1. The classical solutions assume a perfect geometry and a perfectly elastic material. They do not account for:
   - yielding
   - geometric imperfections
   - residual stresses
   - accidental eccentricities

   ...

   All of the above reduce the capacity below the value predicted by the theory, which is therefore unsafe.

2. $$\bar{\lambda} = \sqrt{\frac{P_y}{P_E}}$$

   P_y: capacity accounting for yielding, but ignoring overall instability = cross-sectional capacity

   P_E: capacity accounting for overall instability, but ignoring yielding = elastic buckling load

   For columns, this definition follows from the theory of Perry & Robertson. For other instabilities we use it 'by analogy'.

3. <u>Flexural buckling of columns</u>

   $$\bar{\lambda} = \sqrt{\frac{P_y}{P_E}}$$

   P_y → cross-sectional capacity in compression ≡ <u>Squash Load</u> $A.f_y$

   P_E → elastic flexural buckling load of perfect column ≡ <u>Euler Load</u> $\frac{\pi^2 EI}{L^2}$

   We use this slenderness in combination with the <u>column curves</u> (a_0, a, b, c, d) to determine χ:

   $$\underbrace{P_d}_{\text{design capacity}} = \underbrace{\chi\, P_y}_{\text{buckling coeff.} \times \text{cross-sectional capacity}} / \underbrace{\gamma_{M1}}_{\text{safety factor}}$$

LTB of beams

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_c}{M_{cr}}}$$

M_c: cross-sectional capacity in bending $\equiv \begin{cases} M_y & \text{for Class 3} \\ M_{pl} & \text{for Class 1, 2} \end{cases}$

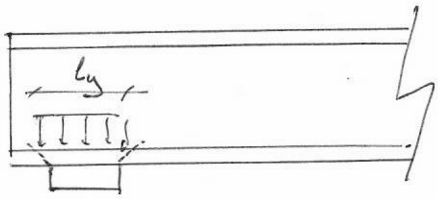
M_{cr}: elastic LTB moment of perfect beam

We use the same column curves (a_0, a, b, c, d) to determine χ_{LT}:

$$\underbrace{M_d}_{\text{design capacity}} = \underbrace{\chi_{LT}\, M_c}_{\text{buckling coeff.} \times \text{cross-sectional capacity}} / \gamma_{M1}$$

γ_{M1} → safety factor

Web crippling



$$\bar{\lambda}_F = \sqrt{\frac{F_y}{F_{cr}}}$$

$F_y = l_y \cdot t \cdot f_y$ yield capacity of the effective area of the web

F_{cr}: elastic buckling load of the web under vertical stresses

$$\rightarrow \chi_F = \frac{0.5}{\bar{\lambda}_F} \quad \text{(strength curve)}$$

$$\underbrace{F_d}_{\text{design capacity}} = \underbrace{\chi_F\, F_y}_{\text{buckling coeff} \times \text{yield capacity}} / \gamma_{M1}$$

γ_{M1} → safety factor

b. (1) RHS 150 x 100 x 5.0

$A_g = 2370 \text{ mm}^2$

$A_n = 2370 - 4(22)(5.0) = 1930 \text{ mm}^2$

* Net section fracture

the load is introduced concentrically

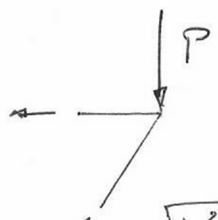
$\rightarrow$ no reduction for shear lag

$N_1 = (0.9) A_n f_u / \gamma_{M2} = (0.9)(1930)(490)/(1.1) = \underbrace{774 \text{ kN}}_{\text{critical}}$

