

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 6 May 2026 09.30 to 11.10

Module 3D1

GEOTECHNICAL ENGINEERING I

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

Engineering Data Book

CUED approved calculator allowed

Attachment: 3D1 & 3D2: Geotechnical Engineering Data Book (21 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Explain how soil particles are formed from parent rocks. [10%]

(b) How do soil particles get transported and deposited by flowing water? How are varved deposits formed from the soil particles transported by water? [15%]

(c) A silty sand layer of 20 m thickness overlying a gravel bed was found at a site. The D_{10} size for this soil is 0.006 mm and in-situ tests reveal that the soil layer has a void ratio of 0.68, and the particles have a specific gravity of 2.61. The surface tension of water and unit weight of water can be taken as $7.42 \times 10^{-2} \text{ N m}^{-1}$ and 10 kN m^{-3} respectively. Calculate the vertical effective stress at a depth of 14 m, if:

- (i) the water table is at a depth 16 m below the ground surface; and
- (ii) the water table is at the ground surface.

Is the change in the effective stress at this depth of significance? [30%]

(d) Explain how a standard Proctor test is conducted on a soil sample collected from a site. How is the data from this test plotted? [15%]

(e) A standard Proctor test yielded the data in Table 1. Determine the maximum dry density and optimum moisture content for this soil sample. Calculate the saturation ratio and air void ratio when this soil is compacted to the maximum dry density. [30%]

Table 1

Void ratio, e [–]	0.77	0.65	0.58	0.54	0.55	0.60	0.68
Water content, w [%]	3.5	6	8	10	16	18	20

- 2 (a) Explain briefly how an oedometer test is conducted on a soil sample. [10%]
- (b) How would you define the one-dimensional compressive strain, shear strain and volumetric strain in a soil sample? Which of these can be obtained from an oedometer test? [15%]
- (c) An oedometer ring with an internal diameter of 50.8 mm contains a soil sample of 20 mm thickness. The void ratio of the sample was 1.38. A platen was placed on top of this sample and the sample was loaded and then unloaded, generating the data in Table 2 and Table 3, respectively. Calculate the slopes of the normal compression and unload-reload lines, λ and κ , respectively, where σ'_v is the vertical effective stress and e is the void ratio. By comparing these values to the data in the 3D1 & 3D2: Geotechnical Engineering Data Book, can you identify the soil being tested? What kind of compression regime is this soil likely to experience when loaded as a consequence of its likely mineralogy? [40%]
- (d) What was the past maximum vertical effective stress that the soil was subjected to? Calculate the thickness of the soil sample, when it is first loaded to 8 MPa and then when it is unloaded to 4 MPa. What are the corresponding volumetric strains? [20%]
- (e) Estimate the one-dimensional stiffness of the soil during the loading and unloading phases. [15%]

Table 2

σ'_v [MPa]	e [–]
0.15	1.367
0.25	1.360
0.4	1.365
0.5	1.360
1	1.350
2	1.130
4	0.900
6	0.760
8	0.660
16	0.424

Table 3

σ'_v [MPa]	e [–]
16	0.424
8	0.430
6	0.433
4	0.436

3 A 12 m thick layer of normally consolidated saturated clay overlies a sandy aquifer. The clay is capped by a thin, low-permeability surface crust that may be considered impermeable for the purposes of this analysis. The clay drains freely into the aquifer below. Due to groundwater extraction, the pore water pressure at the base of the clay layer is instantaneously reduced by 80 kPa and maintained at this level. The clay has a coefficient of consolidation $c_v = 2.5 \text{ m}^2 \text{ year}^{-1}$ and a coefficient of volume compressibility $m_v = 0.25 \text{ MPa}^{-1}$. The ratio of the slopes of the unload-reload line and the normal compression line is $\kappa/\lambda = 0.2$. Using the parabolic isochrone method:

- (a) Sketch the pore pressure distribution in the clay layer at $t = 0$, $t = 4$ years, and $t = 12$ years. Indicate clearly whether consolidation is in the early or late phase at each time. [25%]
- (b) Calculate the degree of consolidation and the surface settlement at $t = 4$ years. [25%]
- (c) Groundwater extraction is halted after 4 years and the aquifer pressure recovers rapidly. Explain why swelling will occur but will not fully recover the settlement experienced in the preceding four years. Hence estimate the permanent settlement. [30%]
- (d) Sketch the expected settlement-time response from $t = 0$ to $t = 10$ years covering both settlement and swelling periods. [20%]

4 A six-storey residential building is supported on a rectangular raft foundation with plan dimensions $B = 12$ m and $L = 20$ m. The total vertical load is $V = 19$ MN. The raft foundation is founded at the surface of a firm clay soil with undrained shear strength $s_u = 40$ kPa and bulk unit weight $\gamma = 18$ kN m⁻³. Assume that tension can be sustained at the soil-foundation interface.

