Version JPH/5

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 25 April 2024 14.00 to 15.40

Module 3D2

GEOTECHNICAL ENGINEERING II

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Engineering Data Book
CUED approved calculator allowed
Attachment: 3D1 and 3D2 Geotechnical Engineering Data Book (21 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A 10 m deep excavation is to be constructed in a soft clay soil with a bulk density of 16 kN m^{-3}, an undrained shear strength of 25 kPa and a K_0 value of 0.4. The water table can be assumed to be at the ground surface.

(a) Calculate the vertical and horizontal total and effective stresses at 5 m depth before construction of the excavation. [20%]

(b) What is the friction angle mobilised within the soil in the *in situ* conditions (prior to excavation)? [20%]

(c) During excavation the horizontal stress reduces. Plot the total and effective stress paths at 5 m depth in σ_H, σ_V space together with the failure criteria. What is the minimum horizontal stress that must be provided by the excavation support system to prevent collapse of the excavation, and what would be the pore pressure measured in the soil with this required minimum horizontal stress? [30%]

(d) If the clay was fissured, what concerns would you have about the assumption of undrained behaviour? [10%]

(e) After construction of the excavation, dewatering leads to an equilibrium pore pressure at 5 m depth of 25 kPa. In the long term, if the excavation system provides a horizontal stress of 50 kPa at 5 m depth, what is the minimum critical state friction angle at this location within the soil mass required for stability of the excavation? [20%]

2 A sample of London Clay is isotropically consolidated to 200 kPa before being allowed to swell in a triaxial device to an effective stress of 150 kPa. The sample is then subjected to a triaxial compression test by reducing the cell pressure on the sample while keeping the vertical stress constant. For all calculations, use the parameters for London Clay given in the 3D1 and 3D2 Geotechnical Engineering Data Book.

(a) Calculate the initial specific volume of the clay sample prior to shearing. [15%]

(b) If the sample is sheared under undrained conditions, calculate the deviatoric stresses and pore pressures observed in the sample at both yield and failure. [30%]

(c) If the sample is sheared under drained conditions, calculate the deviatoric stresses and volumetric strains observed in the sample at both yield and failure. [40%]

(d) If the sample is first loaded under drained conditions to yield before then being loaded undrained to failure, calculate the deviatoric stress at failure. [15%]

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3 (a) An infinite slope of dry sand is shown in Fig. 1. Calculate the safety factor SF for the slope. [20%]

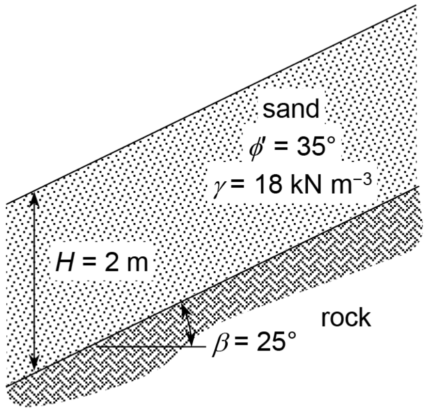

Fig. 1

(b) Two different remediation strategies to help stabilise the slope in Fig. 1 are being considered. The first, shown in Fig. 2(a), applies uniform vertical stress σ_v to the slope surface. The second, shown in Fig. 2(b), applies uniform normal stress σ_n to the slope surface. Will these strategies help to stabilise the slope? Justify your answers. [20%]

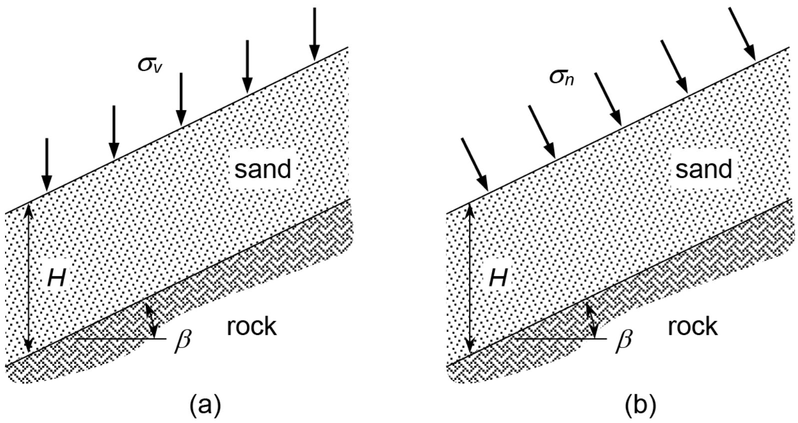

Fig. 2

(c) For a real slope, how could the vertical stress σ_v in Fig. 2(a) be applied? How could the normal stress σ_n shown in Fig. 2(b) be applied? [10%]

(d) An infinite slope of saturated clay is shown in Fig. 3. Considering short-term stability of the slope, derive the formula that expresses the safety factor SF in terms of undrained shear strength s_u, saturated unit weight γ_s, layer height H, and slope angle β. If γ_s = 18 kN m^{-3}, s_u = 15 kPa, and β = 40°, at what height H will the slope fail? [20%]

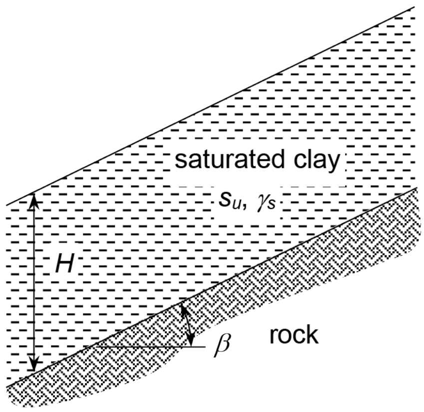

Fig. 3

(e) A finite slope of saturated clay is shown in Fig. 4. Assuming that failure occurs along a plane inclined at angle θ from the horizontal, derive a formula for the safety factor SF in terms of θ, considering short-term stability. Explain how one would determine the value of θ for which failure is most likely. How realistic is this approach? How conservative is it? [30%]