* Gross section yielding

$N_2 = A_g \cdot f_y / \gamma_{M1} = (2370)(350)/(1.1) = 841 \text{ kN}$

(2) <u>Segment BD</u>



CHS 193.7 x 8.0

$A = \pi (193.7 - 8.0)(8.0)$

$= 4667 \text{ mm}^2$

$P \frac{\sqrt{4^2+2^2}}{4} = \frac{4.47}{4} P = 1.12 P$

$P_y = A . f_y = (4667)(355) = 1657 \text{ kN}$

$\lambda = \sqrt{\frac{1657}{(1.12)(250)}} = 2.44$ curve (c) $\rightarrow \alpha = 0.49$

$\phi = \frac{1}{2}[1 + 0.49(\lambda - 0.2) + \lambda^2] = 4.01$ $\chi = \frac{1}{\phi + \sqrt{\phi^2 + \lambda^2}} = 0.115$

$N_d = \chi . P_y / \gamma_{M1} = 190 \text{ kN}$

$P_d = \frac{190}{1.12} = 170 \text{ kN}$

<u>Segment AB</u>

CHS 355.6 x 12.5 $A = \pi (355.6 - 12.5)(12.5) = 13474 \text{ mm}^2$

$P_y = A . f_y = 4783 \text{ kN}$

$\lambda = \sqrt{\frac{4783}{500}} = 3.09 \rightarrow \phi = 6.0 \rightarrow \chi = 0.090$

$N_d = 429 \text{ kN} > 2 \times 170 \text{ kN}$

$\rightarrow P_d = \underline{170 \text{ kN}}$

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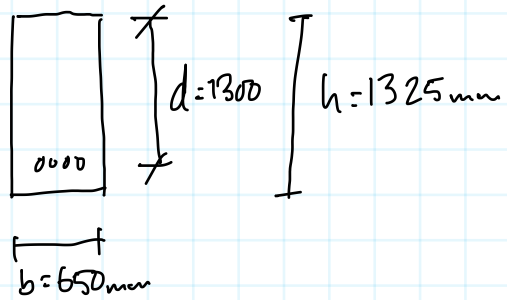
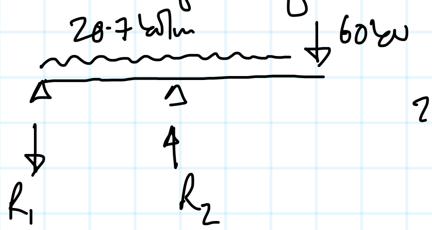
Q2(a)(i) cantilever, choose $L/d = 6$ $\therefore d = \frac{8000}{6} = \underline{1300\text{mm}}$

choose $b = d/2 = \underline{650\text{mm}}$

$h = d + \phi/2 +$ link + cover
$= 1300 + 25/2 + 12 = 1325\text{mm}$ (estimate)



(ii) beam self-weight $= 0.65 \times 1.325 \times 24 = 20.7$ kN/m

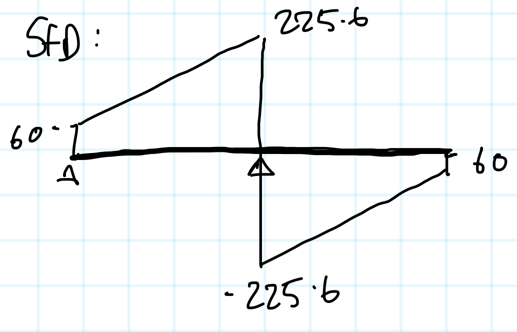


$20.7 \times 16 \times 8 + 60 \times 16 = R_1 \cdot 8$

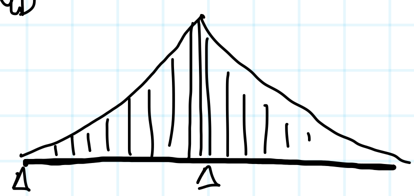
$R_1 = 451.2$ kN
$\underline{R_2 = 60 \text{ kN}}$

$\Delta BE = 451$ kN
$\Delta AD = -60$ kN

(iii) SFD:



