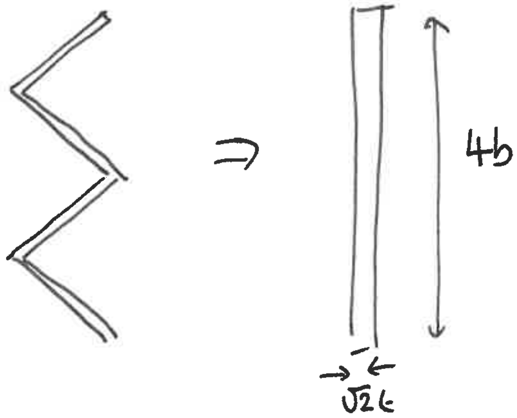
1(a)(i) The x-axis is an axis of symmetry, hence centroid lies on this.

The y-axis has material equally disposed ~~either~~ on either side (for every δA at $+x$, there is δA at $-x$)

$\therefore$ $(x, y) = (0, 0)$ is centroid.

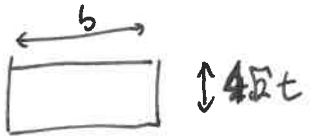
(ii) Consider 'squeezing' material along x-dim (doesn't change I_{xx})



hence

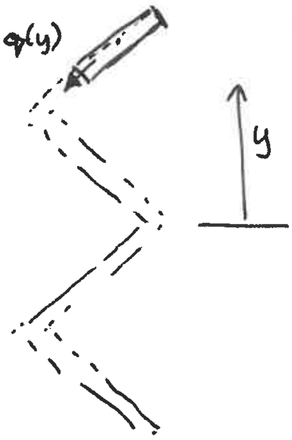
$$I_{xx} = \frac{(4b)^3 \cdot \sqrt{2}t}{12} = \frac{16\sqrt{2}}{3} b^3 t$$

(similarly for I_{xx} used in (b)



$$I_{yy} = \frac{b^3 \cdot 4\sqrt{2}t}{12} = \frac{\sqrt{2}}{3} b^3 t$$)

Use $q = \frac{FA\bar{y}}{I}$ – cut at y



$A = (2b - y) . \sqrt{2}\, t$

$\bar{y} = (2b + y) . \frac{1}{2}$

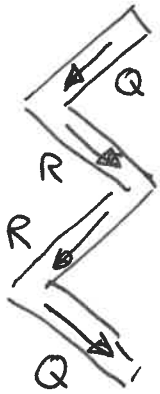
$I_{xx} = \frac{16\sqrt{2}\, b^3 t}{3}$

$\therefore q(y) = (4b^2 - y^2) . \frac{3}{32b^3} . F$

Shear 'flows' along section, hence $\tau = q/t$

$$\tau(y) = \frac{3}{32} . \frac{(4b^2 - y^2)}{b^3 t} . F$$

(iii) Consider force in section as four forces, where each is $\int q\, ds$, where $ds = \sqrt{2}\, dy$



For outside sections

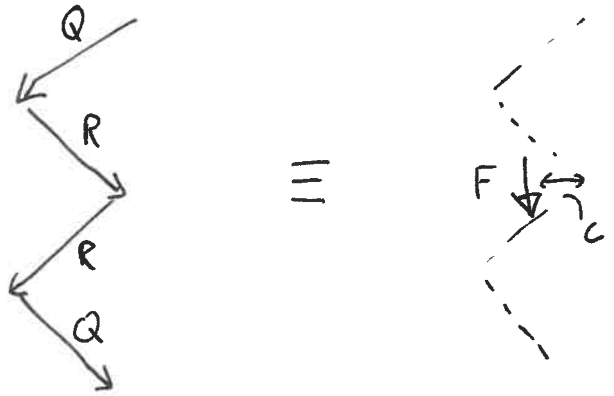
$$Q = F \int_{b}^{2b} \frac{3}{32b^3} . (4b^2 - y^2) . \sqrt{2}\, dy$$

$$Q = \frac{3\sqrt{2}F}{32b^3} \left[4b^2 y - \frac{y^3}{3} \right]_{b}^{2b}$$

$$= \frac{3\sqrt{2}F}{32} \left(\left(8 - \frac{8}{3}\right) - \left(4 - \frac{1}{3}\right) \right)$$

$$= \frac{5\sqrt{2}}{32} F$$

Consider moments about C



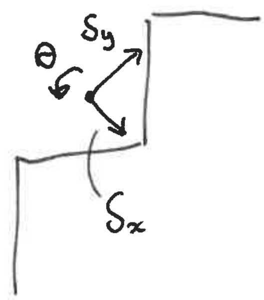
$$2Q \cdot \sqrt{2}\, b = Fc$$

$$c = \frac{5b}{8}$$

i.e. shear centre is at $x = \frac{b}{2} - \frac{5b}{8} = -\frac{b}{8}$

$y = 0$ (by symmetry)

