Q.1.(a) - Initial state and yield surface [25%]

A sample of London Clay is isotropically consolidated to an effective stress of 500 kPa and then allowed to swell to an effective stress of 100 kPa before being placed in a triaxial apparatus.

London Clay Parameters (from Data Book):

- $\lambda = 0.161$
- $\kappa = 0.062$
- $M = 0.89$
- $\Gamma = 2.759$ (specific volume at $p' = 1$ kPa on CSL)

Specific volume calculation:

First, we need to determine N (the specific volume on the NCL at $p' = 1$ kPa) from the given value of Γ (the specific volume on the CSL at $p' = 1$ kPa). For Original Cam-Clay, the NCL and CSL are parallel with vertical separation:

$$N - \Gamma = \lambda - \kappa$$

This relationship arises because the CSL passes through the critical state point at the top of the yield surface, which occurs at $p' = p'_c/e$ for Cam-Clay.

$$\begin{aligned} N &= \Gamma + (\lambda - \kappa) \\ &= 2.759 + (0.161 - 0.062) \\ &= 2.759 + 0.099 \\ &= 2.858 \end{aligned}$$

[5%]

The sample is first consolidated isotropically to $p'_0 = 500$ kPa. On the normal compression line (NCL):

$$\begin{aligned} v_{500} &= N - \lambda \ln(p') \\ &= 2.858 - 0.161 \times \ln(500) \\ &= 2.858 - 0.161 \times 6.215 \\ &= 2.858 - 1.001 \\ &= 1.857 \end{aligned}$$

[5%]

The sample is then allowed to swell to $p' = 100$ kPa. During unloading, the sample follows the swelling line (κ-line):

$$\begin{aligned} v &= v_{500} + \kappa \ln\left(\frac{p'_0}{p'}\right) \\ &= 1.857 + 0.062 \times \ln\left(\frac{500}{100}\right) \\ &= 1.857 + 0.062 \times 1.609 \\ &= 1.857 + 0.100 \\ &= \mathbf{1.957} \end{aligned}$$

[5%]

The size of the yield surface is defined by p'_c, which equals the maximum past consolidation pressure:

$$p'_c = 500 \text{ kPa}$$

The overconsolidation ratio is:

$$\text{OCR} = \frac{p'_c}{p'} = \frac{500}{100} = 5$$

Sketches:

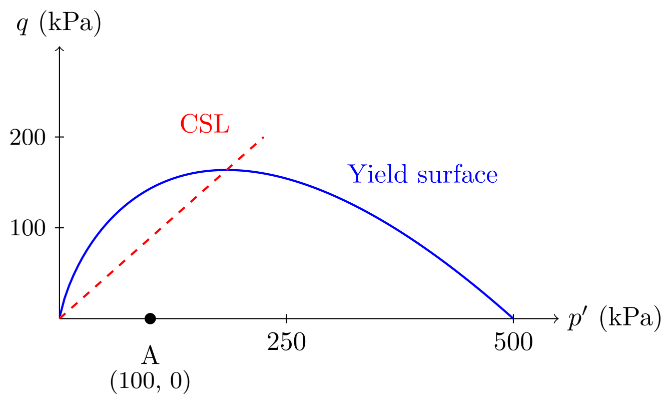
In p'-q space:

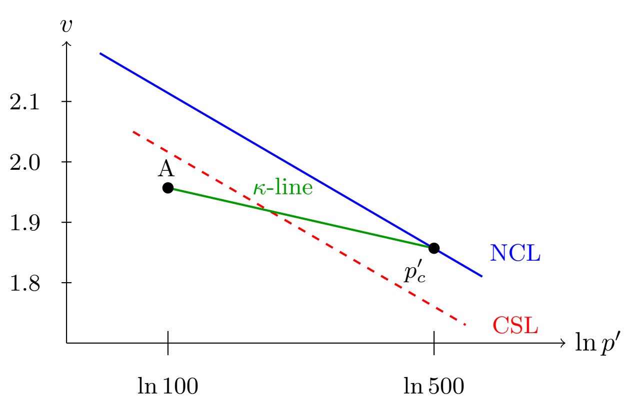
- Yield surface passing through origin and $p'_c = 500$ kPa
- Initial state marked at (100, 0)
- Critical state line $q = Mp' = 0.89p'$ passing through top of yield surface

In v-$\ln p'$ space:

- NCL with equation $v = 2.858 - 0.161 \ln p'$
- CSL parallel to NCL at $v = 2.759 - 0.161 \ln p'$
- Swelling line through initial state with slope $-\kappa$
- Initial state marked at $(\ln(100), 1.957)$ below the CSL

[10%]





[Total 25%]

Q.1.(b) - Drained triaxial compression [40%]

Stress path:

In drained triaxial compression with constant cell pressure (σ_r constant) and increasing axial stress (σ_a increasing):

$$\Delta q = \Delta\sigma_a - \Delta\sigma_r = \Delta\sigma_a$$
$$\Delta p' = \frac{\Delta\sigma_a + 2\Delta\sigma_r}{3} = \frac{\Delta\sigma_a}{3}$$

The stress path has slope:

$$\frac{\Delta q}{\Delta p'} = 3$$

Starting from $(p', q) = (100, 0)$, the stress path is:

$$q = 3(p' - 100)$$

[5%]

Yield point:

The Cam-Clay yield surface is:

$$q = Mp' \ln\left(\frac{p'_c}{p'}\right)$$

Substituting the stress path equation $q = 3(p' - 100)$:

$$3(p' - 100) = 0.89 \times p' \times \ln\left(\frac{500}{p'}\right)$$

[5%]

This equation must be solved numerically. A sensible starting guess can be found by noting that yield must occur between the initial state ($p'_0 = 100$ kPa) and the peak of the yield surface at $p'_c/e = 500/2.718 = 184$ kPa. Taking the midpoint:

$$p'_{guess} = \frac{100 + 184}{2} = 142 \text{ kPa}$$

Testing values from this estimate:

- At $p' = 142$: LHS $= 126$, RHS $= 159$ $\rightarrow$ RHS $>$ LHS, try higher
- At $p' = 152$: LHS $= 156$, RHS $= 161$ $\rightarrow$ RHS $>$ LHS, try higher
- At $p' = 154$: LHS $= 162$, RHS $= 161$ $\rightarrow$ Close match