BMD



$M_{max} = \frac{1}{2}(60 + 225.6)8$

$= \underline{1142 \text{ kNm}}$

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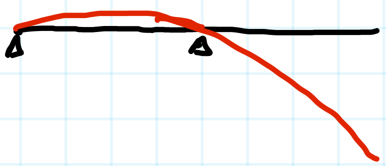
(iv)



(v) Check singly reinforced:

$M_u = 0.225bd^2 f_{cu} = 0.225(650)(1300^2)(60) = 14\,MNm$ ✓ ok

initial guess, $z = 0.9d = 1170\,mm$

$A_s = \frac{1142 \times 10^6}{500/1.15} = 1986\,mm^2$

$\Rightarrow$ provide $5\phi 25\,mm^2 = 2454\,mm^2$

$F_s = 2454 \times \frac{500}{1.15} = 1067\,kN$

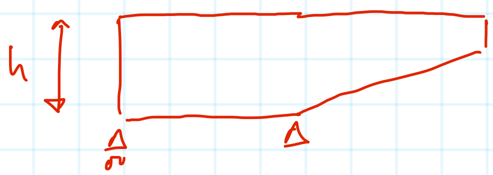
$F_c = 0.6 \frac{f_{cu}}{1.5} x \Rightarrow x = 44.5\,mm$

Check $\varepsilon_s = \frac{0.0035}{x} \cdot (d - x) = 0.09 >$ yield $\therefore$ ok ✓

$\therefore z = 1300 - \frac{44.5}{2} = 1278\,mm$

$M_u = 1067 \times 10^3 \cdot 1278 = 1363\,kNm > 1142 \therefore$ ok

(vi) The beam could be tapered:



Q2(b)(i) $R_2 = 451\,kN$

1) Bearing: $\sigma_B = \frac{f_k}{\gamma_m} = \frac{5}{3} \approx 1.67\,MPa$

$$P_B = \sigma_B (1000)(t) = 451\,kN$$

$$t \geq 270\,mm$$

2) Buckling $P = \beta \frac{f_k}{\gamma_m} b.t$

$e_x \approx 0.05t$ (assumed)

say $t = 270\,mm$, then $h/t = 14.8$ & $\beta = 0.88$

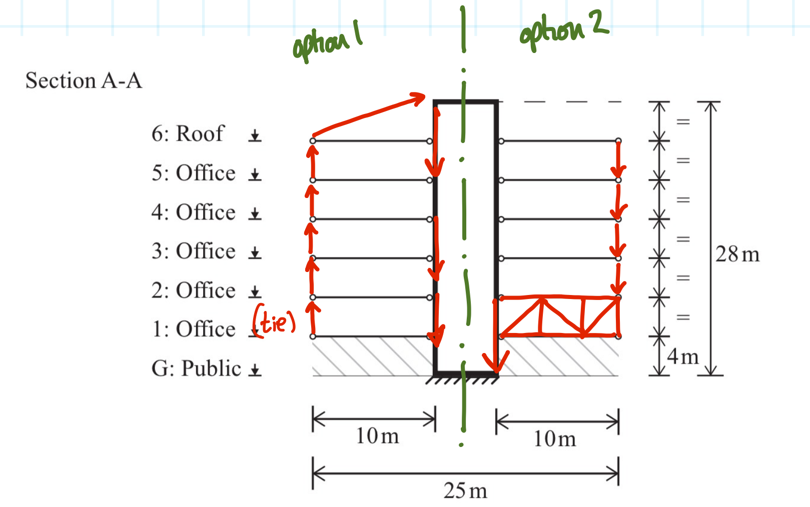
$P = 396\,kN < 451\,kN.$

$t = 300\,mm$ $h/t = 13.3$; $\beta = 0.92$; $P = 414\,kN$

$t = 333\,mm$ $h/t = 12$ $\beta = 0.93$ $P = 516\,kN$ $\therefore$ ok.