(b)



Consider deflection of shear centre (use '$\frac{WL^3}{3EI}$' and '$\frac{TL}{GJ}$')

$$\delta_x = \frac{W}{\sqrt{2}} \cdot \frac{L^3}{3E} \frac{3}{\sqrt{2} b^3 t} = \frac{WL^3}{Eb^3 t} \cdot \frac{1}{2}$$

$$\delta_y = -\frac{W}{\sqrt{2}} \cdot \frac{L^3}{3E} \cdot \frac{3}{16\sqrt{2} b^3 t} = -\frac{WL^3}{Eb^3 t} \cdot \frac{1}{32}$$

$$T = \frac{5b}{8} \cdot \frac{1}{\sqrt{2}} \cdot W = \frac{5\sqrt{2}\,Wb}{16}$$

$$J = 4 \cdot \frac{\sqrt{2}\,b t^3}{3}$$

$$\theta = -\frac{5\sqrt{2}}{16} Wb \cdot L \cdot \frac{3}{4\sqrt{2}\,b t^3} = -\frac{15}{64} \frac{WL}{Gt^3}$$

Work in N, mm

$$\frac{WL^3}{Eb^3 t} = \frac{20 \times 100^3}{70 \times 10^3 \times 10^3 \times 1} = 0.2857 \text{ mm}$$

$$\frac{WL}{Gt^3} = \frac{20 \times 100}{\cancel{70\times} \cdot 26 \times 10^3 \times 1^3} = 0.07692$$

$$\therefore \delta_x = 0.143 \text{ mm} \; ; \; \delta_y = -0.0089 \text{ mm} \; ; \; \theta = -0.0180$$

$$\text{Vertical displacement of } C = \underbrace{\frac{\delta_x}{\sqrt{2}} - \frac{\delta_y}{\sqrt{2}}} - \theta \times \overbrace{\frac{5\sqrt{2}\,b}{16}}^{= 4.42 \text{ mm}}$$

$$= 0.107 \text{ mm} + \cancel{0.\ldots}\; 0.080 \text{ mm}$$

$$= 0.187 \text{ mm} \downarrow$$

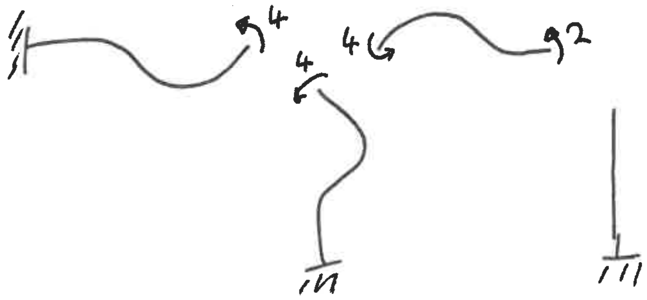
$$\text{Horizontal displacement of } C = \underbrace{\frac{\delta_x}{\sqrt{2}} + \frac{\delta_y}{\sqrt{2}}} + \theta \times \frac{5\sqrt{2}\,b}{16}$$

$$= 0.095 \text{ mm} - 0.080 \text{ mm}$$

$$= 0.015 \text{ mm} \rightarrow$$

2 Consider a rotation of 1 at B, 0 at C

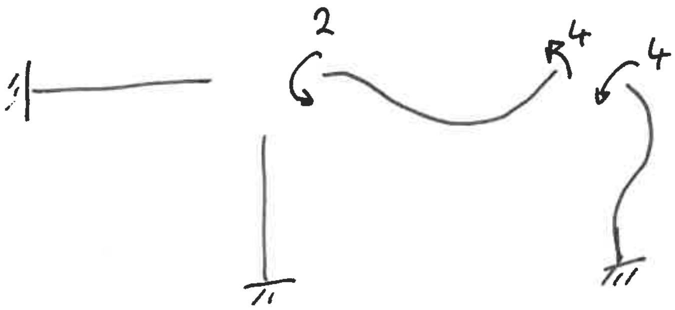
Data Book shows



All $\times \frac{EI}{L}$

i.e. $Q_B = 12$, $Q_C = 2$

Consider at rotn. of 0 at B, 1 at C



All $\times \frac{EI}{L}$

i.e. $Q_B = 2$, $Q_C = 8$

superpose:

$$\Rightarrow \begin{bmatrix} Q_B \\ Q_C \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$$

