_c = \frac{4 \times 7.42 \times 10^{-2}}{10 \times 10^3 \times \frac{0.006}{1000}} = 4.946 \text{ m} \simeq 4.95 \text{ m}$$

i) when water table is at 16 m below ground surface, there will be a capillary rise of 4.95 m above this height that keep soil saturated.

$$\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.61 \times 10}{1.68} = 15.54 \text{ kN/m}^3$$

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.61 + 0.68)}{1.68} \times 10 = 19.58 \text{ kN/m}^3.$$

Total stress $\Big\}$ $\sigma_{VA} = \gamma_d \times 11.05 + \gamma_{sat} \times 2.95$

$= 15.54 \times 11.05 + 19.58 \times 2.95 = 229.48$ kPa.

$u_A = -(4.95 - 2.95)\gamma_w = -20$ kPa (suction)

$\therefore \sigma'_{VA} = 229.48 - (-20) = 249.48$ kPa

$\simeq$ 250 kPa.

(ii) When water table is at ground surface

$\sigma_{VA} = \gamma_{sat} \times 14 = 274.12$ kPa

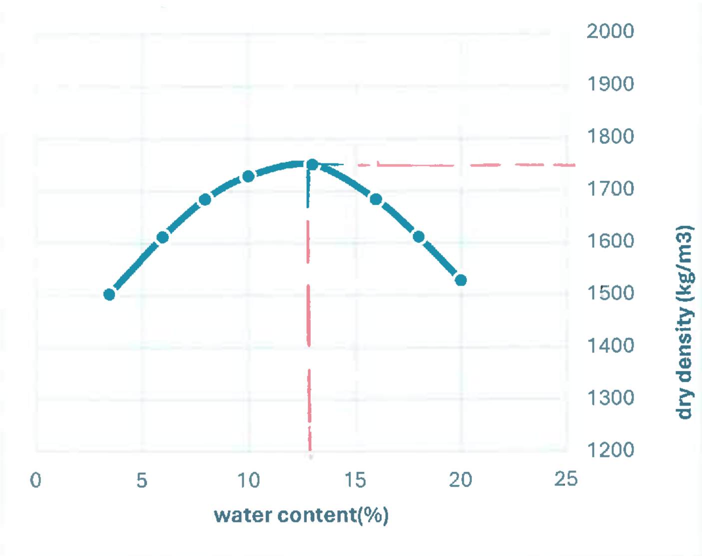
$u_A = \gamma_w \times 14 = 140$ kPa.

$\therefore \sigma'_{VA} = 134.12$ kPa

Obviously this is a significant drop in effective stress, which will nearly 46% drop in effective stress in the soil. Changes in effect the soil stiffness ($\propto \sigma'_v$) in the soil. Changes in water table must be considered carefully in Soil Mechanics. [30%]

1d) A standard Proctor test uses a 2.5 kg hammer to drop from a 300 mm (1 foot) height. The soil sample is mixed with a certain % of water. A small sample of soil is taken and dried in an oven @105°C for 24 hours to determine the actual moisture content later. The mixed soil sample is divided into 3 parts and each part is placed in the Proctor mould and 27 blows are applied to each layer. The mould volume is known (1 litre normally) and knowing the weight of the soil the density is determined. This is clearly the bulk density, the dry density is later calculated. [15%]

e)



void ratio e	water content w (%)	Dry density (kg/m3)	Sr (%)	Air void ratio
0.77	3.5	1503.14	11.86	0.38
0.65	6	1612.45	24.09	0.30
0.58	8	1683.89	36.00	0.23
0.54	10	1727.63	48.33	0.18
0.52	**13**	**1750.36**	**65:25**	**0.12**
0.58	16	1683.89	72.00	0.10
0.65	18	1612.45	72.28	0.11
0.74	20	1529.05	70.54	0.13

$$\rho_d = \frac{G_s \gamma_w}{1+e} = \frac{2.61 \times 10 \times 10^3 / 9.81}{1.77} = 1503.14 \ kg/m^3$$

$$w G_s = e S_r \Rightarrow S_r = \frac{3.5 \times 2.61}{0.77} = 11.86\%$$

$$A_r = 1 - S_r \times \frac{e}{1+e} = \left(1 - \frac{11.86}{100}\right) \cdot \frac{0.77}{1.77} = 0.38$$

Maximum Dry Density (from graph) = 1750.36 kg/m^3.