(a) Calculate the uniaxial ultimate vertical capacity V_{ult} , ultimate horizontal capacity H_{ult} and ultimate moment capacity M_{ult} , of the foundation. In computing the uniaxial moment capacity M_{ult} , show that the uniaxial ultimate moment capacity is $M_{ult} = 0.69B^2Ls_u$ using an upper bound solution. [40%]

(b) Idealise the building mass as a lumped mass at height $h = 10$ m above foundation level. Under design earthquake loading, the lumped mass experiences a horizontal acceleration of $0.35g$. Express the foundation loads V , H , and M in terms of the earthquake acceleration, and hence determine whether the foundation can safely resist the design earthquake. [40%]

(c) Discuss the implications of the assumption that tension can be sustained at the soil-foundation interface. [20%]

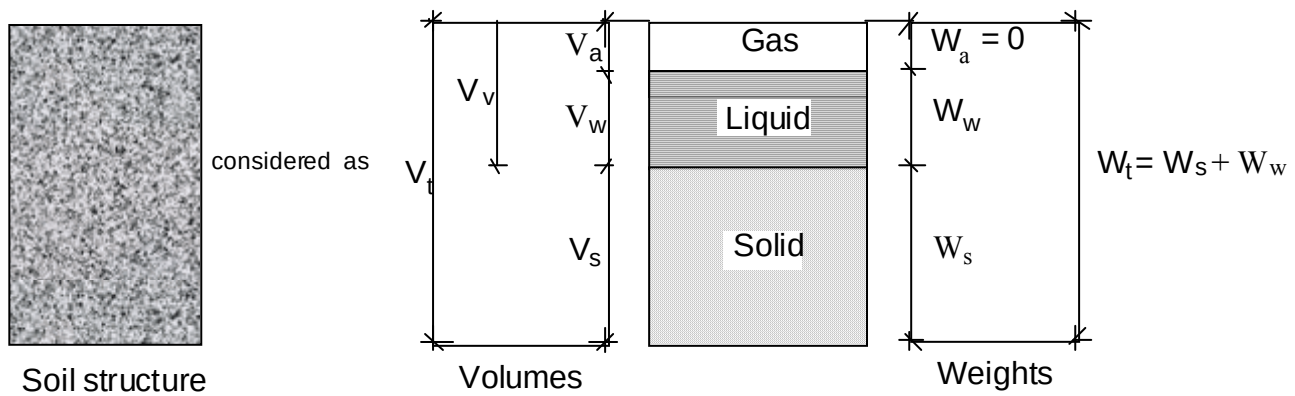
END OF PAPER

Engineering Tripos Part IIA

3D1 & 3D2 Geotechnical Engineering Data Book

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General definitions



Specific gravity of solid

$$G_s$$

Voids ratio

$$e = V_v / V_s$$

Specific volume

$$v = V_t / V_s = 1 + e$$

Porosity

$$n = V_v / V_t = e / (1 + e)$$

Water content

$$w = (W_w / W_s)$$

Degree of saturation

$$S_r = V_w / V_v = (w G_s / e)$$

Unit weight of water

$$\gamma_w = 9.81 \text{ kN/m}^3$$

Unit weight of soil

$$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e} \right) \gamma_w$$

Buoyant saturated unit weight

$$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w$$

Unit weight of dry solids

$$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e} \right) \gamma_w$$

Air volume ratio

$$A = V_a / V_t = \left(\frac{e(1 - S_r)}{1 + e} \right)$$

Soil classification (BS1377)Liquid limit w_L Plastic Limit w_P Plasticity Index $I_P = w_L - w_P$ Liquidity Index $I_L = \frac{w - w_P}{w_L - w_P}$ Activity = $\frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2 \mu\text{m}}$ Sensitivity = $\frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$ (at the same water content)*Classification of particle sizes:–*

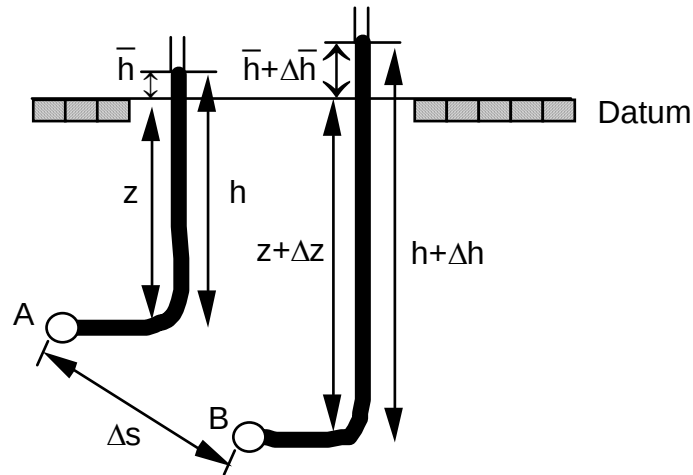
Boulders	larger than	200 mm	
Cobbles	between	200 mm	and 60 mm
Gravel	between	60 mm	and 2 mm
Sand	between	2 mm	and 0.06 mm
Silt	between	0.06 mm	and 0.002 mm
Clay	smaller than	0.002 mm (two microns)	

D equivalent diameter of soil particle

 D_{10} , D_{60} etc. particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains. C_U uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w (\bar{h} + z)$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta \bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta \bar{u} = \gamma_w (\bar{h} + \Delta \bar{h})$

Hydraulic gradient A $\rightarrow$ B $i = - \frac{\Delta \bar{h}}{\Delta s}$

Hydraulic gradient (3D) $i = - \nabla \bar{h}$

Darcy's law $V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10 \text{ mm}$: non-laminar flow
 $10 \text{ mm} > D_{10} > 1 \mu\text{m}$: $k \cong 0.01 (D_{10} \text{ in mm})^2 \text{ m/s}$
 clays : $k \cong 10^{-9} \text{ to } 10^{-11} \text{ m/s}$