2b Additional stiffness due to members BF and CG of $\frac{GJ}{L}$ at B and C

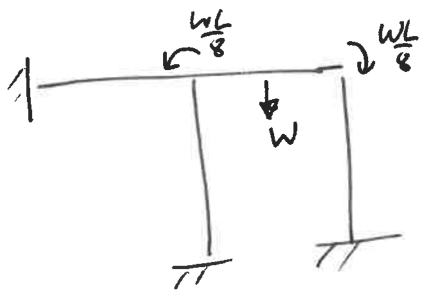
$$\Rightarrow \begin{bmatrix} Q_B \\ Q_C \end{bmatrix} = \frac{EI}{L}\begin{bmatrix} 12 & 2 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} + \frac{GJ}{L}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$$

for $\frac{GJ}{EI} = 0.611$

$$= \frac{EI}{l}\begin{bmatrix} 12.611 & 2 \\ 2 & 8.611 \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$$

Consider superposition of two solutions.

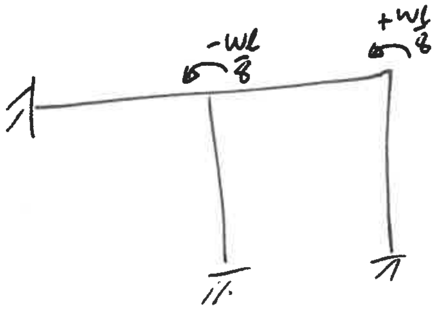
(1)



No rotn. of joints.

+

(2)



Cancel moments at joints.

(i) Analyse system (2) to find θ_B & θ_C

2b(i).

$$\begin{bmatrix}\theta_B\\ \theta_C\end{bmatrix} = \frac{L}{EI}\cdot\frac{1}{(12.611\times 8.611)-4}\begin{bmatrix}8.611 & -2\\ -2 & 12.611\end{bmatrix}\begin{bmatrix}-1\\ +1\end{bmatrix}\frac{WL}{8}$$

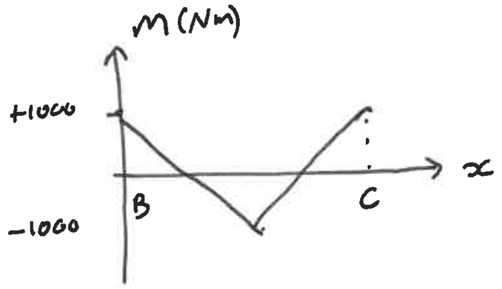
$$= \frac{WL^2}{EI}\cdot\begin{bmatrix}-0.01268\\ 0.01746\end{bmatrix}$$

for SHS, $\frac{WL^2}{EI} = \frac{2\times 10^3\,N \times (4m)^2}{210\times 10^9\,N/m^2 \times 137\times 10^{-6}\,m^4}$

$= 1.112\times 10^{-3}$

$$\therefore \begin{bmatrix}\theta_B\\ \theta_C\end{bmatrix} = \begin{bmatrix}-14.1\\ 19.4\end{bmatrix}\times 10^{-6}$$

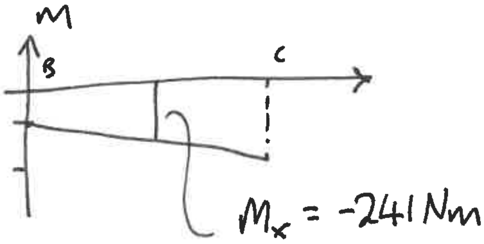
(ii) Consider BC. For system ① $\frac{WL}{8} = 1000$ Nm



For system ②, $\frac{EI}{L} = 7.1925\times 10^6$ Nm

$M_B = (4\theta_B + 2\theta_C)\frac{EI}{L} = -126.6$ Nm

$M_C = -(2\theta_B + 4\theta_C)\frac{EI}{L} = -355.3$ Nm



2b(ii)

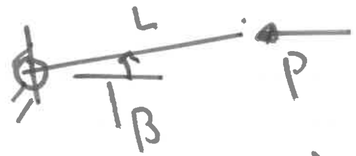
At centre of BC, $M = (-1000 - 241)$ Nm

$= -1241$ Nm.