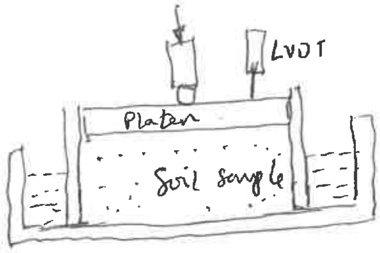
OMC = 13 %.

S_r = 65.25 %.

A_r = 0.12

[30%]

Q2) a) The soil sample is placed in the oedometer ring. A stiff loading platen is placed on top of the soil sample.



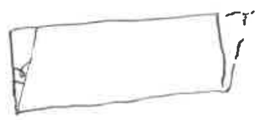
The applied load is measured using a load cell or dial gauge. The settlement of the sample at each load increment is measured using a LVDT or a displacement gauge. If the soil sample is saturated the oedometer ring is placed in a water bath. In case of clay samples, each load increment may be held for several hours to allow for consolidation to take place. For coarser soils with higher permeability, the load increments can be applied more quickly.

[10%]

b) In the oedometer soil sample undergoes 1-d compression.

If the sample compresses by δh, then 1-d compressional strain is defined as

$$\varepsilon_h = \frac{\delta h}{h}.$$



If the soil sample is sheared to form an oblong shape, then it is said to undergo shear strain. γ.

Volumetric strain is defined as $\varepsilon_v = \frac{\delta V}{V}$; that is change in volume of the soil sample to its initial volume. Using an oedometre we can only get ε_h (compressional strain) and volumetric strain, but not shear strain, as the soil sample cannot be sheared in the oedometre apparatus.

[15%]

c) Plot the data given on a semi-log plot (see next page) $e - \ln \sigma_v'$.

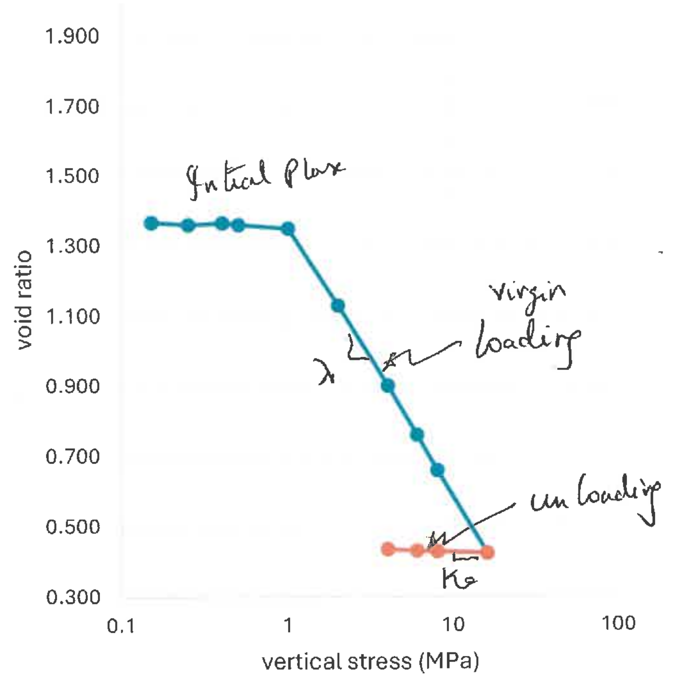
The sample compresses at gentle slope until 1MPa. From 1MPa to 16MPa it follows the virgin compression line.

$$\lambda = \frac{(1.35 - 0.424)}{\ln 16 - \ln 1} = 0.334$$

Similarly for unloading phase

$$\kappa_e = \frac{0.436 - 0.424}{\ln (16/4)} = 0.00865 \simeq 0.009.$$

1c)