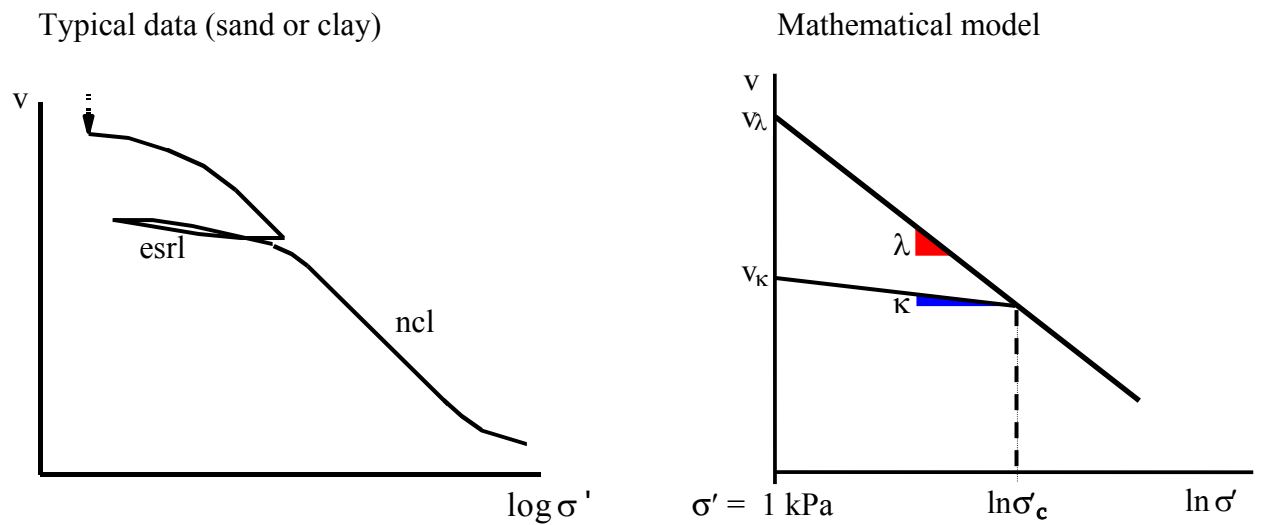
Saturated capillary zone

$h_c = \frac{4T}{\gamma_w d}$: capillary rise in tube diameter d , for surface tension T

$h_c \approx \frac{3 \times 10^{-5}}{D_{10}} \text{ m}$: for water at 10°C ; note air entry suction is $u_c = - \gamma_w h_c$

One-Dimensional Compression

• Fitting data



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$
 $= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

• Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta \sigma' / \Delta \epsilon)_o$

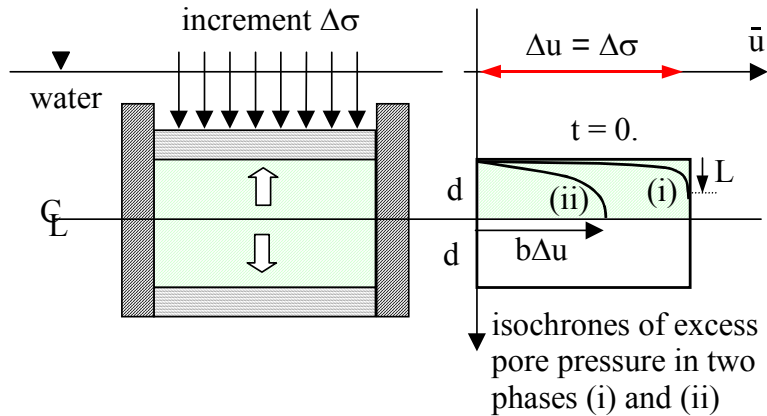
Tangent 1D plastic compression modulus $E_o = v \sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v \sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u}) dz$	$= \int (\Delta u - \bar{u}) / E_o dz$
Coefficient of consolidation	c_v	$= \frac{k}{m_v \gamma_w}$	$= \frac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \frac{c_v t}{d^2}$	
Relative settlement	R_v	$= \frac{\rho}{\rho_{ult}}$	

• Solutions for initially rectangular distribution of excess pore pressure



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12 c_v t$

$$R_v = \sqrt{\frac{4T_v}{3}} \quad \text{for } T_v < 1/12$$

Phase (ii) $b = \exp(1/4 - 3T_v)$

$$R_v = [1 - 2/3 \exp(1/4 - 3T_v)] \quad \text{for } T_v > 1/12$$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

- **Principle of effective stress (saturated soil)**

$$\text{total stress } \sigma = \text{effective stress } \sigma' + \text{pore water pressure } u$$

- **Principal components of stress and strain**

sign convention	compression positive
total stress	$\sigma_1, \sigma_2, \sigma_3$
effective stress	$\sigma'_1, \sigma'_2, \sigma'_3$
strain	$\varepsilon_1, \varepsilon_2, \varepsilon_3$

- **Simple Shear Apparatus (SSA)** ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

$$\text{work increment per unit volume} \quad \delta W = \tau \delta \gamma + \sigma' \delta \varepsilon$$

- **Biaxial Apparatus - Plane Strain (BA-PS)** ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ'_2 , in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma'_1 + \sigma'_3)/2 = s - u$
shear stress	$t = (\sigma'_1 - \sigma'_3)/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma'_1 \delta \varepsilon_1 + \sigma'_3 \delta \varepsilon_3$
	$\delta W = s' \delta \varepsilon_v + t \delta \varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