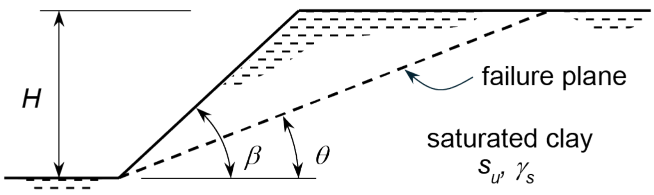

Fig. 4

4 (a) An excavation in sand is supported by an embedded cantilever retaining wall that is propped at the top and embedded in clay to a depth of 2 m, as shown in Fig. 5. Effective friction angle ϕ' of the sand and undrained shear strength s_u of the clay are indicated in Fig. 5, where γ and γ_s are the unit weights above and below the water table, respectively. Behind the wall, the water table is at a depth of 4 m below the ground surface, and in front of the wall, the water table is at the ground surface, as indicated in Fig. 5. Assuming the wall is smooth and that the embedment depth is just sufficient to maintain short-term stability, determine the pressure distribution on the front and back of the wall, and sketch the calculated distributions. [25%]

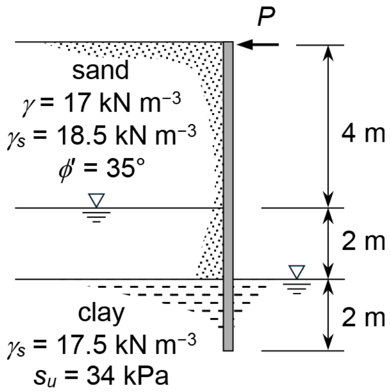

Fig. 5

(b) Based on your results from Part (a), calculate the prop force P shown in Fig. 5 necessary to maintain short-term stability. [10%]

(c) Due to site constraints, engineers are considering the option of propping the wall by connecting it to a second wall driven at distance L from the excavation, as shown in Fig. 6. The wall will be driven to a depth of 3 m. As indicated, cables transfer force P to the top of each wall, where P is equal in magnitude on each wall but opposite in direction. The configuration is identical to the one in Fig. 5 apart from the addition of the second wall and cables. Without doing the calculations, explain how one could determine the maximum value of P that the second wall can sustain. Assume that the second wall is smooth and that the distance L is sufficiently large that no interaction is expected between the walls. [30%]

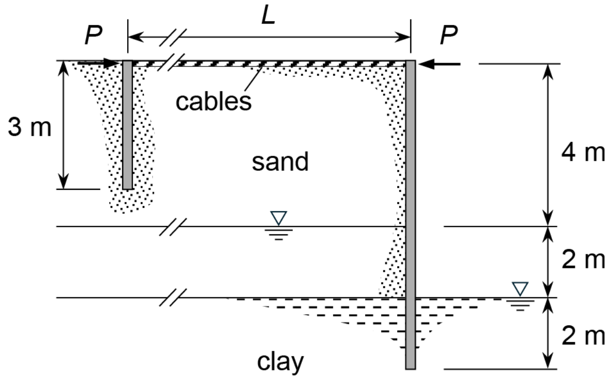

Fig. 6

(d) As a method for temporarily storing dry granular material on a saturated soft clay site, engineers are considering the system of L-cantilever walls shown in Fig. 7. The L-cantilever walls rest on the clay, and the dry granular material is also supported by the clay in the region between the walls, as shown. Material properties and dimensions are indicated in the figure. Calculate the maximum value of L for which the system is stable in the short term. Neglect the weight and thickness of the walls, and clearly state any other assumptions needed for your calculations. [35%]

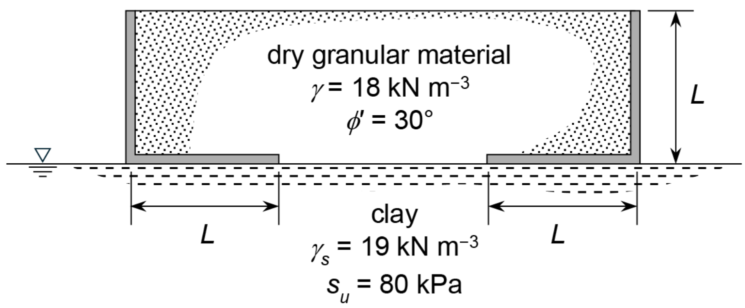

Fig. 7

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Engineering Tripos Part IIA

3D1 & 3D2
Geotechnical Engineering

Data Book 2023-2024

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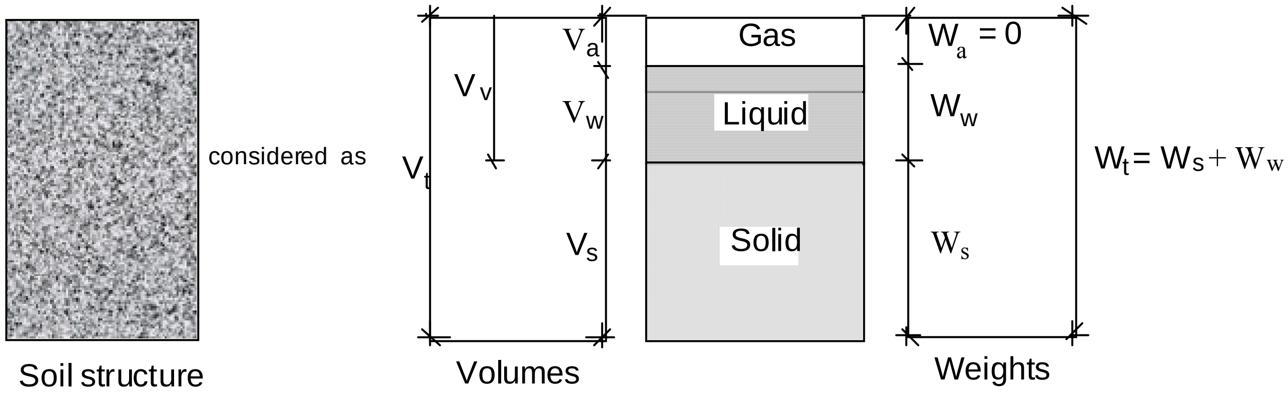
General definitions