Looking at 3DI data sheets, the λ and Ko ~~values~~ we obtained are close to Dog's bay Sand. This material has a large Carbonate fractions made of shells etc that are friable. So this soil will undergo 'clastic compression' with associated particle breakage, changes in particle arrangements and void filling. [40%]

1d) The initial void ratio of soil sample is 1.38. ; initial height = 20mm
During the initial phases of loading upto 1MPa, the soil sample follows more or less. the Ko line. This suggests that the soil sample has seen a previous vertical stress of 1MPa.

In oedometer $\varepsilon_v = \frac{\delta V}{V} = \frac{\delta h}{h}$ as it is 1-D with constant cross-section.

$$\frac{\delta h}{h} = \frac{\delta e}{1+e}$$

Void ratio at end of initial phase (upto 1MPa) $= \frac{1.38 - 1.35}{1.38} = 0.0217 = \frac{\delta h}{20}$.

$\delta h = 0.4347$ mm   so sample thickness $= 20 - 0.4345 = 19.565$ mm

Corresponding volumetric strain $= \varepsilon_v = \frac{0.4348}{20} = 2.17\%$

Loading along virgin compression line

$$\delta e = \frac{1.35 - 0.424}{1 + 1.35} = 0.394 = \frac{\delta h}{h}$$

$\therefore \delta h = 0.394 \times 19.565 = 7.709$ mm.

$\therefore$ height of the Sample $= 19.565 - 7.709 = \underline{\underline{11.826 \text{ mm}}}$

$$\varepsilon_{zz} = \frac{7.709}{19.565} = 39.4\ \% \ !!$$

Unloading along K_0 line.

$$\frac{\delta e}{1+e} = \frac{(0.436 - 0.424)}{1.424} = -8.427 \times 10^{-3} = \frac{\delta h}{h}$$

$\delta h = 0.0996$

$\therefore$ height of the sample $= 11.826 + 0.0996$

increases to $= \underline{\underline{11.926 \text{ mm}}}$

$\varepsilon_{zz} = -0.842\ \%$ (swelling). [20%]

1e) One-dimensional Stiffness $E_0 = (1+e)\dfrac{\sigma'_v}{\lambda}$ or $\dfrac{\nu\,\sigma'_v}{\lambda}$

So @ 16 MPa of virgin loading

$$E_0 = (1.424)\frac{16}{0.334} = \underline{\underline{68.22 \text{ MPa}}}$$

During unloading

$$E_0 = (1+e)\frac{\sigma'_v}{K_0} = \frac{1.424 \times 16}{0.009} = \underline{\underline{2531.56 \text{ MPa}}}$$

Unloading/Reloading stiffness $\gg$ Virgin stiffness.

[15%]

Q.3.(a) - Pore pressure isochrones [25%]

The problem describes drainage at the base only (upper surface impermeable due to capping layer), so the drainage path length is $L = 12$ m. The early phase ends when the consolidation front reaches the impermeable boundary. This occurs at:

$$
\begin{aligned}
t_{transition} &= \frac{L^2}{12c_v} \\
&= \frac{12^2}{12 \times 2.5} \\
&= 4.8 \text{ years}
\end{aligned}
$$

[5%]

At $t = 0$:

- Uniform excess pore pressure $u' = u'_i = 80$ kPa throughout the layer
- No consolidation has occurred

[5%]

At $t = 4$ years:

$$\tilde{T} = \frac{c_v t}{L^2} = \frac{2.5 \times 4}{144} = 0.069$$

Since $\tilde{T} < 1/12$, we are in the **early phase**. Depth of consolidation front:

$$
\begin{aligned}
\ell &= \sqrt{12c_v t} \\
&= \sqrt{12 \times 2.5 \times 4} \\
&= 10.95 \text{ m}
\end{aligned}
$$

The pore pressure distribution is:

- Parabolic from $u' = 0$ at $z = 0$ to $u' = u'_i = 80$ kPa at $z = \ell = 10.95$ m
- Constant $u' = u'_i = 80$ kPa from $z = 10.95$ m to $z = L = 12$ m

[5%]

At $t = 12$ years:

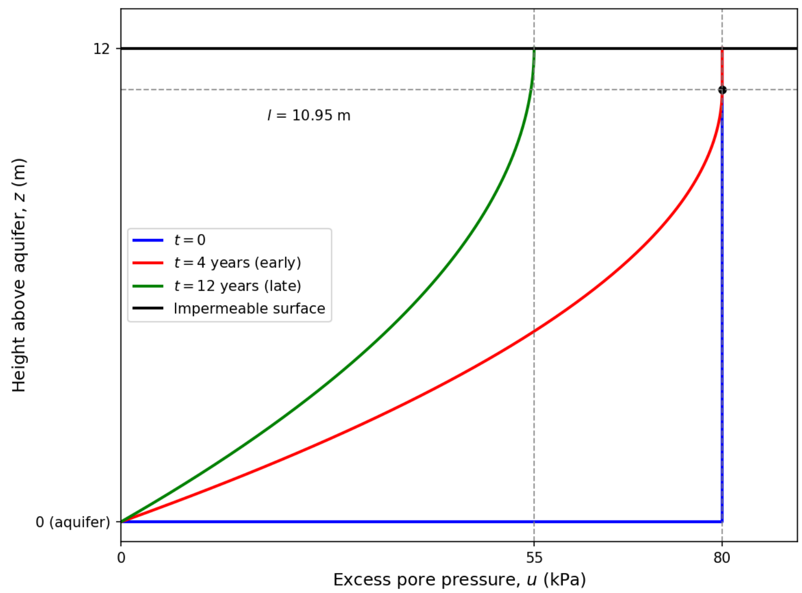
$$\tilde{T} = \frac{c_v t}{L^2} = \frac{2.5 \times 12}{144} = 0.208$$

Since $\tilde{T} > 1/12$, we are in the **late phase**. Maximum pore pressure (at impermeable boundary):

$$
\begin{aligned}
u'_L &= u'_i \exp\left(-3c_v(t - t_1)/L^2\right) \\
&= 80 \exp\left(-3 \times 2.5 \times (12 - 4.8)/144\right) \\
&= 80 \exp(-0.375) \\
&= 80 \times 0.687 \\
&= 55.0 \text{ kPa}
\end{aligned}
$$

The pore pressure distribution is parabolic from $u' = 0$ at $z = 0$ to $u' = u'_L = 55$ kPa at $z = L = 12$ m. [5%]

Sketch of all three isochrones:



[5%]

[Total 25%]

Q.3.(b) - Degree of consolidation and settlement [25%]

At $t = 4$ years, we established that $\tilde{T} = 0.069$ (early phase) with consolidation front at $\ell = 10.95$ m. The degree of consolidation equals the ratio of dissipated to initial excess pore pressure area:

$$A_{\text{initial}} = u_i' \times L = 80 \times 12 = 960 \text{ kPa·m}$$

[5%]

The area remaining comprises the parabolic region below the consolidation front plus the rectangular region of constant excess pore pressure above it:

$$\begin{aligned} A_{\text{remaining}} &= \frac{2}{3}\ell\, u_i' + (L - \ell)\, u_i' \\ &= \frac{2}{3} \times 10.95 \times 80 + (12 - 10.95) \times 80 \\ &= 584 + 84 = 668 \text{ kPa·m} \end{aligned}$$

[5%]

Therefore:

$$\tilde{S} = \frac{A_{\text{initial}} - A_{\text{remaining}}}{A_{\text{initial}}} = \frac{960 - 668}{960} = 0.304$$

[5%]

The ultimate settlement corresponds to full dissipation of the 80 kPa excess pore pressure:

$$\begin{aligned} s_\infty &= m_v \Delta\sigma' L \\ &= 0.25 \times 10^{-3} \times 80 \times 12 \\ &= 0.24 \text{ m} \\ &= 240 \text{ mm} \end{aligned}$$

[5%]

Settlement at $t = 4$ years:

$$\begin{aligned} s &= \tilde{S} \times s_\infty \\ &= 0.304 \times 240 \\ &= 73 \text{ mm} \end{aligned}$$

[5%]

[Total 25%]

Q.3.(c) - Swelling and permanent settlement [30%]

When pumping stops, the aquifer pressure recovers rapidly (the aquifer is permeable). The pore pressure at the base of the clay layer returns to its original value.