• **Triaxial Apparatus – Axial Symmetry (TA-AS)** (cylindrical element with radial symmetry)

total axial stress	$\sigma_a = \sigma'_a + u$
total radial stress	$\sigma_r = \sigma'_r + u$
total mean normal stress	$p = (\sigma_a + 2\sigma_r)/3$
effective mean normal stress	$p' = (\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	$q = \sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	$\eta = q/p'$
axial strain	ϵ_a
radial strain	ϵ_r
volumetric strain	$\epsilon_v = \epsilon_a + 2\epsilon_r$
triaxial shear strain	$\epsilon_s = \frac{2}{3}(\epsilon_a - \epsilon_r)$
work increment per unit volume	$\delta W = \sigma'_a \delta \epsilon_a + 2\sigma'_r \delta \epsilon_r$
	$\delta W = p' \delta \epsilon_v + q \delta \epsilon_s$

Types of triaxial test include:

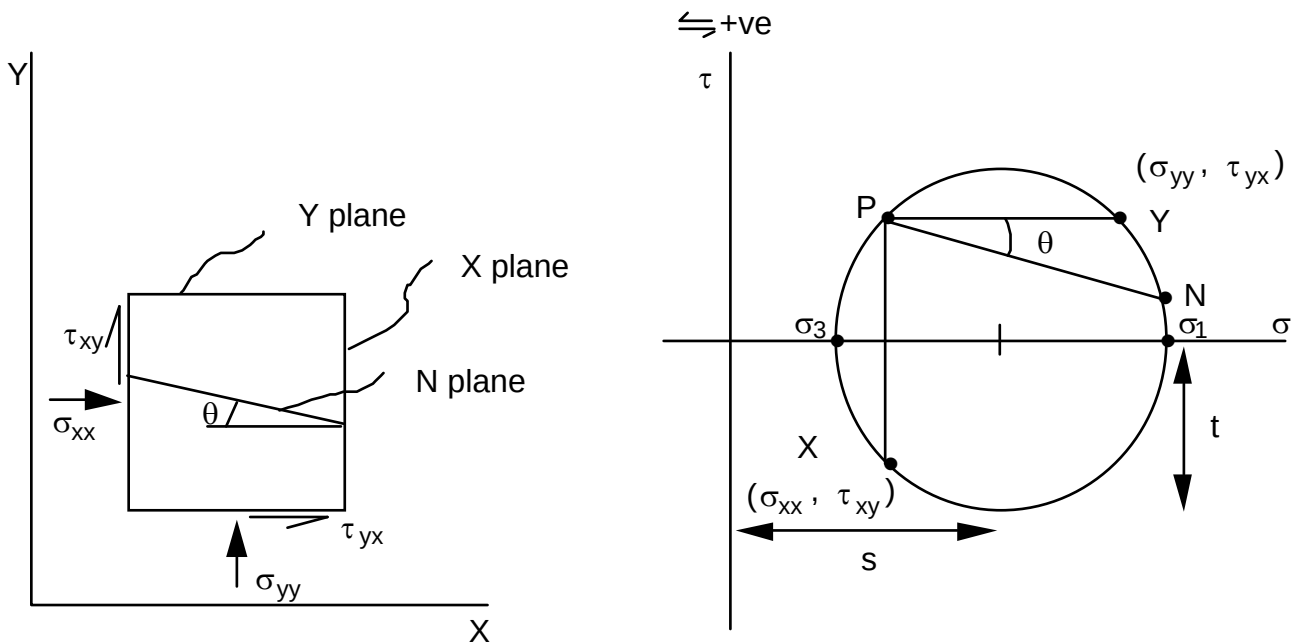
isotropic compression in which p' increases at zero q

triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• **Mohr's circle of stress (1–3 plane)**

Sign of convention: compression, and counter-clockwise shear, positive



Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ -line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

$$\text{compressibility} \quad m_v = d\varepsilon / d\sigma'$$

$$\text{constrained modulus} \quad E_o = 1 / m_v$$

Physically fundamental parameters

$$\text{shear modulus} \quad G' = dt / d\varepsilon_\gamma$$

$$\text{bulk modulus} \quad K' = dp' / d\varepsilon_v$$

Parameters which can be used for constant-volume deformations

$$\text{undrained shear modulus} \quad G_u = G'$$

$$\text{undrained bulk modulus} \quad K_u = \infty \quad (\text{neglecting compressibility of water})$$

Alternative convenient parameters

$$\text{Young's moduli} \quad E' \text{ (effective), } E_u \text{ (undrained)}$$

$$\text{Poisson's ratios} \quad \nu' \text{ (effective), } \nu_u = 0.5 \text{ (undrained)}$$

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

$$\text{Relationships:} \quad G = \frac{E}{2(1 + \nu)}$$

$$K = \frac{E}{3(1 - 2\nu)}$$

$$E_o = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)}$$

Cam Clay

• Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan \phi_{\text{crit}}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin \phi_{\text{crit}}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

• General equations of plastic work

Plastic work and dissipation

$$\sigma^* \delta \varepsilon^* + \tau^* \delta \gamma^* = \mu^*_{\text{crit}} \sigma^* \delta \gamma^*$$

Plastic flow rule – normality

$$\frac{d\tau^*}{d\sigma^*} \cdot \frac{d\gamma^*}{d\varepsilon^*} = -1$$

• General yield surface

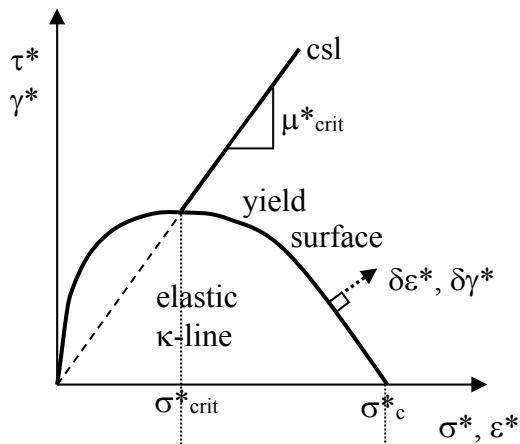
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{\text{crit}} \cdot \ln \left[\frac{\sigma^*_c}{\sigma^*} \right]$$