[5%]

Therefore:

$$p'_Y \approx \mathbf{154} \text{ kPa}, \quad q_Y \approx 3(154 - 100) = \mathbf{162} \text{ kPa}$$

[5%]

Critical state failure:

At critical state, $q = Mp'$. Combined with the stress path:

$$Mp' = 3(p' - 100)$$
$$0.89p' = 3p' - 300$$
$$300 = 3p' - 0.89p' = 2.11p' \therefore p'_F = \frac{300}{2.11} = \mathbf{142.2} \text{ kPa}$$

[5%]

Deviatoric stress at failure:

$$q_F = Mp'_F = 0.89 \times 142.2 = \mathbf{126.6} \text{ kPa}$$

[5%]

Volumetric strain:

The specific volume at critical state lies on the CSL:

$$\begin{aligned} v_F &= \Gamma - \lambda \ln(p'_F) \\ &= 2.759 - 0.161 \times \ln(142.2) \\ &= 2.759 - 0.161 \times 4.958 \\ &= 2.759 - 0.798 \\ &= 1.961 \end{aligned}$$

[5%]

The volumetric strain from initial state to failure:

$$\begin{aligned} \varepsilon_v &= \frac{v_0 - v_F}{v_0} \\ &= \frac{1.957 - 1.961}{1.957} \\ &= \frac{-0.004}{1.957} \\ &= -\mathbf{0.20}\% \text{ (dilation)} \end{aligned}$$

[5%]

Note: The negative volumetric strain indicates dilation, which is expected for heavily overconsolidated clay on the "dry side" of critical state. The sample expands as it shears towards the critical state line.

[Total 40%]

Q.1.(c) - Undrained triaxial compression [25%]

Undrained constraint:

For undrained loading, the specific volume remains constant:

$$v = v_0 = 1.957$$

Yield point:

During elastic (pre-yield) undrained loading, Cam-Clay predicts no change in mean effective stress (assuming isotropic elasticity with no coupling). The effective stress path is vertical in p'-q space until yield. [5%]

At yield, with $p' = 100$ kPa (unchanged from initial), using the Original Cam-Clay yield surface:

$$\begin{aligned} q_Y &= Mp' \ln\left(\frac{p'_c}{p'}\right) \\ &= 0.89 \times 100 \times \ln\left(\frac{500}{100}\right) \\ &= 89 \times \ln(5) \\ &= 89 \times 1.609 \\ &= \mathbf{143.2} \text{ kPa} \end{aligned}$$

[5%]

Excess pore pressure at yield:

Total mean stress: $p = p'_0 + \Delta p = p'_0 + \frac{q}{3}$ (for conventional triaxial compression)

$$\begin{aligned} p_Y &= 100 + \frac{143.2}{3} = 100 + 47.7 = 147.7 \text{ kPa} \\ u_Y &= p_Y - p'_Y = 147.7 - 100 = \mathbf{47.7} \text{ kPa} \end{aligned}$$

[5%]

Critical state failure:

At critical state, $v = \Gamma - \lambda \ln(p'_F)$:

$$\begin{aligned}
1.957 &= 2.759 - 0.161 \ln(p'_F) \\
0.161 \ln(p'_F) &= 2.759 - 1.957 = 0.802 \\
\ln(p'_F) &= 4.981 \\
p'_F &= e^{4.981} = \mathbf{145.6} \text{ kPa}
\end{aligned}$$

Deviatoric stress at failure:

$$q_F = Mp'_F = 0.89 \times 145.6 = \mathbf{129.6} \text{ kPa}$$

[5%]

Excess pore pressure at failure:

$$\begin{aligned}
p_F &= 100 + \frac{129.6}{3} = 100 + 43.2 = 143.2 \text{ kPa} \\
u_F &= p_F - p'_F = 143.2 - 145.6 = -\mathbf{2.4} \text{ kPa}
\end{aligned}$$

[5%]

The negative pore pressure at failure is characteristic of heavily overconsolidated clay. The soil tends to dilate during shearing, but since drainage is prevented, negative (suction) pore pressures develop, increasing the mean effective stress p'.

[Total 25%]

Q.1.(d) - Cam-Clay compared to real soil on the dry side of critical [20%]

Any two of the below:

- Experimental evidence shows that real soils fail on the Hvorslev surface, a frictional failure criterion that "cuts off" the nose of the Cam-Clay bullet at lower deviatoric stresses than the model predicts.
- The yield stresses calculated in parts (b) and (c) are likely overestimates — real London Clay at OCR = 5 would yield at a lower deviatoric stress when the stress path intersects the Hvorslev surface.
- The critical state predictions remain valid, since both Cam-Clay and real soils converge to the same CSL at large strains. The values of p'_F, q_F, ε_v, and u_F are therefore reliable.
- Real OC clay exhibits strain-softening after peak, with strength dropping from the Hvorslev peak towards the critical state.
- For practical design, critical state strength should be used for long-term or residual conditions; Cam-Clay peak strengths are unconservative for heavily OC materials.

[Total 10%]

Q.2.(a) - Initial stress state [20%]

A long strip foundation of width $B = 4$ m is to be constructed on a deposit of normally consolidated clay. The clay has unit weight $\gamma = 20$ kN/m^3, angle of shearing resistance $\phi' = 25°$, and undrained shear strength $s_u = 30$ kPa. The water table is at ground level. The coefficient of earth pressure at rest is estimated using Jaky's formula, $K_0 = 1 - \sin\phi'$. Element A is located directly beneath the centre of the foundation at a depth of 5 m. When the foundation applies a bearing pressure q, the vertical and horizontal stress increments at Element A are $\Delta\sigma_v = 0.50q$ and $\Delta\sigma_h = 0.15q$.