Peak stress $= \dfrac{-My_{max}}{I}$

$$= \frac{1241 \times 40 \times 10^{-3}}{137 \times 10^{-8}} = 36.2 \times 10^{6} \, N/m^2$$

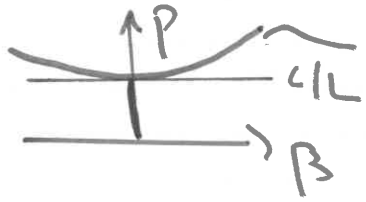
a)



restoring couple $= c\beta$
$= PL\sin\beta$ from P.

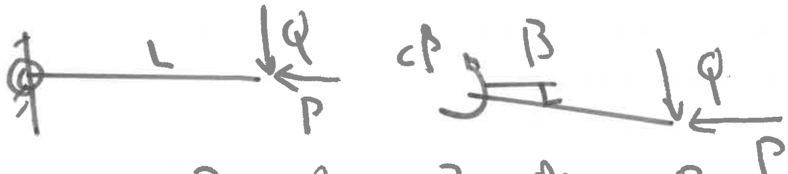
$\Rightarrow PL\sin\beta = c\beta$

$\Rightarrow P = \frac{c}{L}\cdot\beta/\sin\beta \simeq \underline{\underline{c/L}}$ for $\beta \simeq 0$



(symmetrical buckling w.r.t. β)

b)(i)



let this be +ve rotn dirn of β

$\Rightarrow c\beta - QL\cos\beta - PL\sin\beta = 0$

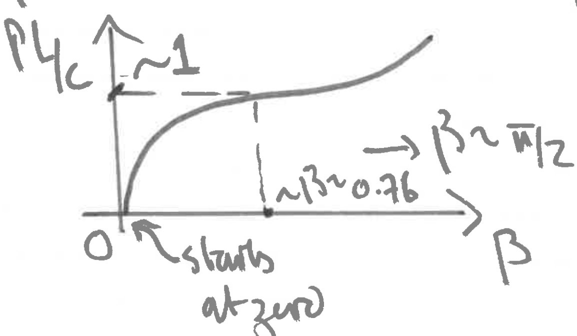
$\Rightarrow PL\sin\beta = c\beta - QL\cos\beta$ let $Q = \rho P$.

$\Rightarrow PL\sin\beta = c\beta - \rho PL\cos\beta$

$PL\sin\beta + \rho L\cos\beta P = c\beta$

$\Rightarrow \underline{\frac{PL}{c} = \beta/\sin\beta + \rho\cos\beta}$ (1)

When Q acts in same dirn as β



$\frac{PL}{c} \simeq 1$ when $\beta = 0.76$

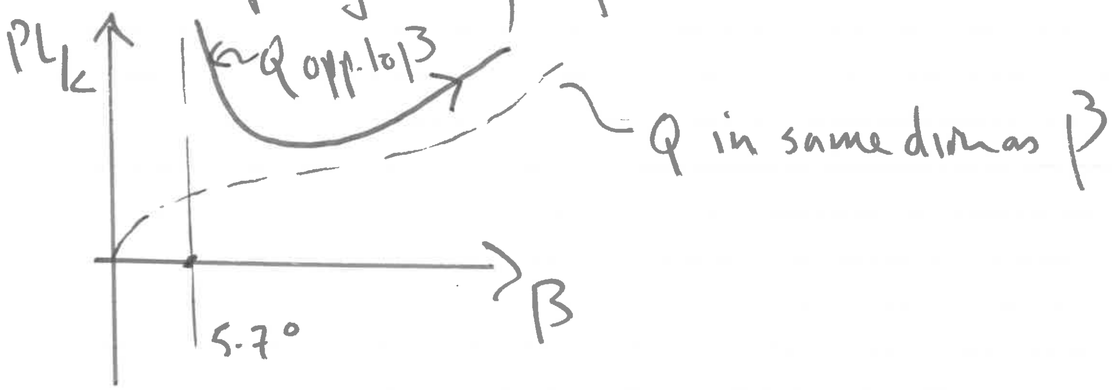
rises as $\beta \to \pi/2$
starts at zero.

b) (ii) when Q opposite to rotatn, treat ρ −ve in (1)