• Parameter values which fit soil data

	London Clay	Weald Clay	Kaolin	Dog's Bay Sand	Ham River Sand
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c, \text{ virgin}}$ kPa	1	1	1	Loose 500 Dense 1500	Loose 2500 Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	-----	-----
w_P	0.26	0.18	0.42	-----	-----
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^* , κ^* , Γ^* , $\sigma^*_{c, \text{ virgin}}$ should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

- The yield surface in (σ^*, τ^*, v) space



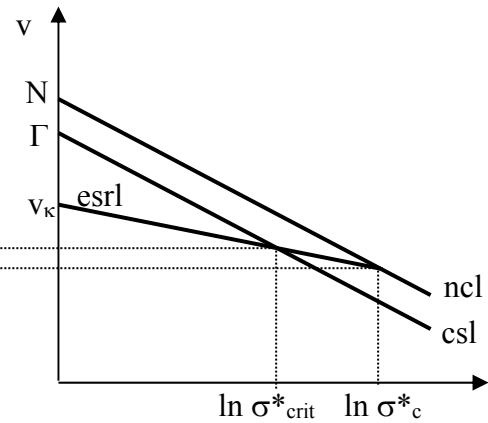
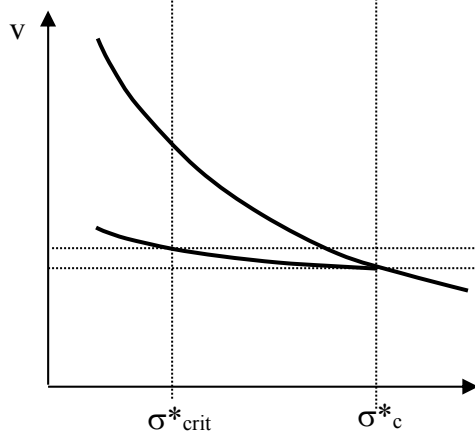
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

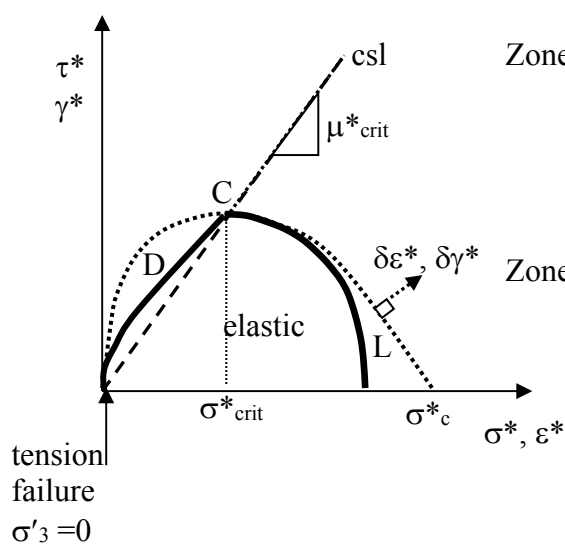
cs1: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



- **Regions of limiting soil behaviour**



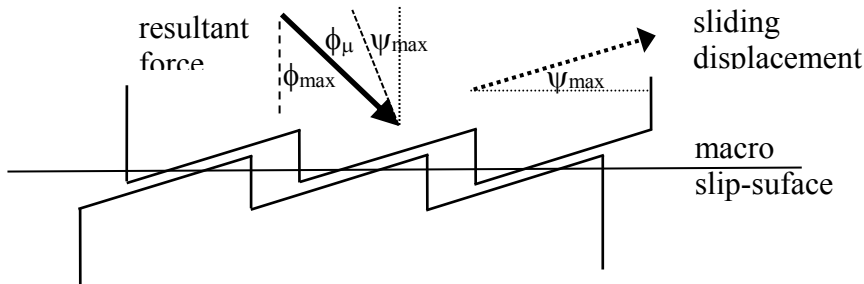
Variation of Cam Clay yield surface

Zone D: denser than critical, “dry”,
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, “wet”,
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

• Friction and dilatancy: the saw-blade model of direct shear

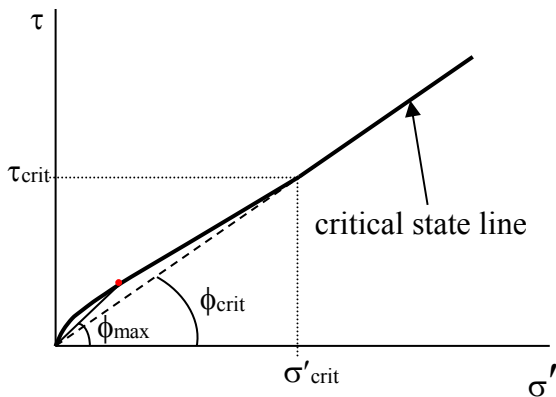


Intergranular angle of friction at sliding contacts ϕ_{μ}

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_{\mu} + \psi_{max}$

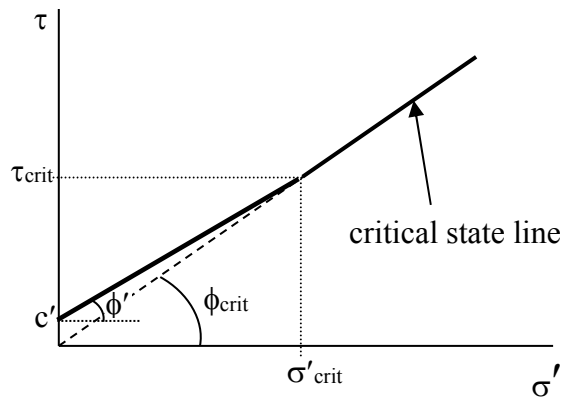
• Friction and dilatancy: secant and tangent strength parameters