Total and effective stresses at 5m depth:

Vertical total stress:

$$\sigma_v = \gamma z = 20 \times 5 = \mathbf{100} \text{ kPa}$$

Pore water pressure (water table at surface):

$$u_0 = \gamma_w z = 10 \times 5 = 50 \text{ kPa}$$

Vertical effective stress:

$$\sigma'_v = \sigma_v - u = 100 - 50 = \mathbf{50} \text{ kPa}$$

Coefficient of earth pressure at rest (Jaky's formula):

$$K_0 = 1 - \sin\phi' = 1 - \sin 25° = 1 - 0.423 = \mathbf{0.58}$$

Horizontal effective stress:

$$\sigma'_h = K_0\sigma'_v = 0.58 \times 50 = \mathbf{29} \text{ kPa}$$

Horizontal total stress:

$$\sigma_h = \sigma'_h + u = 29 + 50 = \mathbf{79} \text{ kPa}$$

[10%]

Plane strain stress invariants:

The stress invariants are defined as:

$$s = \frac{\sigma_v + \sigma_h}{2} = \frac{100 + 79}{2} = \mathbf{89.5} \text{ kPa}$$

$$t = \frac{\sigma_v - \sigma_h}{2} = \frac{100 - 79}{2} = \mathbf{10.5} \text{ kPa}$$

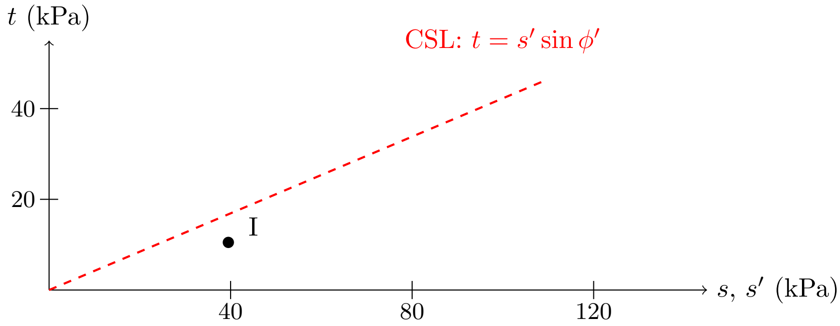
$$s' = s - u = 89.5 - 50 = \mathbf{39.5} \text{ kPa}$$

Alternatively, $s' = (\sigma'_v + \sigma'_h)/2 = (50 + 29)/2 = 39.5$ kPa.

[5%]

Sketch of initial stress path:

With $K_0 < 1$, the initial state has $\sigma'_v > \sigma'_h$, so $t > 0$. The initial point lies below the critical state line. The initial state plots at point $A_{initial} = (s' = 39.5, t = 10.5)$. The critical state line (CSL) has equation $t = s' \sin\phi' = 0.423s'$.



[5%]

[Total 20%]

Q.2.(b) - Element A stress paths and yield [40%]

Total Stress Path (TSP):

For Element A, the stress increments are $\Delta\sigma_v = 0.50q$ and $\Delta\sigma_h = 0.15q$.
Changes in stress invariants:

$$\Delta s = \frac{\Delta\sigma_v + \Delta\sigma_h}{2} = \frac{0.50q + 0.15q}{2} = 0.325q$$

$$\Delta t = \frac{\Delta\sigma_v - \Delta\sigma_h}{2} = \frac{0.50q - 0.15q}{2} = 0.175q$$

The TSP gradient is:

$$\frac{\Delta t}{\Delta s} = \frac{0.175q}{0.325q} = \frac{7}{13} \approx 0.54$$

The TSP starts at $(s = 89.5, t = 10.5)$ and moves with gradient 0.54. [10%]

Effective Stress Path (ESP):

For undrained loading, no drainage occurs, so there is no change in water content or volume. In the elastic range, this means the mean effective stress s' remains constant.
The ESP is therefore **vertical** in t-s' space, starting at $(s' = 39.5, t = 10.5)$ and moving upward as t increases. [10%]
Yield condition:

Yield occurs when the ESP intersects the yield surface. At the yield surface with $s' = 39.5$ kPa:

$$t_y = s' \sin\phi' = 39.5 \times \sin 25° = 39.5 \times 0.423 = \mathbf{16.7} \text{ kPa}$$

[10%]

Pore pressure at yield:

The increase in t to reach yield is $\Delta t = t_y - t_0 = 16.7 - 10.5 = 6.2$ kPa. From the TSP gradient:

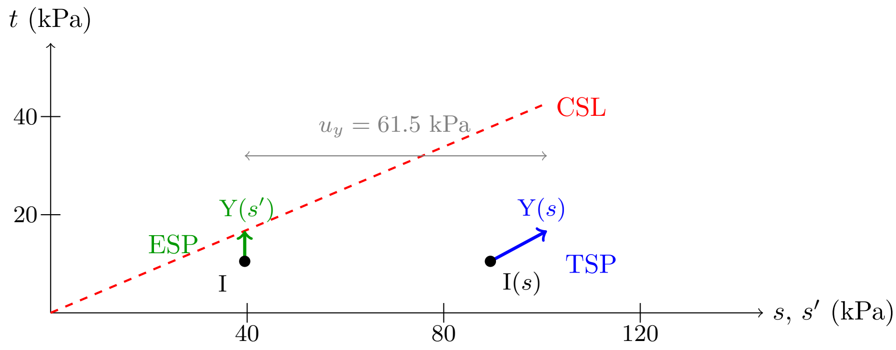
$$\Delta s = \frac{\Delta t}{0.54} = \frac{6.2}{0.54} = 11.5 \text{ kPa}$$

Since $\Delta s' = 0$ for undrained loading, the excess pore water pressure equals the change in total mean stress:

$$\Delta u = \Delta s = \mathbf{11.5} \text{ kPa}$$

[10%]

Stress path diagram:



[Total 40%]

Q.2.(c) - Ultimate failure [25%]

After yield, the ESP follows the critical state line (CSL) as plastic straining occurs. The ESP moves up and to the right along the line $t = s' \sin\phi'$ until the shear stress reaches the undrained shear strength (Tresca cap) at $t = s_u = 30$ kPa. [5%]

At ultimate failure, $t = s_u = 30$ kPa and the ESP lies on the CSL:

$$s'_f = \frac{t_f}{\sin\phi'} = \frac{30}{0.423} = \mathbf{70.9} \text{ kPa}$$

[5%]

Total stress at failure:

From the TSP, with gradient $\Delta t/\Delta s = 0.54$:

$$s_f = s_0 + \frac{t_f - t_0}{0.54} = 89.5 + \frac{30 - 10.5}{0.54} = 89.5 + 36.1 = \mathbf{125.6} \text{ kPa}$$

[5%]

Pore pressure at failure:

Pore water pressure at failure:

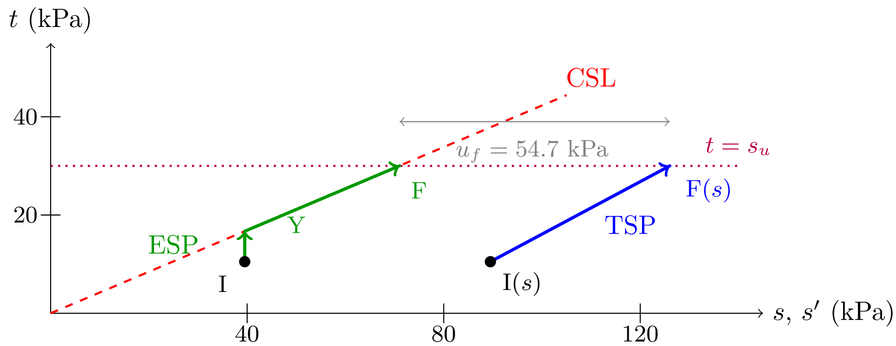
$$u_f = s_f - s'_f = 125.6 - 70.9 = \mathbf{54.7} \text{ kPa}$$

Excess pore water pressure:

$$\Delta u = u_f - u_0 = 54.7 - 50 = \mathbf{4.7} \text{ kPa}$$

[5%]

Sketch of complete stress path:



[5%]

[Total 25%]

Q.2.(d) - Shear strain rate in each phase [15%]

(i) Pre-yield elastic phase: Elastic behaviour; shear strain rate $\dot{\gamma} = \dot{t}/G'$ is finite and proportional to the stress rate. [5%]

(ii) Post-yield phase on the yield surface: Elasto-plastic behaviour requiring three ingredients:

- **Yield surface** ($t = s' \sin \phi'$): boundary of elastic behaviour
- **Flow rule**: direction of plastic straining (normal to yield surface for associative flow)
- **Consistency condition** ($\dot{f} = 0$): stress remains on yield surface, determining the magnitude of plastic strain

The strain rate is found by combining the constitutive law with the flow rule direction and solving for the plastic multiplier using the consistency condition.

[5%]

(iii) At ultimate failure (Tresca cap): Perfectly plastic behaviour at $t = s_u$. Shear stress cannot increase further, so $\dot{\gamma} \to \infty$ (unlimited deformation at constant stress).

[5%]

[Total 15%]

Question ~~4~~ 3 - 3D2 Exam AY25-26

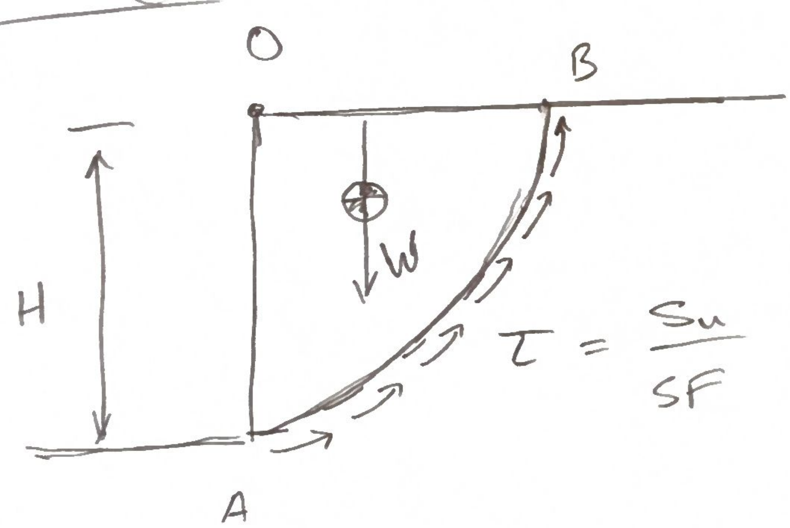
Part (a)

Yes, there is sufficient information in that no vertical cut is stable in the long term, when undrained conditions prevail. The excavation is unstable for any ϕ'.

Part (b)

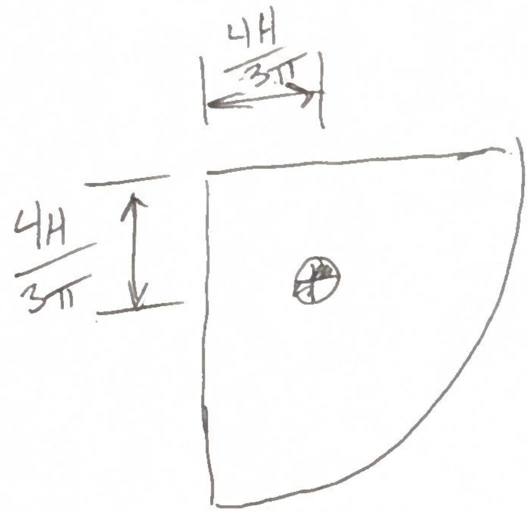
No, this will not cause the trench to become unstable in the short term. In fact, it has a stabilizing effect. For short-term stability, we consider undrained conditions and total stress. The normal stress (pressure) exerted by the water will increase the forces/moments resisting collapse.