Specific gravity of solid G_s

Voids ratio $e = V_v / V_s$

Specific volume $v = V_t / V_s = 1 + e$

Porosity $n = V_v / V_t = e/(1 + e)$

Water content $w = (W_w / W_s)$

Degree of saturation $S_r = V_w / V_v = (w\, G_s/e)$

Unit weight of water $\gamma_w = 9.81$ kN/m^3

Unit weight of soil $\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right) \gamma_w$

Buoyant saturated unit weight $\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right) \gamma_w$

Unit weight of dry solids $\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1 + e}\right) \gamma_w$

Air volume ratio $A = V_a/V_t = \left(\dfrac{e\,(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit	w_L
Plastic Limit	w_P
Plasticity Index	$I_P = w_L - w_P$
Liquidity Index	$I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}} \quad \text{(at the same water content)}$$

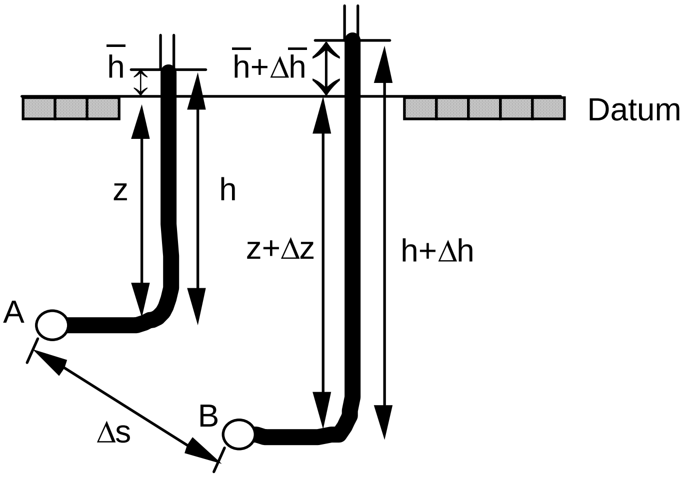
Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D	equivalent diameter of soil particle
D_{10}, D_{60} etc.	particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.
C_U	uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w (\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
$10 \text{ mm} > D_{10} > 1\mu\text{m}$	:	$k \cong 0.01 (D_{10} \text{ in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

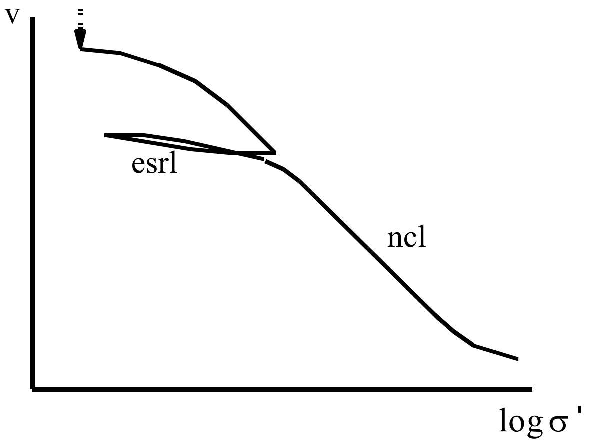
$h_c = \dfrac{4T}{\gamma_w d}$: capillary rise in tube diameter d, for surface tension T

$h_c \approx \dfrac{3\times10^{-5}}{D_{10}}$ m : for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

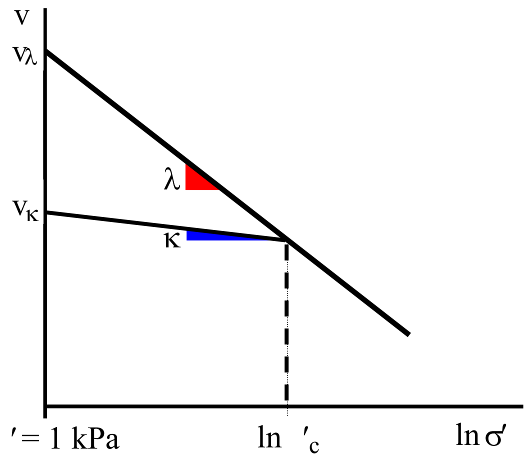
One-Dimensional Compression

• Fitting data

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

• Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

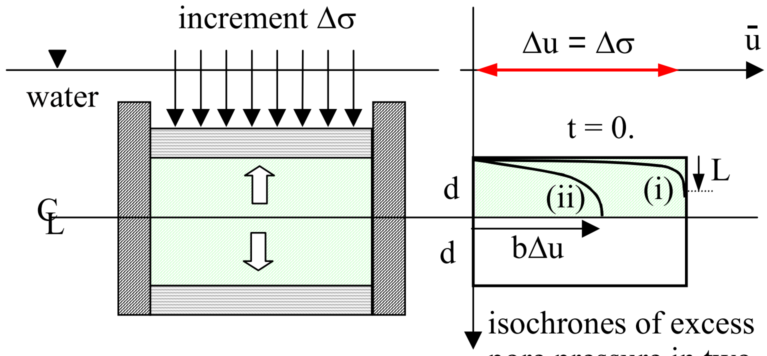
Tangent 1D plastic compression modulus $E_o = v \sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v \sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz$	$= \int (\Delta u - \bar{u}) / E_o \, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w}$	$= \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$	
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$	

- **Solutions for initially rectangular distribution of excess pore pressure**



Approximate solution by parabolic isochrones:

Phase (i) $\quad L^2 = 12\, c_v t$

$$R_v = \sqrt{\frac{4T_v}{3}} \qquad \text{for } T_v < {}^1/_{12}$$

Phase (ii) $\quad b = \exp(\tfrac{1}{4} - 3T_v)$

$$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)] \qquad \text{for } T_v > {}^1/_{12}$$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