Secant angle of internal friction

$$\begin{aligned}\tau &= \sigma' \tan \phi_{max} \\ \phi_{max} &= \phi_{crit} + \Delta\phi \\ \Delta\phi &= f(\sigma'_{crit}/\sigma')\end{aligned}$$

typical envelope fitting data:
power curve
 $(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$



Tangent angle of shearing envelope

$$\begin{aligned}\tau &= c' + \sigma' \tan \phi' \\ c' &= f(\sigma'_{crit})\end{aligned}$$

typical envelope:
straight line
 $\tan \phi' = 0.85 \tan \phi_{crit}$
 $c' = 0.15 \tau_{crit}$

• Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26^\circ$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2^\circ$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $I_D = \frac{(e_{\text{max}} - e)}{(e_{\text{max}} - e_{\text{min}})}$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test

e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $I_C = \ln(\sigma_c / p')$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{\text{max}} - \phi_{\text{crit}}) = f(I_R)$

Relative dilatancy index $I_R = I_D I_C - 1$ where:

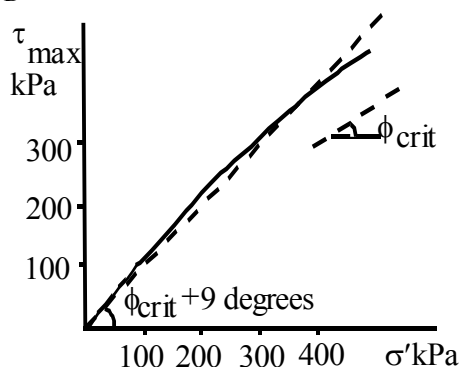
$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state

$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

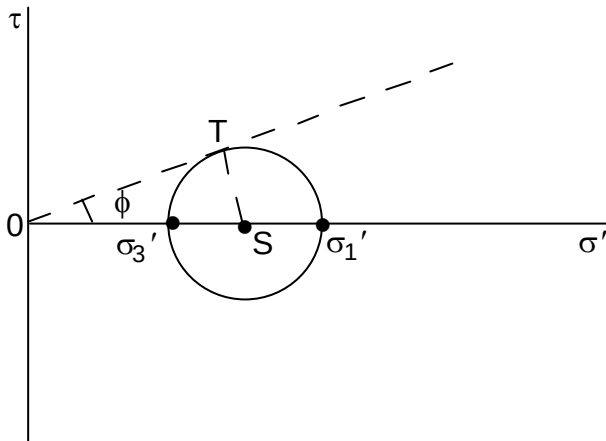
plane strain conditions	$(\phi_{\text{max}} - \phi_{\text{crit}})$	$= 0.8 \psi_{\text{max}}$	$= 5 I_R$ degrees
triaxial strain conditions	$(\phi_{\text{max}} - \phi_{\text{crit}})$	$= 3 I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{\text{max}}$	$= 0.3 I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$$\phi_{\text{max}} > \phi_{\text{crit}} + 9^\circ \quad \text{for } I_D = 1, \sigma' = 400 \text{ kPa}$$

• Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



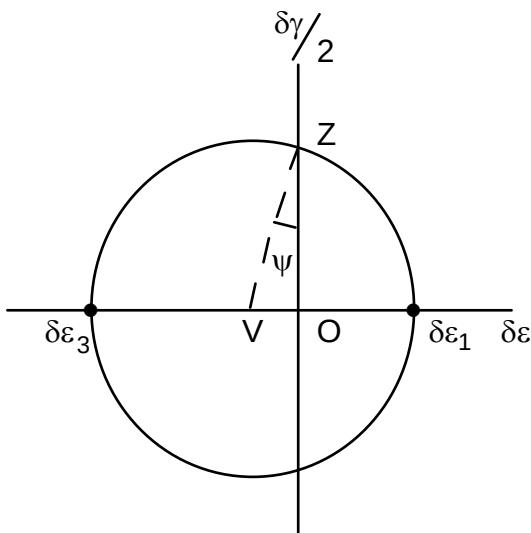
$$\begin{aligned}\sin \phi &= TS/OS \\ &= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2} \\ \left[\frac{\sigma_1'}{\sigma_3'} \right] &= \frac{(1 + \sin \phi)}{(1 - \sin \phi)}\end{aligned}$$

Angle of shearing resistance:

at peak strength $\phi_{\max}$ at $\left[\frac{\sigma_1'}{\sigma_3'} \right]_{\max}$

at critical state ϕ_{crit} after large shear strains

• Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\begin{aligned}\sin \psi &= VO/VZ \\ &= -\frac{(\delta \epsilon_1 + \delta \epsilon_3)/2}{(\delta \epsilon_1 - \delta \epsilon_3)/2} \\ &= -\frac{\delta \epsilon_v}{\delta \epsilon_\gamma}\end{aligned}$$

$$\left[\frac{\delta \epsilon_1}{\delta \epsilon_3} \right] = -\frac{(1 - \sin \psi)}{(1 + \sin \psi)}$$

at peak strength $\psi = \psi_{\max}$ at $\left[\frac{\sigma_1'}{\sigma_3'} \right]_{\max}$