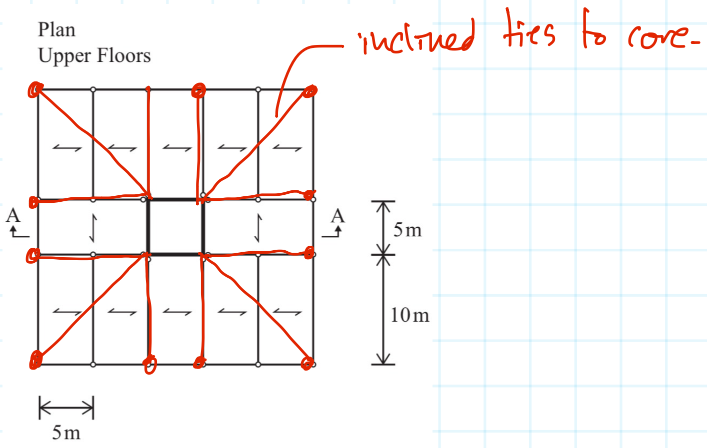
$t_{req} \approx \underline{\underline{333\,mm}}$

Q3



<u>Vertical</u>: most sensible seems to be option 1 above - tension columns at perimeter and hung from core. Alternatives include a truss at level 1, or bracing each level up to top of core.

note in option 1, beam at roof level must carry significant axial load.



(option 2 would be considerably less efficient.)

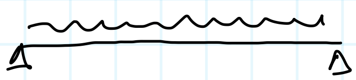
<u>Lateral</u>: tie floors back to core using RC slabs as diaphragms. span = 5m, choose thickness 200mm.

(b) Design primary beam, span 10m.

Loading: slab = 200mm = $0.2 \times 24 = 4.8\,kN/m^2$
q_k $= 2\,kPa$

$1.35G + 1.5Q = 9.5\,kPa$ (note excludes beam swt)

assume 5m tributary area $= 47.5\,kN/m$


47.5kN/m

SLS limit: span/180 = 10,000/180 = 56mm

ULS limit: σ_y = 355MPa (assume $\gamma = 1.0$)

$$M_{max} = \frac{47.5(10^2)}{8} = 594\,kNm$$

ULS:

$Z_p \geq M/\sigma_y$
$\geq 594 \times 10^6 / 355$
$\geq 1673\,cm^3$
$\downarrow$
457 × 191 × 82

SLS

$\delta_{max} \leq 5wL^4/384EI$

$I \geq 5wL^4/384E\delta_{max}$

$$\geq \frac{5(47.5)(10{,}000^4)}{384(200{,}000)(56)}$$

$\geq 55222\,cm^4$
$\downarrow$
533 × 210 × 92 UB

($Z_p = 2360\,cm^3$; $I_{xx} = 55230\,cm^4$) OK ✓

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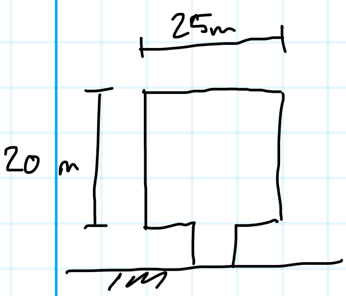
Q3(c) Dead load per floor: 200mm slab $= 0.2 \times 24 = 4.8kPa$
140m beam $= 92 \times 14 \times 10 \times g = 126kN$

Area $= 25^2 - 5^2 = 600m^2$

Load $= 4.8(600) + 126 \approx 3006kN = \underline{\underline{3MN}}$

$\times 6$ floors $= \underline{\underline{18MN}} \downarrow$

Wind load



Area $= 25(20) = 500m^2$

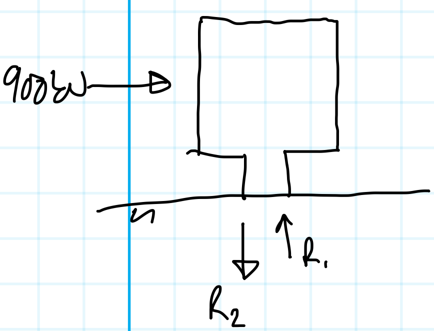
$q = 1.2(1.5) = 1.8kPa$

$P = 500(1.8) = 900kN$.