Part (c)



Consider moments inducing and resisting collapse, calculating moments about O

Centroid



$$\text{Weight:} \quad W = \left(\frac{\pi H^2}{4}\right) \gamma_{sat}$$

Moment equilibrium :

$$W\left(\frac{4H}{3\pi}\right) = \int_A^B \tau r \, ds$$

$$\left(\frac{\cancel{\pi} H^2}{\cancel{4}} \gamma_{sat}\right)\left(\frac{\cancel{4}H}{3\cancel{\pi}}\right) = \int_0^{\frac{\pi}{2}} \tau r (r \, d\theta)$$

$$\frac{\gamma_{sat} H^3}{3} = \int_0^{\frac{\pi}{2}} \left(\frac{S_u}{SF}\right) H^2 \, d\theta$$

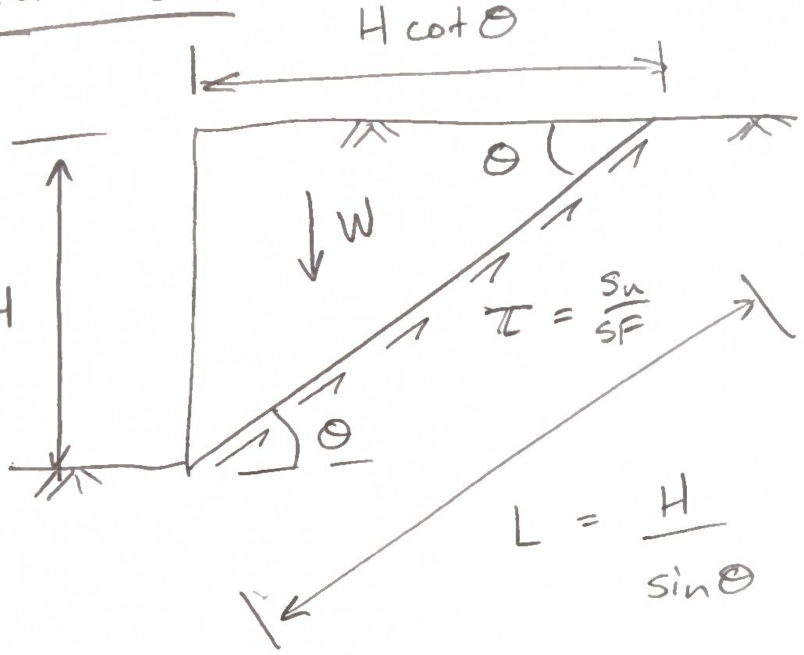
$$\vdots \quad = \frac{S_u H^2}{SF} \int_0^{\frac{\pi}{2}} d\theta$$

$$\frac{\gamma_{sat} H^3}{3} = \frac{\pi}{2} \frac{S_u H^2}{SF}$$

Solve for SF :

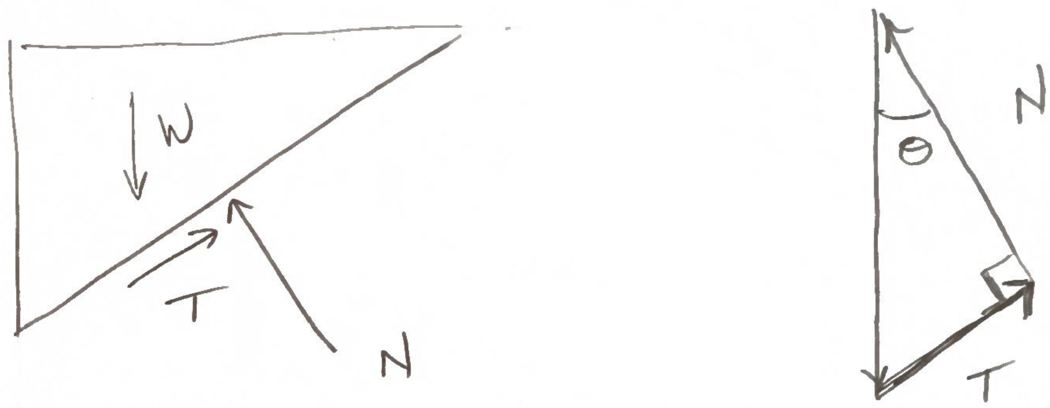
$$\boxed{SF = \frac{3\pi}{2} \frac{S_u}{\gamma_{sat} H}}$$

Part (d)



$$W = \frac{1}{2}(H)(H\cot\theta)\gamma_{sat}$$
$$= \frac{1}{2}\frac{H^2}{\tan\theta}\gamma_{sat}$$

Equilibrium of forces:



$$T = W\sin\theta$$

$$\Rightarrow \quad \tau = \frac{T}{L} = \frac{(W\sin\theta)}{(H/\sin\theta)} = \frac{W}{H}\sin^2\theta$$

Substitute for W from above:

$$\tau = \frac{W}{H}\sin^2\theta = \left(\frac{1}{2}\frac{H^2}{\tan\theta}\gamma_{sat}\right)\frac{\sin^2\theta}{H}$$
$$= \frac{1}{2}\frac{H\gamma_{sat}\sin^2\theta}{(\sin\theta/\cos\theta)} = \frac{1}{2}\gamma_{sat}H\sin\theta\cos\theta$$

Substitute this expression for τ into $\tau = \dfrac{S_u}{SF}$

and solve for SF :

$$SF = \frac{S_u}{\tau} = \frac{S_u}{\frac{1}{2}\gamma_{sat} H \sin\theta \cos\theta}$$

$$= 2 \frac{S_u}{\gamma_{sat} H} \frac{1}{\sin\theta\cos\theta}$$

Minimise SF :

$$\frac{\partial(SF)}{\partial\theta} = 2 \frac{S_u}{\gamma_{sat} H} \frac{\partial}{\partial\theta} (\sin\theta\cos\theta)^{-1}$$

$$= 2 \frac{S_u}{\gamma_{sat} H} (\sin\theta\cos\theta)^{-2} (\cos^2\theta - \sin^2\theta)$$

$$= 2 \frac{S_u}{\gamma_{sat} H} \frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta\cos^2\theta} = 0$$

Since $\sin^2\theta \cos^2\theta$ is positive, we need

$$\sin^2\theta - \cos^2\theta = -\cos(2\theta) = 0$$

$$\cos(2\theta) = 0 \quad \text{for} \quad 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$$

Angle of failure plane leading to least SF is 45°.