• Principle of effective stress (saturated soil)

total stress σ = effective stress σ' + pore water pressure u

• Principal components of stress and strain

sign convention	compression positive
total stress	$\sigma_1, \sigma_2, \sigma_3$
effective stress	$\sigma_1', \sigma_2', \sigma_3'$
strain	$\varepsilon_1, \varepsilon_2, \varepsilon_3$

• Simple Shear Apparatus (SSA) ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

• Biaxial Apparatus - Plane Strain (BA-PS) ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

• Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	=	$\sigma'_a + u$
total radial stress	σ_r	=	$\sigma'_r + u$
total mean normal stress	p	=	$(\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	=	$(\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	=	$\sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	=	q/p'
axial strain	ε_a		
radial strain	ε_r		
volumetric strain	ε_v	=	$\varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	=	$\frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	=	$\sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	=	$p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

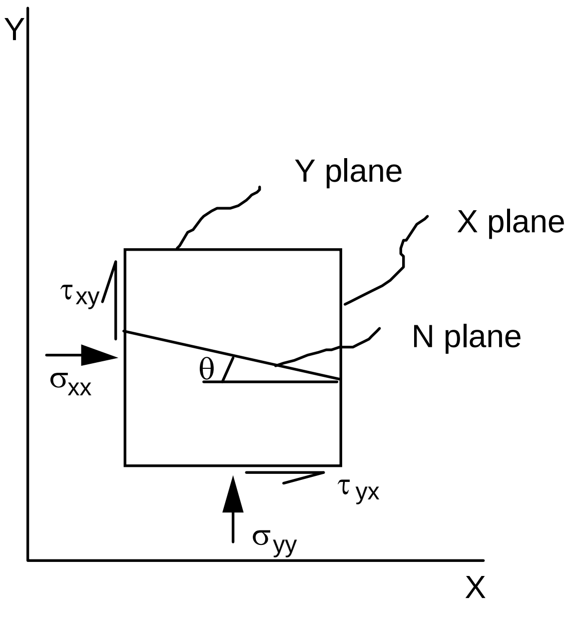
isotropic compression in which p' increases at zero q

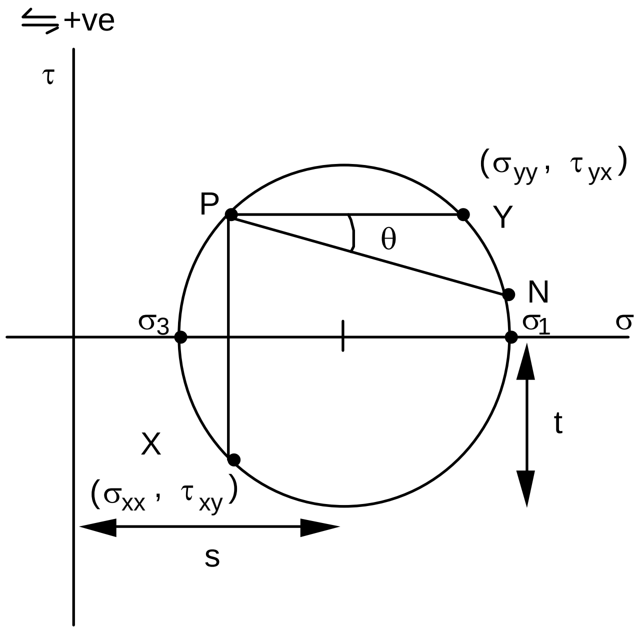
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

Cam Clay

• Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

• General equations of plastic work

Plastic work and dissipation $\quad \sigma^*\,\delta\varepsilon^* + \tau^*\,\delta\gamma^* = \mu^*_{crit}\,\sigma^*\,\delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*}\cdot\dfrac{d\gamma^*}{d\varepsilon^*} = -1$

• General yield surface

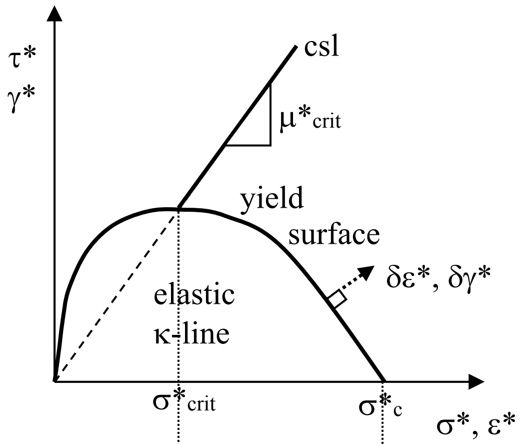
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit}\,.\ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

• Parameter values which fit soil data

	London Clay	**Weald Clay**	**Kaolin**	**Dog's Bay Sand**	**Ham River Sand**
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\ virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