Assume acts at mid height, $\frac{20}{2} + 4 = 14m$ from ground.

$M_{max} = 900 \times 14 = 12.6MNm.$

Then resolve into push-pull at base of core:



$R_1 \cdot 5 = 900(14)$

$R_1 = 2.5MN \quad R_2 = 2.5MN \downarrow$

(d)
- Increase core height to decrease hange force.
- don't build it
- don't use concrete
- shape to be more aerodynamic
- reduce number of floors
- etc..

Q4(a)(i) UDL:

ULS $= 1.35G + 1.5Q$

$= 1.35(0.75) + 1.5(4) = 7\text{kPa}$

Point load:

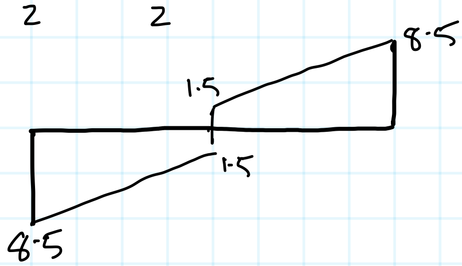
ULS $= 1.5(4) = 6$ kN

$6/2 = 3$ kN

$7\text{kPa} \times 0.5 = 3.5\text{kN/m}$

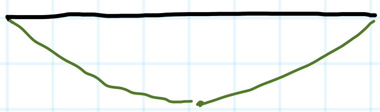
(ii) $R = \frac{2.7}{2} + \frac{3.5(4)}{2} = 8.5$ kN

SFD:



(kN)

BMD:



$M_{max} = 10\text{kNm}$

(iii) Bending – at midspan

$f_{m,d} = k_{mod}\, k_h\, k_{crit}\, k_{ls}\, f_{m,k} / \gamma_m$

$k_{mod} \Rightarrow$ SC2, medium term $= 0.80$

$k_{crit} = 1$ (assume restraint from deck)

$f_{m,d} = 0.8(1)(1)(1)(24)/1.3 = \underline{14.8}$ MPa

$\sigma = \frac{M}{Z} = \frac{10 \times 10^6}{bd^2/6}$

$\therefore d^2 = \frac{10 \times 10^6 \times 6}{14.8(100)} \Rightarrow d = 201$ mm

$d_{,req} = \underline{200\text{mm}}$

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(iii) contd Shear, at supports, $V = 8.5\ kN$

$f_{vd} = k_{mod}\, k_{ls}\, f_{v,k}\, k_{cr} / \gamma_m$ (k_{cr} from question)

$= 0.8(1)(2.5)(0.67)/1.3 = \underline{1\ MPa}$

$\sigma = \frac{3}{2}\frac{V}{bd} = 1\ MPa \quad \therefore\ d_{req} = 127.5\ mm$

$\therefore$ bending governs.

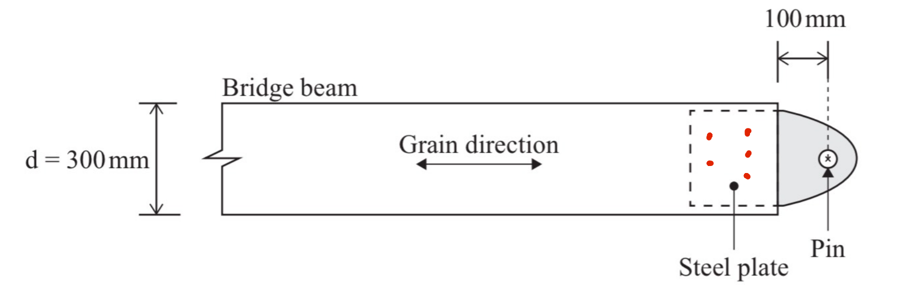
(iv) For example: dynamics, fire, durability, carbon, the deck construction, etc...