[5%]

At this instant, the clay near the base has consolidated - it has reduced void ratio and is now at a higher effective stress than the equilibrium condition. Relative to the new (recovered) boundary condition, the clay has *negative* excess pore pressure. Water is drawn back into the clay from the aquifer, pore pressures rise, effective stresses fall, and the clay swells.

[5%]

The clay is normally consolidated, meaning it was on the virgin compression line (λ-line) during loading. On unloading, the clay follows the much stiffer swelling line (κ-line).

[5%]

For a given change in effective stress $\Delta\sigma'$:

- Settlement (loading on λ-line): $\delta s = m_v \Delta\sigma' L$
- Heave (unloading on κ-line): $\delta h = (\kappa/\lambda) \times m_v \Delta\sigma' L$

Since $\kappa/\lambda = 0.2$, the heave is only 20% of the settlement for the same stress change. [5%]

The settlement at $t = 4$ years was 73 mm, achieved by consolidation on the λ-line.
When the aquifer recovers, the clay swells on the κ-line. The maximum heave (as $t \to \infty$) is:

$$\begin{aligned} h_\infty &= \frac{\kappa}{\lambda} \times s(t = 4) \\ &= 0.2 \times 73 \\ &= 15 \text{ mm} \end{aligned}$$

[5%]

Therefore the permanent settlement is:

$$\begin{aligned} s_{permanent} &= s(t=4) - h_\infty \\ &= 73 - 15 \\ &= 58 \text{ mm} \end{aligned}$$

[5%]

[Total 30%]

Q.3.(d) - Settlement-time response [20%]

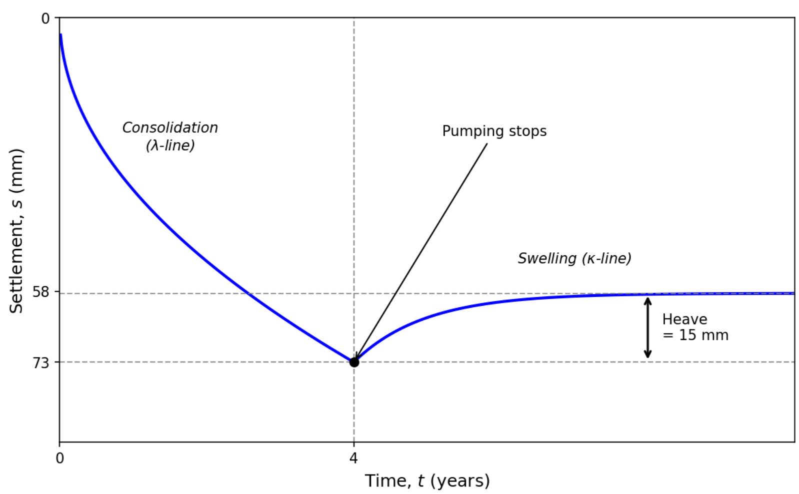
The settlement-time curve should show:

- Rapid increase from $t = 0$, curving as rate decreases ($\sqrt{t}$ behaviour in early phase)
- Settlement reaches 73 mm at $t = 4$ years

[10%]

- After $t = 4$ years: settlement *decreases* (heave) as clay swells
- Swelling is rapid initially then slows, asymptotic behaviour towards a permanent settlement of 58 mm
- Swelling completes faster than consolidation due to higher $c_{v,swell}$

[10%]



The curve shows a clear peak at $t = 4$ years followed by partial recovery. The permanent settlement of 58 mm represents the irrecoverable plastic strain from loading on the virgin compression line.