Evalute SF with $\theta = \frac{\pi}{4}$:

$$SF = 2 \frac{S_u}{\gamma_{sat} H} \frac{1}{\sin(\frac{\pi}{4})(\cos \frac{\pi}{4})}$$

$$= 2 \frac{S_u}{\gamma_{sat} H} (2)$$

$$= 4 \frac{S_u}{\gamma_{sat} H}$$

$$SF = 4 \frac{S_u}{\gamma_{sat} H}$$

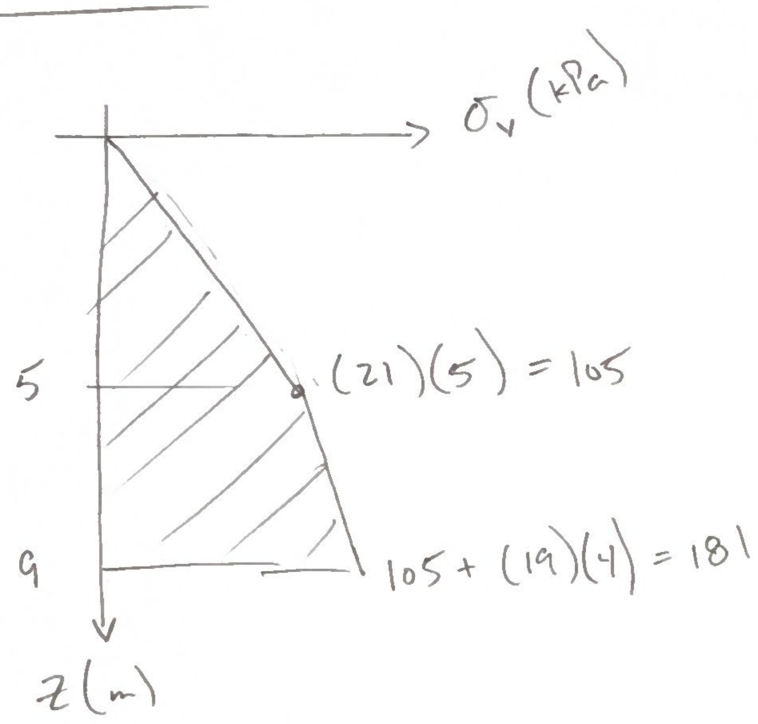
Part (e)

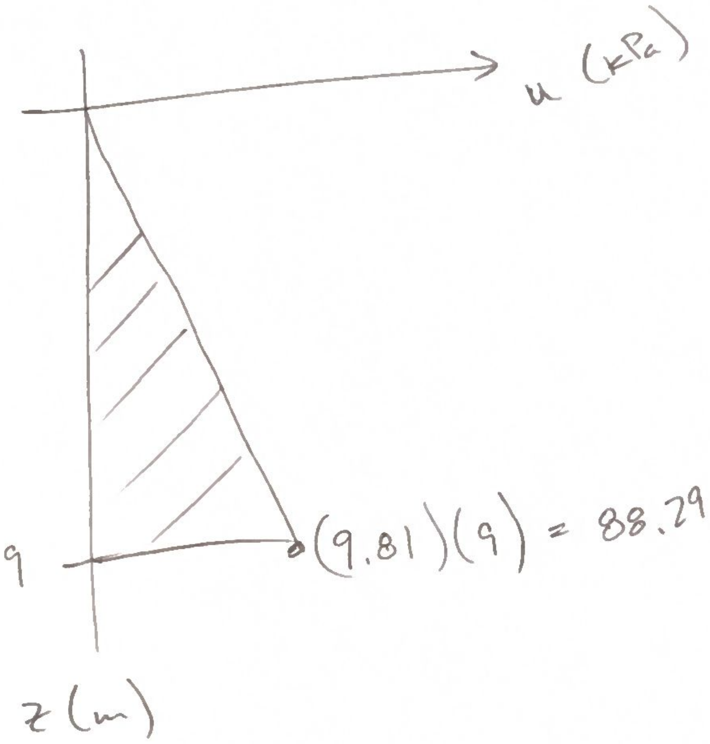
The safety factor SF for the planar surface is less than that for the circular failure surface, for which $SF = \frac{3\pi}{2} \frac{S_u}{\gamma_{sat} H} \approx 4.71 \frac{S_u}{\gamma_{sat} H}$.

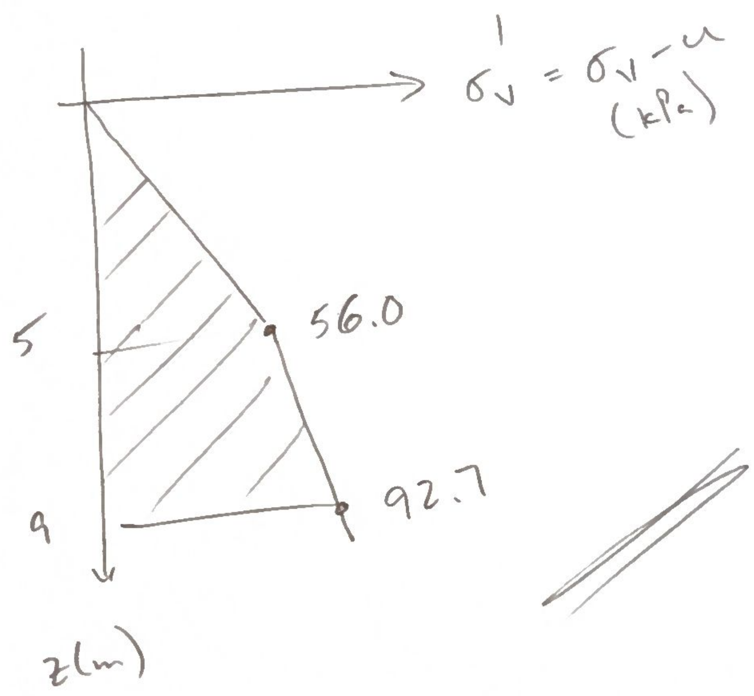
This directly suggests that a planar surface controls, with lower stability number $\frac{\gamma H}{S_u}$ occuring for the same desired factor of safety.