(b)(i) $k_{mod} = 0.8$

bolt design strength $= \frac{5 \times 0.8}{1.3} = 3\ kN$ ($\perp$ grain)

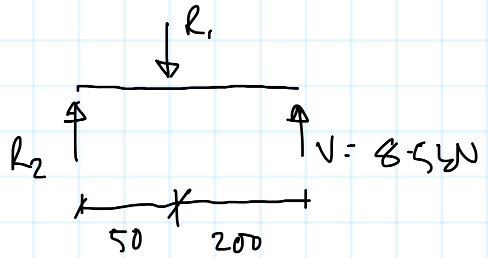
$\frac{7 \times 0.8}{1.3} = 4.3\ kN$ (// grain)

(ii) Start with minimum spacings:



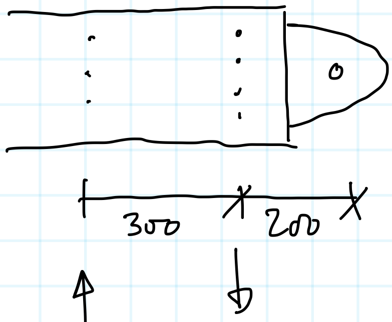
end distance = 8d = 96 mm ≈ $\underline{100\ mm}$
Min. spacing = 4d = $\underline{50\ mm}$

b(ii) cont'd



$R_1 = 43.kN \rightarrow$ 14 bolts – would not fit since
$14 \times 4 \times 12 = 672mm > d$

∴ increase spacing:

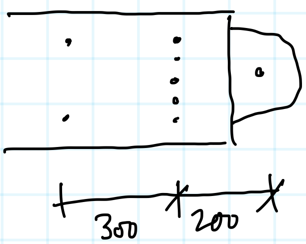


$R = 5.5\ kN$ $\quad$ $R = 14kN$

↳ 5 bolts = 15kN ✓

check spacing $5 \times 4 \times 12 = 240mm < d$ ∴ ok

second row: 2 bolts = 6 kN ✓



Engineering Tripos Part IIA 2024
Assessors Report
Module 3D3 Concrete and Prestressed Concrete

This module was taken by 23 IIA students. The target number of candidates per class was achieved without scaling.

Examination

Question 1

In part (a) most candidates could adequately answer (i) and (ii), in (iii) answers were less convincing, often being reading like a regurgitation of the wrong part of a textbook. In part (b) many candidates struggled with both parts, undertaking irrelevant calculations.

Question 2

Many candidates spent far too long answering part (a)(i). Many candidates forgot about self weight when calculating the change in axial load in (a)(ii). In (a)(iii) the quality of shear force and bending moment diagrams was poor generally, with many impossible diagrams presented. Deflected shapes were generally adequate, good candidates annotated their sketches to clarify what kind of lines they were drawing. Longitudinal reinforcement design was complicated for some candidates who had derived strange values of moment in (iii), but generally completed well. In part (b) the majority of candidates checked both bearing and buckling.

Question 3

In part (a) nearly every candidate came up with the same solution – diagonal bracing at each floor level, which is a feasible solution but an inefficient one – good candidates realised that such bracing imposes a compression load into the floor beams. In part (b) candidates adequately considered ULS, some candidates forgot to check for SLS. Part (c) was generally rushed, many candidates calculated only one or two of the requested values, and often got confused with the load case being considered. Part (d) was well answered.

Question 4

This question was not particularly well answered overall. There were a range of solutions for ULS loading, and resulting shear force and bending moment diagrams were often incorrect or impossible. Many candidates could use equations to derive a beam size, but when such results were either very big or very small, very few made a comment on this. The connection design was poorly done overall – very few candidates even attempted (b)(ii).

J Orr
3 May 2024