• The yield surface in (σ*, τ*, v) space



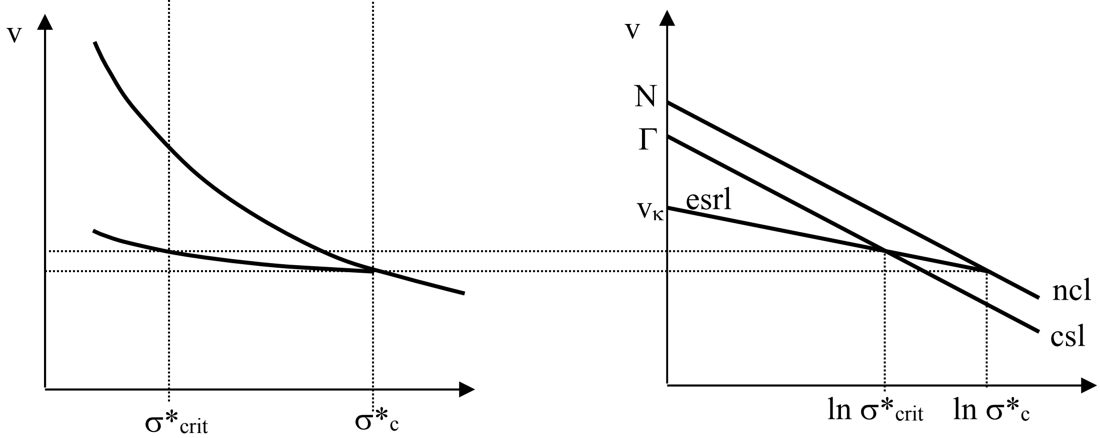
ncl: normal compression line

$v = N - \lambda \ln \sigma^*$

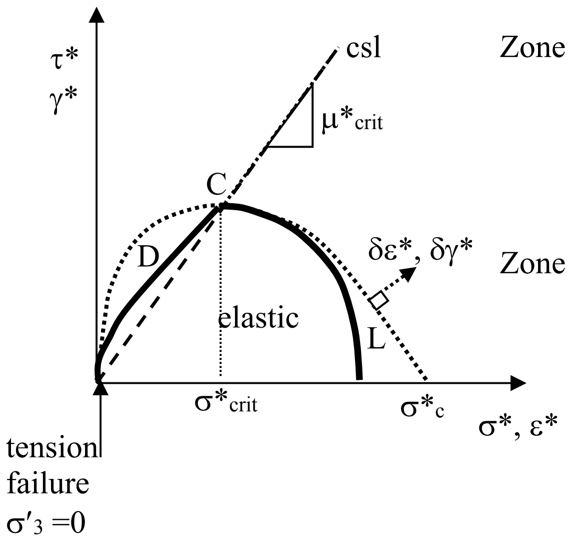
csl: critical state line

$v = \Gamma - \lambda \ln \sigma^*$

where $N = \Gamma + \lambda - \kappa$



• Regions of limiting soil behaviour



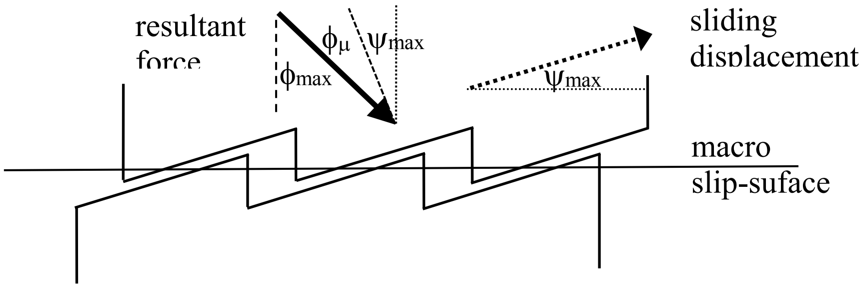
Variation of Cam Clay yield surface

Zone D: denser than critical, “dry”,
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, “wet”,
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

• **Friction and dilatancy: the saw-blade model of direct shear**

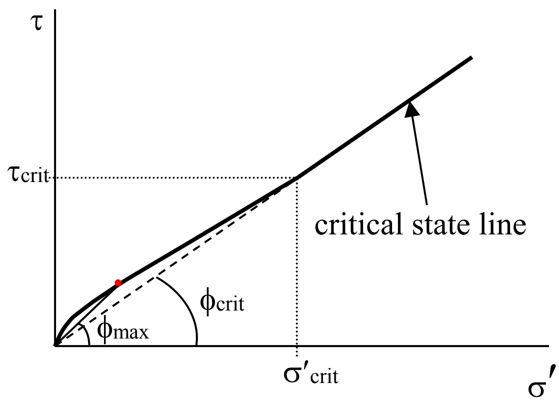


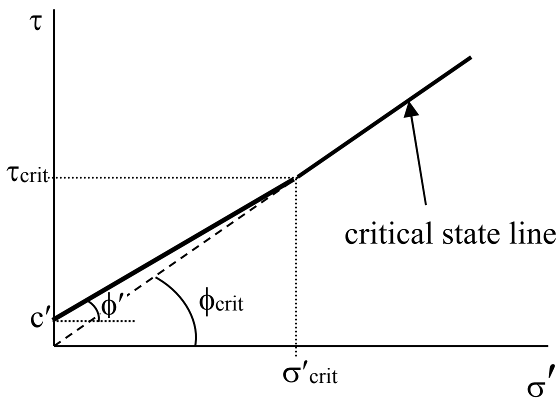
Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

• **Friction and dilatancy: secant and tangent strength parameters**





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^{\alpha}$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

• Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2^o$) are:

well-graded, angular quartz or feldspar sands	40º
uniform sub-angular quartz sand	36º
uniform rounded quartz sand	32º

Relative density $\quad I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$ $\quad$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $\quad I_C = \ln(\sigma_c / p')$ $\quad$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

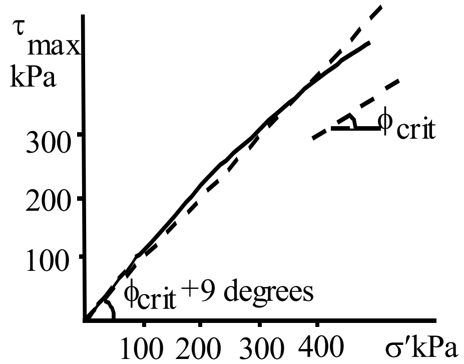
Relative dilatancy index $I_R = I_D I_C - 1$ $\quad$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

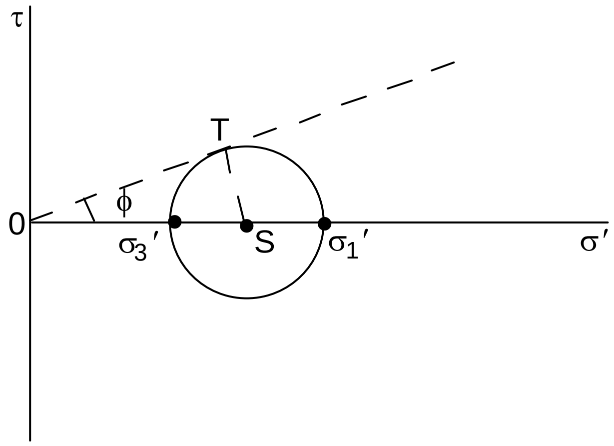
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ $\quad$ for $I_D = 1, \sigma' = 400$ kPa