[Total 20%]

Q.4.(a) - Uniaxial capacities [40%]

From the databook, the ultimate vertical bearing capacity for undrained loading of a surface foundation is:

$q_f = N_c \cdot s_c \cdot s_u$

Bearing capacity factor:

$N_c = 5.14$

Shape factor:

$$\begin{aligned} s_c &= 1 + 0.2\frac{B}{L} \\ &= 1 + 0.2 \times \frac{12}{20} \\ &= 1.12 \end{aligned}$$

[5%]

Ultimate bearing pressure:

$$\begin{aligned} q_f &= 5.14 \times 1.12 \times 40 \\ &= 230 \text{ kPa} \end{aligned}$$

Ultimate vertical capacity:

$$\begin{aligned} V_{ult} &= q_f \times B \times L \\ &= 230 \times 12 \times 20 \\ &= 55200 \text{ kN} \\ &= 55.2 \text{ MN} \end{aligned}$$

[5%]

Horizontal capacity, H_{ult}:

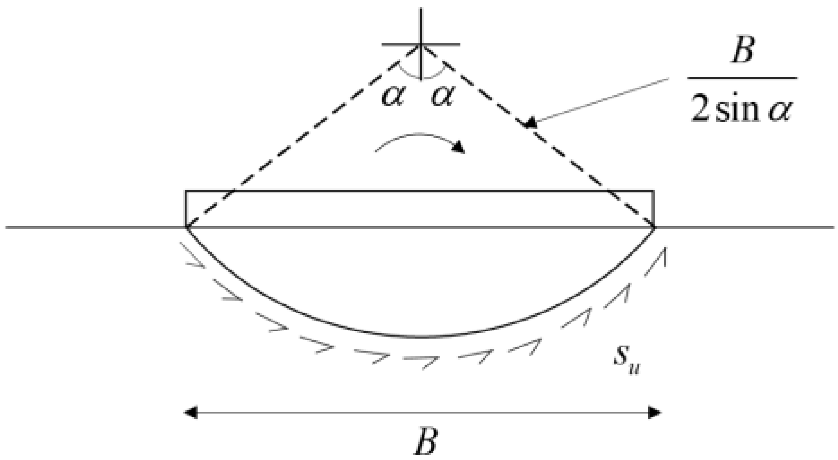
The horizontal capacity is governed by sliding at the soil-foundation interface, assuming full adhesion:

$$\begin{aligned} H_{ult} &= B \times L \times s_u \\ &= 12 \times 20 \times 40 \\ &= 9600 \text{ kN} \\ &= 9.6 \text{ MN} \end{aligned}$$

[5%]

Moment capacity, M_{ult}:

Consider a scoop mechanism rotating about an axis along the long edge of the foundation. The scoop has half-angle α and radius $r = B/(2\sin\alpha)$: [5%]



For a virtual rotation $\delta\theta$ about the scoop centre:
Arc length of slip surface:

$$L_{arc} = 2\alpha \cdot r = \frac{\alpha B}{\sin\alpha}$$

Displacement along the arc:

$$\delta s = r \cdot \delta\theta = \frac{B}{2\sin\alpha} \cdot \delta\theta$$

Energy dissipated along the slip surface:

$$\begin{aligned}\delta W_{int} &= s_u \times L_{arc} \times L \times \delta s \\ &= s_u \times \frac{\alpha B}{\sin\alpha} \times L \times \frac{B}{2\sin\alpha} \cdot \delta\theta \\ &= \frac{s_u B^2 L\alpha}{2\sin^2\alpha} \cdot \delta\theta\end{aligned}$$

[5%]

External work done by the moment:

$$\delta W_{ext} = M \cdot \delta\theta$$

Equating internal and external work:

$$M_{ult} = \frac{s_u B^2 L\alpha}{2\sin^2\alpha}$$

[5%]