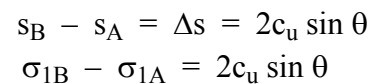
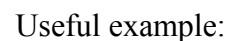
at critical state $\psi = 0$ since volume is constant

Tresca $|\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \delta \varepsilon_\gamma$$

$$D = A c_u x$$
Rotation of major principal stress θ 
$$ds = 2c_u d\theta$$


$$\theta = 30^\circ$$

$$\sigma_{1B} - \sigma_{1A} = C_u$$

$$\tau_D / c_H = 0.87$$

 σ_{1A} = major principal stress in zone A σ_{1B} = major principal stress in zone B

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

Passive pressure:

$$\sigma'_h > \sigma'_v$$

$$\sigma'_1 = \sigma'_h \text{ (assuming principal stresses are horizontal and vertical)}$$

$$\sigma'_3 = \sigma'_v$$

$$K_p = (1 + \sin \phi)/(1 - \sin \phi) = 1 / K_a$$

Diagram illustrating the geometry of a dislocation line (discontinuity) and the associated angles and stresses. The dislocation line is labeled "discontinuity". The angle between the dislocation line and the slip plane is labeled $(\Omega + \delta)/2$. The angle between the dislocation line and the slip plane is also labeled $(\Omega - \delta)/2$. The angle between the dislocation line and the slip plane is also labeled θ . The stress components are labeled σ_{1A} , σ_{1B} , σ_D , and τ_D .

Integration gives $s'_B/s'_A = \exp (2\theta \tan \phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin \phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc} \left[1 + \frac{(n-1)(n_{max}^\alpha - 1)}{(n_{max} - 1)} \right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max} / \sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max} / \sigma'_{v,min}$

α is to be taken as $1.2 \sin \phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta \sigma_c = \sigma_c - \sigma_o$

At radius r : small displacement $\rho = \frac{\delta A}{2\pi r}$

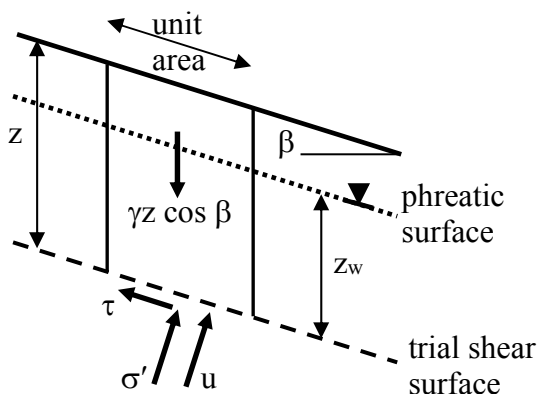
small shear strain $\gamma = \frac{2\rho}{r}$

Radial equilibrium: $r \frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta \sigma_c = G \frac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta \sigma_c = s_u \left[1 + \ln \frac{G}{s_u} + \ln \frac{\delta A}{A} \right]$

Infinite slope analysis



$$\begin{aligned} u &= \gamma_w z_w \cos^2 \beta \\ \sigma &= \gamma z \cos^2 \beta \\ \sigma' &= (\gamma z - \gamma_w z_w) \cos^2 \beta \\ \tau &= \gamma z \cos \beta \sin \beta \end{aligned}$$

$$\tan \phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan \beta}{\left(1 - \frac{\gamma_w z_w}{\gamma z}\right)}$$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f , of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$$N_c = 2 + \pi \quad (\text{Prandtl, 1921})$$

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_c = 1 + 0.2 B / L$$

The exact solution for a rough circular foundation ($D = B = L$) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h , is:

$$d_c = 1 + 0.33 \tan^{-1} (h/B) \quad (\text{or } h/D \text{ for a circular foundation})$$

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

$$\text{If } V/V_{ult} > 0.5: \quad \frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2 \frac{V}{V_{ult}} - 1 \right)^2$$

$$\text{If } V/V_{ult} < 0.5: \quad H = H_{ult} = B s_u$$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

$$\text{Without lift-off:} \quad \left(\frac{V}{V_{ult}} \right)^2 + \left[\frac{M}{M_{ult}} \left(1 - 0.3 \frac{H}{H_{ult}} \right) \right]^2 + \left| \left(\frac{H}{H_{ult}} \right)^3 \right| - 1 = 0 \quad (\text{Taiebet \& Carter 2000})$$

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f , of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2) e^{(\pi \tan \phi)} \quad (\text{Prandtl 1921})$$

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2 (N_q - 1) \tan \phi$$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

$$\text{Rough base:} \quad N_\gamma = 0.1054 e^{9.6\phi}$$

$$\text{Smooth base:} \quad N_\gamma = 0.0663 e^{9.3\phi}$$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin \phi) / L$$

$$s_\gamma = 1 - 0.3 B / L$$

For circular footings take $L = B$.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