• **Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane**



$$\sin\phi = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

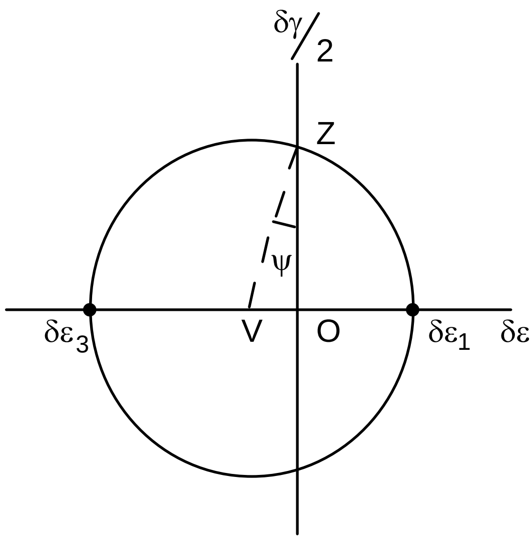
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ_{crit} after large shear strains

• **Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane**



$$\sin\psi = VO/VZ$$

$$= -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2}$$

$$= -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: **Cohesive material** $\tau_{max} = c_u$ (or s_u)

- **Limiting stresses**

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

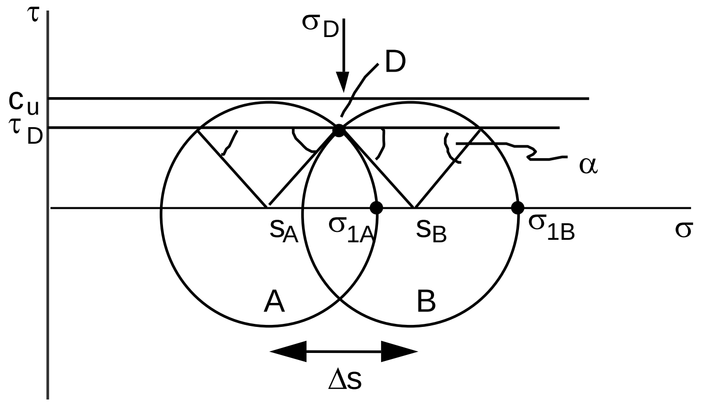
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = Ac_u x$$

- **Stress conditions across a discontinuity**



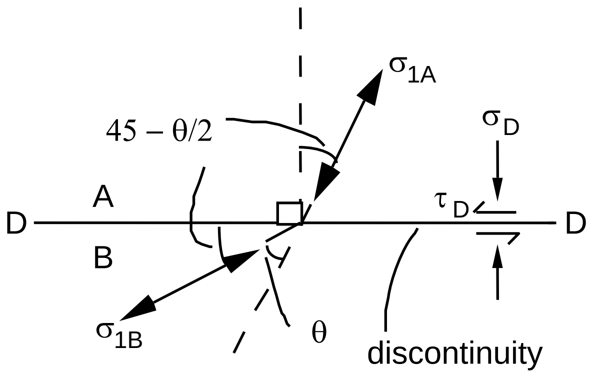
Rotation of major principal stress θ

$s_B - s_A = \Delta s = 2c_u \sin\theta$

$\sigma_{1B} - \sigma_{1A} = 2c_u \sin\theta$

In limit with $\theta \rightarrow 0$

$$ds = 2c_u\, d\theta$$



Useful example:

$\theta = 30^\circ$

$\sigma_{1B} - \sigma_{1A} = c_u$

$\tau_D / c_u = 0.87$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Frictional material $(\tau/\sigma')_{max} = \tan\phi$

• Limiting stresses

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

Passive pressure:

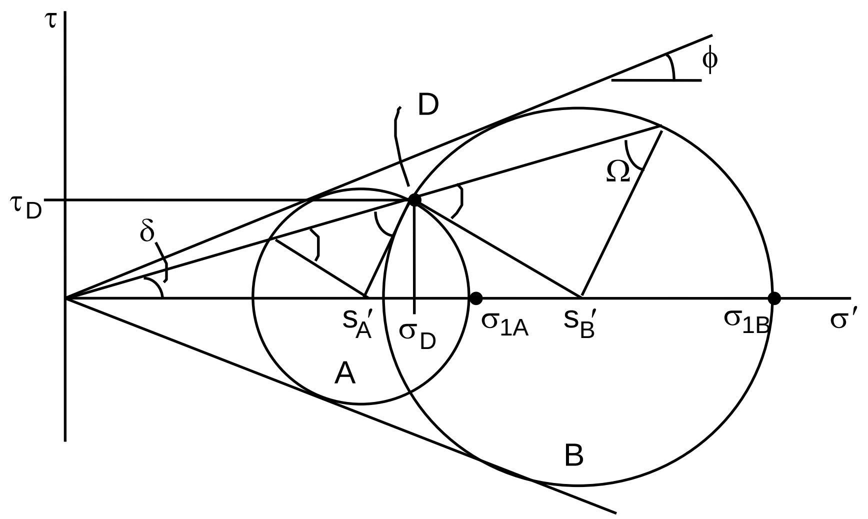
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi)/(1 - \sin\phi) = 1/K_a$

• Stress conditions across a discontinuity



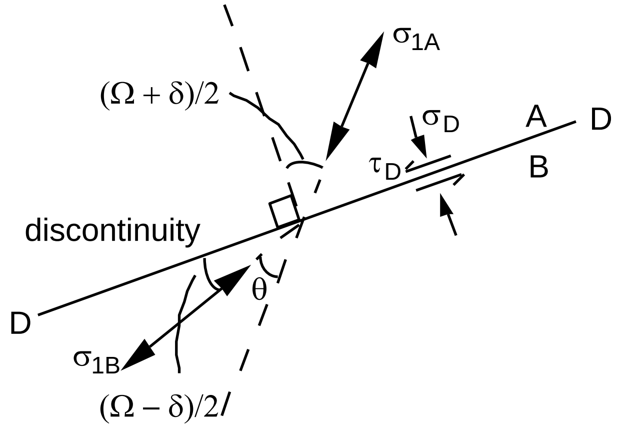
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi$