To find the optimal mechanism, minimise M_{ult} with respect to α:

$$\begin{aligned}
\frac{d}{d\alpha}\left(\frac{\alpha}{\sin^2\alpha}\right) &= 0 \\
\frac{\sin^2\alpha - 2\alpha\sin\alpha\cos\alpha}{\sin^4\alpha} &= 0 \\
\sin\alpha &= 2\alpha\cos\alpha \\
\tan\alpha &= 2\alpha
\end{aligned}$$

Solving numerically gives $\alpha = 67°$ (1.17 rad). Substituting:

$$\frac{\alpha}{2\sin^2\alpha} = \frac{1.17}{2 \times 0.921^2} = \frac{1.17}{1.70} = 0.69$$

Therefore:

$$M_{ult} = 0.69 \cdot B^2 L \cdot s_u \qquad Q.E.D.$$

[5%]

Numerical value:

$$\begin{aligned}
M_{ult} &= 0.69 \times 12^2 \times 20 \times 40 \\
&= 79500 \text{ kN·m} \\
&= 79.5 \text{ MN·m}
\end{aligned}$$

[5%]

[Total 40%]

Q.4.(b) - Design earthquake check [40%]

Vertical load (unchanged):

$$V = mg = 19 \text{ MN}$$

[5%]

Horizontal load:

$$H = ma = \frac{V}{g} \cdot a = V \cdot \frac{a}{g}$$

[5%]

Overturning moment about foundation level:

$$M = ma \cdot h = V \cdot h \cdot \frac{a}{g}$$

[5%]

Loads at $a = 0.35g$:

$$V = 19 \text{ MN}$$
$$H = 19 \times 0.35 = 6.65 \text{ MN}$$
$$M = 19 \times 10 \times 0.35 = 66.5 \text{ MN·m}$$

Normalised loads:

$$\frac{V}{V_{ult}} = \frac{19}{55.2} = 0.344$$
$$\frac{H}{H_{ult}} = \frac{6.65}{9.6} = 0.693$$
$$\frac{M}{M_{ult}} = \frac{66.5}{79.5} = 0.836$$

[5%]

Apply Taiebat & Carter failure surface:

From the databook, the VHM failure surface with interface tension and some rearrangement is:

$$\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\frac{H}{H_{ult}}\right|^3 = 1$$

[5%]

Substituting:

$$\begin{aligned} \text{LHS} &= (0.344)^2 + [0.836 \times (1 - 0.3 \times 0.693)]^2 + (0.693)^3 \\ &= 0.118 + [0.836 \times 0.792]^2 + 0.333 \\ &= 0.118 + (0.662)^2 + 0.333 \\ &= 0.118 + 0.438 + 0.333 \\ &= 0.889 < 1 \end{aligned}$$

[10%]

Since LHS < 1, the load state lies inside the failure surface. Therefore the foundation can safely resist the design earthquake with $a = 0.35g$. Therefore the foundation can safely resist the design earthquake with a $= 0.35g$.

[5%]

[Total 40%]

Q.4.(c) - Implications of interface tension assumption [20%]

Any four of the below points.

Physical meaning:

- The foundation is effectively "bonded" to the soil. Tensile stresses can develop at the interface without separation occurring.

Valid assumption when we have:

- Skirted foundations where soil is trapped between skirts and suction can develop
- Suction caissons where negative pore pressures provide tensile capacity
- Foundations with sufficient embedment where the weight of overlying soil prevents separation
- Short-duration loading where there is insufficient time for drainage and separation to occur

Implications for the loss of interface tension:

- The effective contact area reduces as the foundation rocks onto one edge
- The problem reverts to Meyerhof's reduced effective area approach
- The bearing capacity is reduced compared to the tension-sustained case
- The failure surface in V-H-M space contracts significantly

Implications for the earthquake loading problem:

- The bearing capacity would be reduced
- The foundation would experience rocking behaviour
- Energy dissipation through repeated lift-off and impact could occur
- P-δ effects from the tilting foundation could increase the overturning moments
- For a conservative design, the analysis should be repeated using Meyerhof's reduced area approach (without interface tension) and the results compared

[Total 20%]