Question ~~3~~ 4 - 3D2 Exam AY25-26

Part (a)







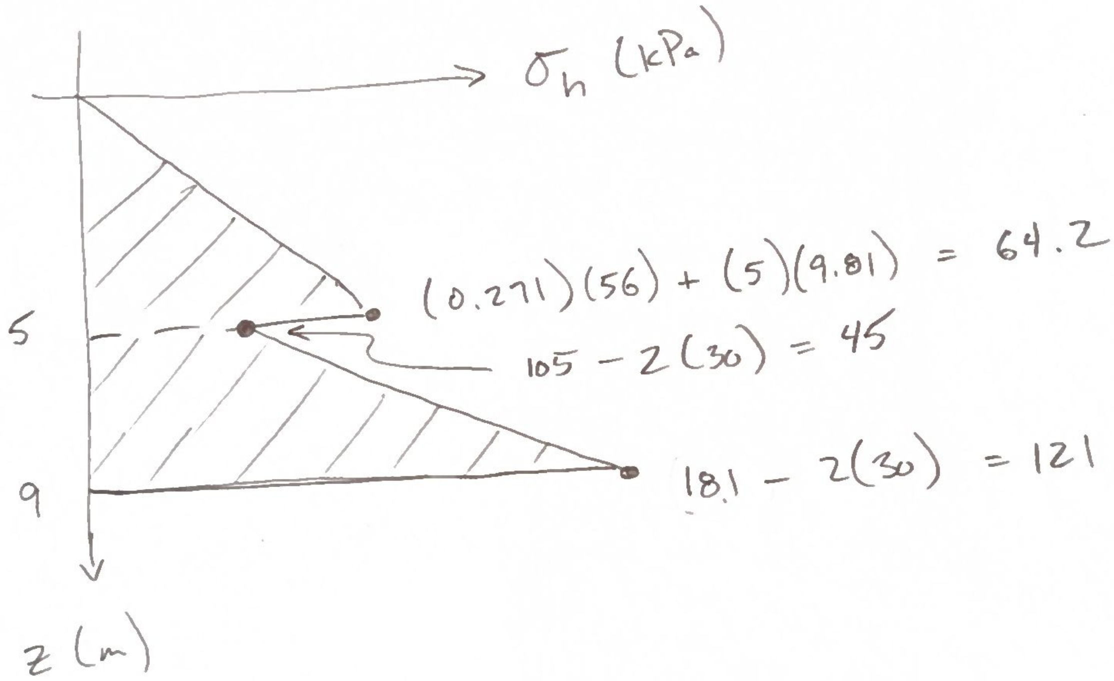
Part (b)

Minimum thrust on wall corresponds to active state

For sand, $\sigma_h' = K_a \sigma_v'$ and $\sigma_h = \sigma_h' + u$

$$K_a = \frac{1 - \sin\phi'}{1 + \sin\phi'} = 0.271$$

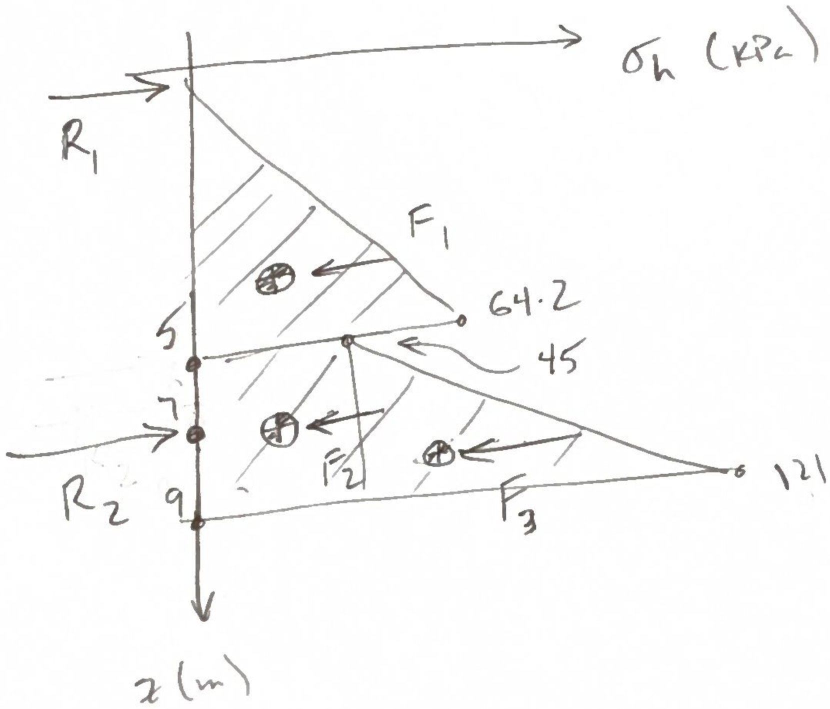
For clay, $\sigma_h = \sigma_v - 2s_u$



$$S_a = \frac{1}{2}(64.2)(5) + (45)(4) + \frac{1}{2}(121-45)(4)$$
$$= 492.5 \text{ kN/m}$$

minimum thrust $S_a = 493$ kN/m

Part (c)



Sum moments (about top):

$$F_1 = \frac{1}{2}(64.2)(5) = 160.5 \text{ kN/m}$$

$$F_2 = (45)(4) = 180 \text{ kN/m}$$

$$F_3 = \frac{1}{2}(121-45)(4) = 152 \text{ kN/m}$$

$$\sum M = -F_1\left(\frac{2}{3}\right)(5) - F_2(5+2) - F_3\left[5+\left(\frac{2}{3}\right)(4)\right] + R_2(7) = 0$$