$ds' = 2s'.\ d\theta \tan\phi$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin\phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha}-1)}{(n_{max}-1)}\right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max}/\sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max}/\sigma'_{v,min}$

α is to be taken as $1.2\sin\phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

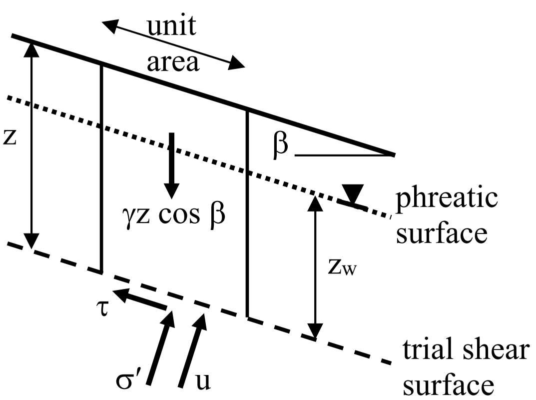
small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Infinite slope analysis



$u = \gamma_w z_w \cos^2\beta$

$\sigma = \gamma z \cos^2\beta$

$\sigma' = (\gamma z - \gamma_w z_w)\cos^2\beta$

$\tau = \gamma z \cos\beta \sin\beta$

$$\tan\phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan\beta}{\left(1 - \dfrac{\gamma_w z_w}{\gamma z}\right)}$$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$N_c = 2 + \pi$ (Prandtl, 1921)

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$s_c = 1 + 0.2\ B / L$

The exact solution for a rough circular foundation (D = B = L) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$d_c = 1 + 0.33 \tan^{-1}(h/B)$ (or h/D for a circular foundation)

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $$\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$$

If $V/V_{ult} < 0.5$: $H = H_{ult} = B s_u$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

Without lift-off: $$\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0 \quad \text{(Taiebet \& Carter 2000)}$$

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

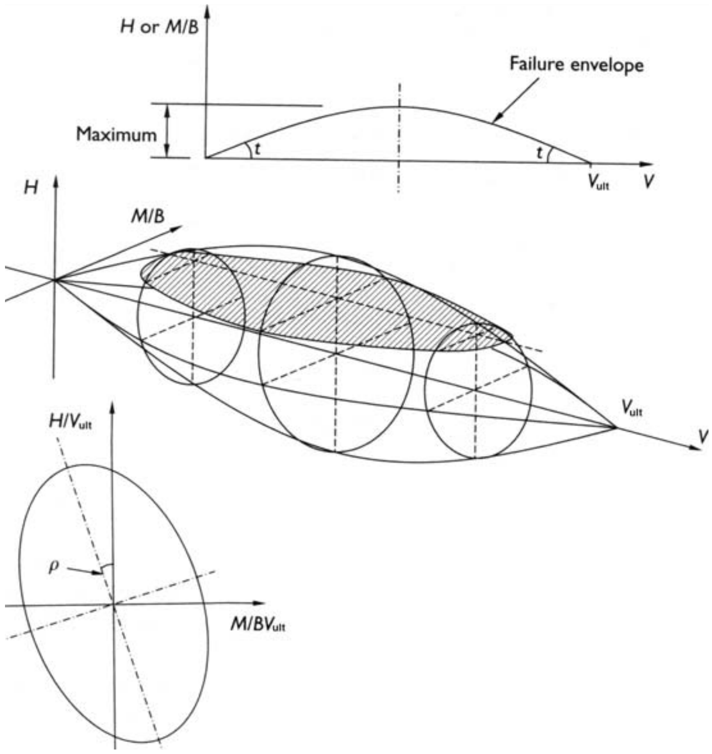
The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)}$ (Prandtl 1921)

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$N_\gamma = 2\,(N_q - 1) \tan \phi$



Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$s_q = 1 + (B \sin \phi) / L$
$s_\gamma = 1 - 0.3\, B / L$

For circular footings take L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

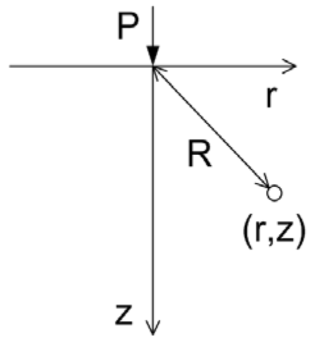
where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi, 1994)

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15°$. Note that t_h is the friction coefficient, $H/V = \tan\phi$, during sliding.

Settlement of Shallow Foundations

Elastic stress distributions below point, strip and circular loads

Point loading (Boussinesq solution)



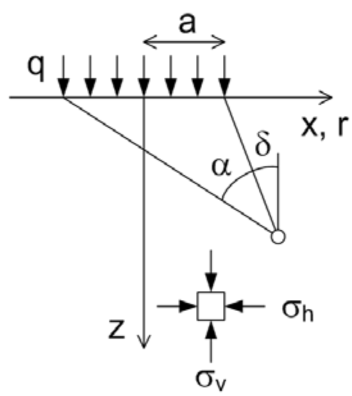
Vertical stress $\sigma_z = \dfrac{3Pz^3}{2\pi R^5}$

Radial stress $\sigma_r = \dfrac{P}{2\pi R^2}\left[\dfrac{3r^2 z}{R^3} - \dfrac{(1-2\nu)R}{R+z}\right]$

Tangential stress $\sigma_\theta = \dfrac{P(1-2\nu)}{2\pi R^2}\left[\dfrac{R}{R+z} - \dfrac{z}{R}\right]$