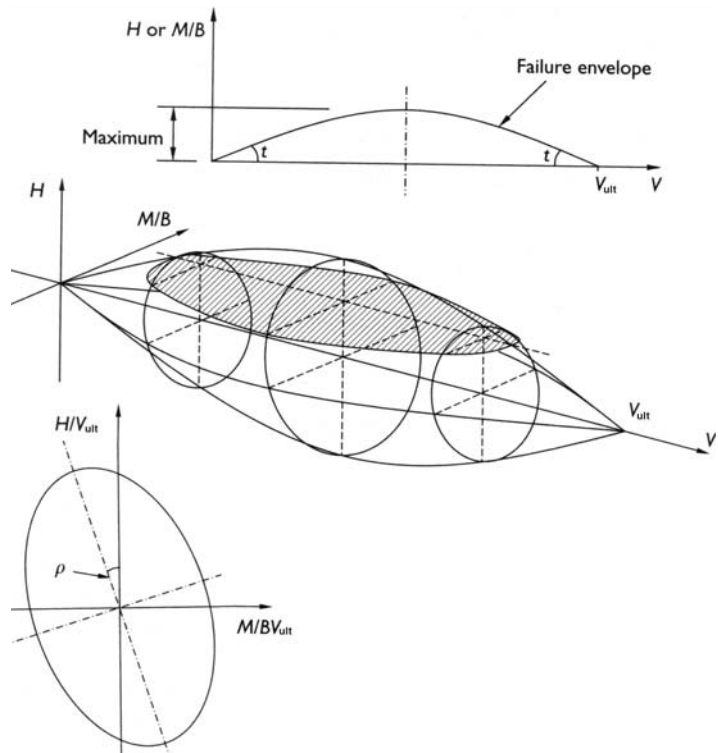
Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h} \right]^2 + \left[\frac{M/BV_{ult}}{t_m} \right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m} \right] = \left[\frac{V}{V_{ult}} \left(1 - \frac{V}{V_{ult}} \right) \right]^2$$

where $C = \tan \left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2t_h t_m} \right)$ (Butterfield & Gottardi, 1994)

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15^\circ$. Note that t_h is the friction coefficient, $H/V = \tan \phi$, during sliding.



Settlement of Shallow Foundations

Elastic stress distributions below point, strip and circular loads

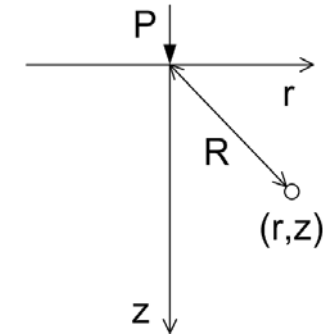
Point loading (Boussinesq solution)

Vertical stress $\sigma_z = \frac{3Pz^3}{2\pi R^5}$

Radial stress $\sigma_r = \frac{P}{2\pi R^2} \left[\frac{3r^2z}{R^3} - \frac{(1-2\nu)R}{R+z} \right]$

Tangential stress $\sigma_\theta = \frac{P(1-2\nu)}{2\pi R^2} \left[\frac{R}{R+z} - \frac{z}{R} \right]$

Shear stress $\tau_{rz} = \frac{3Prz^2}{2\pi R^5}$



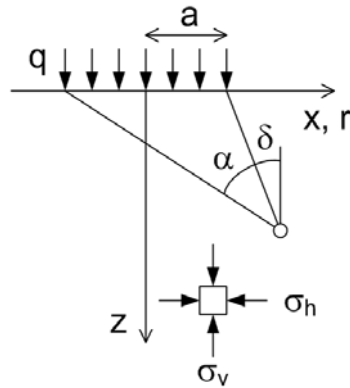
Uniformly-loaded strip

Vertical stress $\sigma_v = \frac{q}{\pi} [\alpha + \sin \alpha \cos(\alpha + 2\delta)]$

Horizontal stress $\sigma_h = \frac{q}{\pi} [\alpha - \sin \alpha \cos(\alpha + 2\delta)]$

Shear stress $\tau_{vh} = \frac{q}{\pi} \sin \alpha \sin(\alpha + 2\delta)$

Principal stresses $\sigma_1 = \frac{q}{\pi} (\alpha + \sin \alpha) \quad \sigma_3 = \frac{q}{\pi} (\alpha - \sin \alpha)$



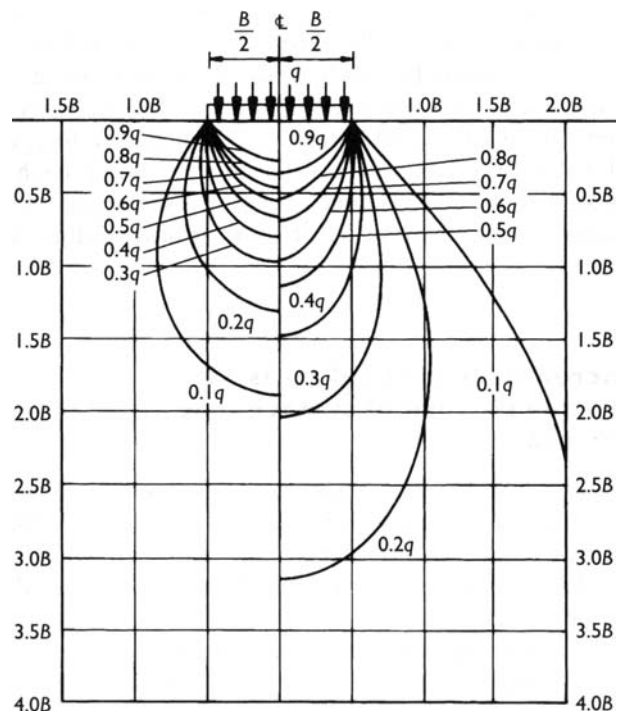
Uniformly-loaded circle radius a (on centerline, r=0)

Vertical stress

$$\sigma_v = q \left[1 - \left(\frac{1}{1 + (a/z)^2} \right)^{\frac{3}{2}} \right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2} \left[(1 + 2\nu) - \frac{2(1 + \nu)z}{(a^2 + z^2)^{1/2}} + \frac{z^3}{(a^2 + z^2)^{3/2}} \right]$$



Contours of vertical stress below uniformly-loaded circular (left) and strip footings (right)