$$-2960 + 7R_2 = 0$$
$$R_2 = 423 \text{ kN/m}$$

$$R_1 + R_2 = S_a \Rightarrow R_1 = S_a - R_2$$

$$= 492.5 \text{ kN/m} - 422.9 \text{ kN/m}$$

$$= 69.6 \text{ kN/m}$$

$$R_1 = 70 \text{ kN/m} \quad , \quad R_2 = 423 \text{ kN/m}$$

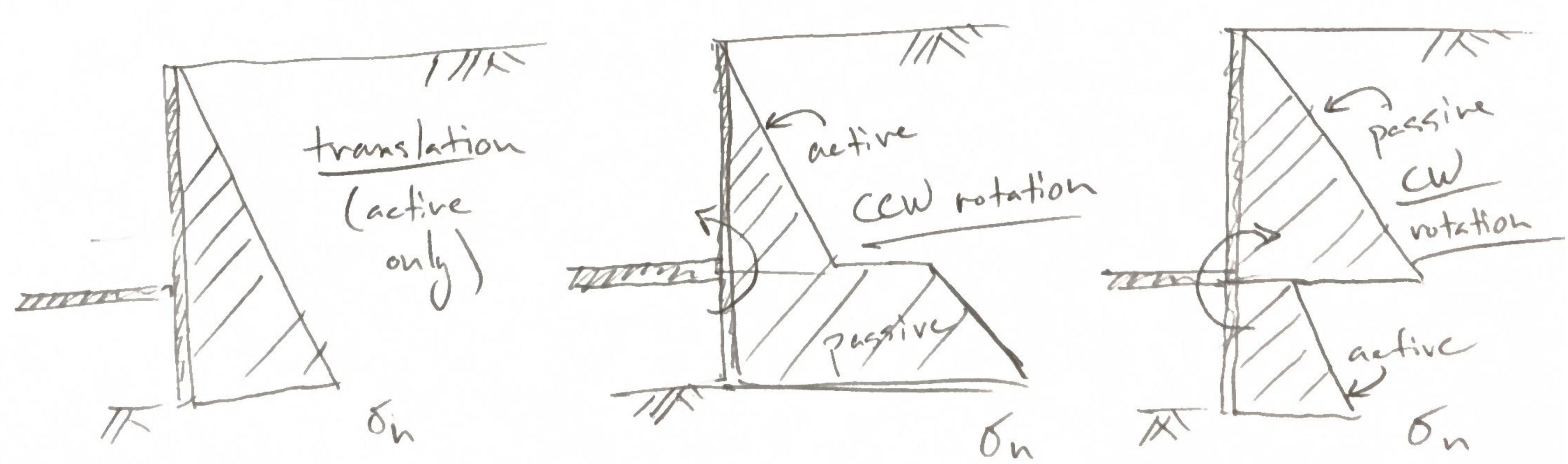
(upper) (lower)

~~Part (d)~~

It would not be reasonable to use these values for design of the props. These forces correspond to the minimum thrust that must be applied for stability. In reality, larger forces will be needed to ensure the wall does not move excessively.

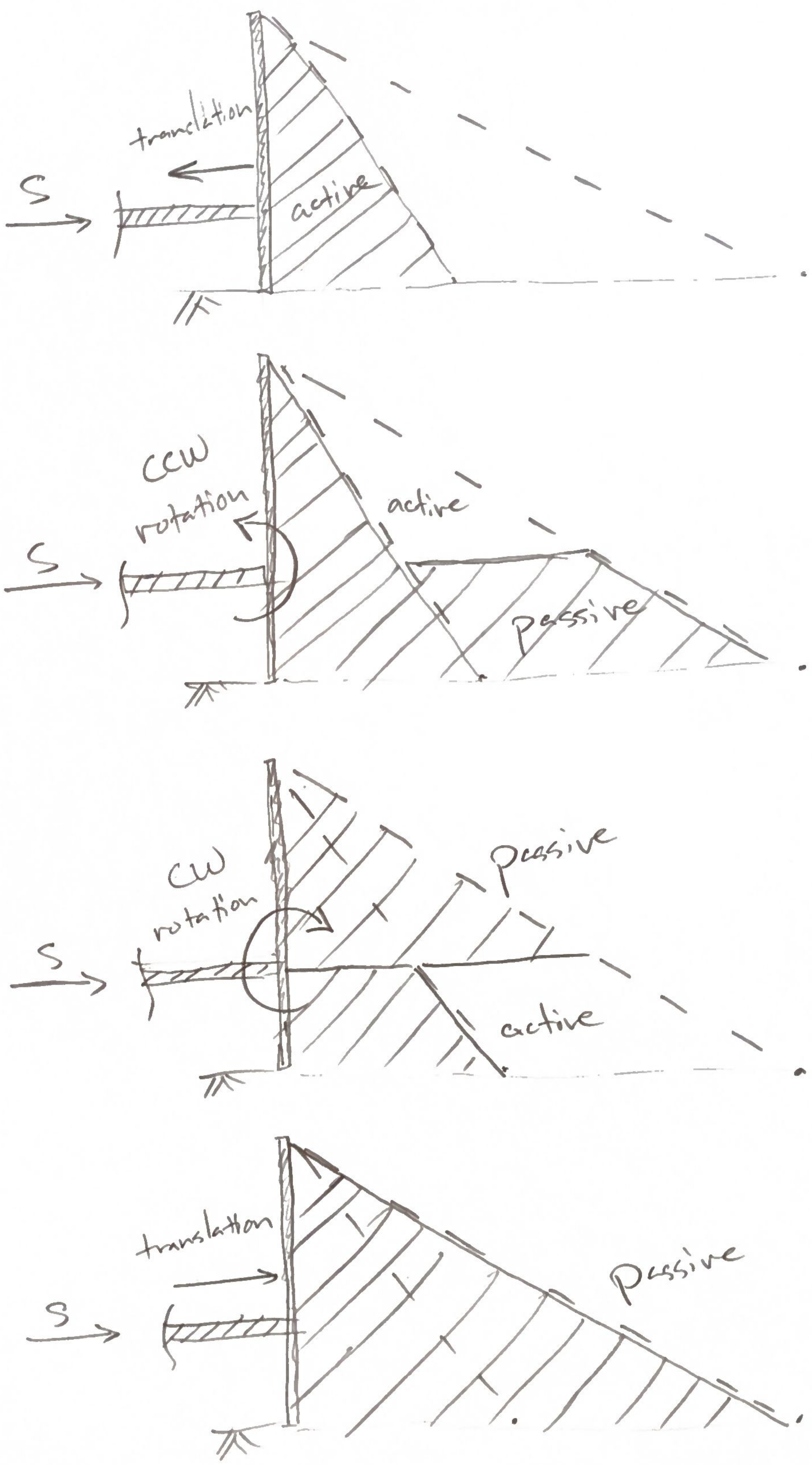
~~Part (e)~~ Part (d)

A total of four states of limiting equilibrium are possible, and three of these are relevant:



The fourth state is translation with passive earth pressure only; this is not practically relevant here but rounds out the picture, as seen below.

S ... prop force



area in this case is least, so this case corresponds to the minimum active thrust on the wall (case considered earlier)