Shear stress $\tau_{rz} = \dfrac{3Prz^2}{2\pi R^5}$

Uniformly-loaded strip



Vertical stress $\sigma_v = \dfrac{q}{\pi}\left[\alpha + \sin\alpha\cos(\alpha + 2\delta)\right]$

Horizontal stress $\sigma_h = \dfrac{q}{\pi}\left[\alpha - \sin\alpha\cos(\alpha + 2\delta)\right]$

Shear stress $\tau_{vh} = \dfrac{q}{\pi}\sin\alpha\sin(\alpha + 2\delta)$

Principal stresses $\sigma_1 = \dfrac{q}{\pi}(\alpha + \sin\alpha)$ $\sigma_3 = \dfrac{q}{\pi}(\alpha - \sin\alpha)$

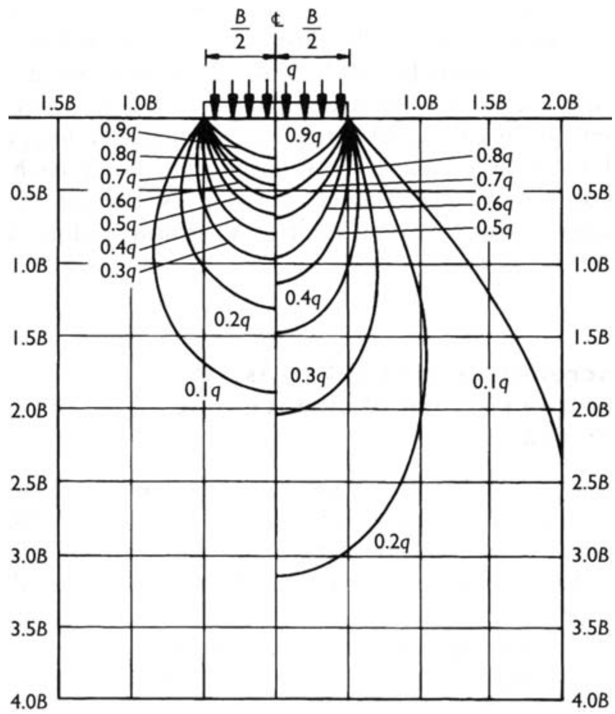
Uniformly-loaded circle radius *a* (on centerline, r=0)

Vertical stress

$$\sigma_v = q\left[1 - \left(\frac{1}{1+(a/z)^2}\right)^{\frac{3}{2}}\right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2}\left[(1+2\nu) - \frac{2(1+\nu)z}{(a^2+z^2)^{1/2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$



Contours of vertical stress below uniformly-loaded circular (left) and strip footings (right)

Elastic solutions for surface settlement
Isotropic, homogeneous, elastic half-space (semi-infinite)

Point load (Boussinesq solution)

Settlement, w, at distance s: $w(s) = \frac{1}{2\pi}\frac{(1-\nu)}{G}\frac{P}{s}$

Circular area (radius a), uniform soil

Uniform load: central settlement: $w_o = \frac{(1-\nu)}{G}qa$

edge settlement: $w_e = \frac{2}{\pi}\frac{(1-\nu)}{G}qa$

Rigid punch: ($q_{avg}= V/\pi a^2$) $w_r = \frac{\pi}{4}\frac{(1-\nu)}{G}q_{avg}a$

Rectangular area, uniform soil

Uniform load, corner settlement:

$$w_c = \frac{(1-\nu)}{G}\frac{qB}{2}I_{rect}$$

Where I_{rect} depends on the aspect ratio, L/B:

L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}
1	0.561	1.6	0.698	2.4	0.822	5	1.052
1.1	0.588	1.7	0.716	2.5	0.835	6	1.110
1.2	0.613	1.8	0.734	3	0.892	7	1.159
1.3	0.636	1.9	0.750	3.5	0.940	8	1.201
1.4	0.658	2	0.766	4	0.982	9	1.239
1.5	0.679	2.2	0.795	4.5	1.019	10	1.272

Rigid rectangle: $w_r = \frac{(1-\nu)}{G}\frac{q_{avg}\sqrt{BL}}{2}I_{rgd}$ where I_{rgd} varies from $0.9 \rightarrow 0.7$ for L/B = 1-10.

Note: $G = \frac{E}{2(1+\nu)}$ where ν= Poisson's ratio, E= Young's modulus.

Answers, 3D2 Exam 2024

Question 1
(a) $\sigma_V = 80$ kPa, $\sigma_V' = 30$ kPa, $\sigma_H = 62$ kPa, $\sigma_H' = 12$ kPa
(b) $\phi = 25.4°$
(c) $\sigma_H = 30$ kPa, $u = 34$ kPa
(d) see crib
(e) $\phi = 22°$

Question 2
(a) $v_i = 2.02$
(b) yield: $q = 38.4$ kPa, $u = -25.6$ kPa; failure: $q = 86$ kPa, $u = -4$ kPa
(c) yield: $q = 58.5$ kPa, $\varepsilon_V = 0.92\%$; failure: $q = 83.8$ kPa, $\varepsilon_V = 0.19\%$
(d) $q = 76.5$ kPa

Question 3
(a) $SF = 1.5$
(b) Strategy (a) does <u>not</u> stabilise slope; Strategy (b) stabilizes slope
(c) Strategy (a) could be achieved by applying a layer of material (fill); Strategy (b) could be achieved with pretensioned anchors mounted to rock
(d) $SF = s_u/(\gamma_s H \cos\beta \sin\beta)$; H = 1.7 m
(e) SF = $2s_u \sin\beta/[\gamma_s H \sin(\beta-\theta)\sin\theta]$; failure most likely for minimum SF, for which derivate w.r.t. θ must be calculated and set equal to zero to find solution; unrealistic and unconservative

Question 4
(a) see crib for sketches with values
(b) $P = 35.9$ kN/m
(c) assume rotation at depth D, calculate active and passive pressure distributions accordingly, and then use force and moment equilibrium to solve for the two unknowns (force P and depth D)
(d) L = 22.9 m