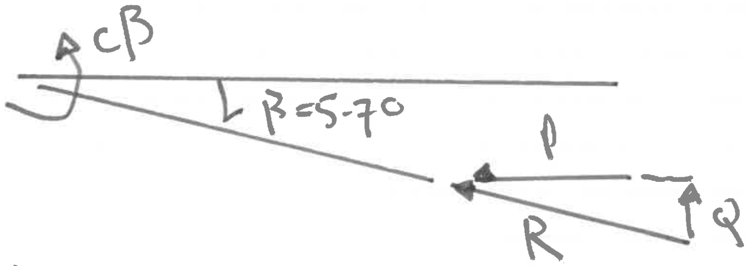
i.e. $\frac{PL}{c} = \beta/\sin\beta - \rho\cos\beta$ $\Rightarrow$ ρ and β take +ve values henceforth

b) (ii) contd: $\sin\beta = \rho\cos\beta$: $\tan\beta = \rho$ = asymptote

for $\rho = 1/10$, $\beta = 5.7°$

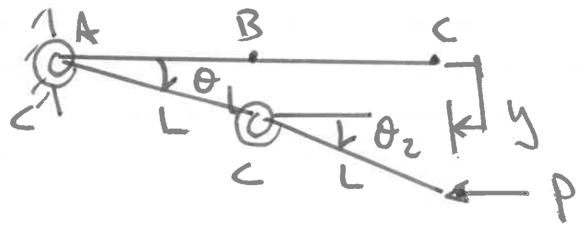


If $\beta = 5.7°$ P = infinite



R: resultant = axial: but $c\beta$ unbalanced ⇒
very large R (P and Q) needed for $\beta > 5.7°$
to bring moment eqlm.

a)



Angles of rotation are absolute w.r.t. horizontal

$\therefore$ rotation at B $= \theta_2 - \theta_1$

$$y = 2L - L\cos\theta_1 - L\cos\theta_2 = L\left[2 - (1 - \theta_1^2/2) - (1 - \tfrac{\theta_2^2}{2})\right]$$

$$= L\left[2 - 1 + \theta_1^2/2 - 1 + \theta_2^2/2\right] = \frac{L}{2}\left[\theta_1^2 + \theta_2^2\right]$$

No need to convert to lateral displacements.

$$\Pi \text{ (potential energy)} = \tfrac{1}{2}c\,\theta_1^2 + \tfrac{1}{2}c(\theta_2 - \theta_1)^2 - Py$$

$$\Pi = \tfrac{1}{2}c\theta_1^2 + \tfrac{1}{2}c\left[\theta_2^2 - 2\theta_2\theta_1 + \theta_1^2\right] - P\frac{L}{2}\left[\theta_1^2 + \theta_2^2\right]$$

$$= \tfrac{1}{2}c\left[\theta_1^2 + \theta_1^2 + \theta_2^2 - 2\theta_2\theta_1\right] - \frac{PL}{2}\left[\theta_1^2 + \theta_2^2\right]$$

$$= \frac{c}{2}\left[\theta_1\ \theta_2\right]\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} - \frac{PL}{2}\left[\theta_1\ \theta_2\right]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

b)

$$\Pi = \left[\theta_1\ \theta_2\right]\begin{bmatrix} c & -c/2 \\ -c/2 & c/2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} - \left[\theta_1\ \theta_2\right]\begin{bmatrix} \frac{PL}{2} & 0 \\ 0 & PL/2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$= \left[\theta_1\ \theta_2\right]\begin{bmatrix} c - PL/2 & -c/2 \\ -c/2 & c/2 - PL/2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \tfrac{1}{2}\,\underset{\sim}{\theta}\,\underset{\sim}{K}\,\underset{\sim}{\theta}$$

$$= \tfrac{1}{2}\left[\theta_1\ \theta_2\right]\begin{bmatrix} 2c - PL & -c \\ -c & c - PL \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

tangent stiffness matrix

$$\underset{\sim}{K} = \begin{bmatrix} 2c - PL & -c \\ -c & c - PL \end{bmatrix}$$

c) $|\underset{\sim}{K} - \lambda \underset{\sim}{I}| = 0$ for eigenvalues.

$\underset{\sim}{K} = \begin{bmatrix} a & -b \\ -b & d \end{bmatrix}$: $a = 2c - PL$; $d = c - PL$, $b = c$

$\Rightarrow \begin{vmatrix} a-\lambda & -b \\ -b & d-\lambda \end{vmatrix} = 0 \qquad (a-\lambda)(d-\lambda) - b^2 = 0.$

$ad - d\lambda - a\lambda + \lambda^2 - b^2 = 0 \qquad \lambda^2 - \lambda(d+a) + ad - b^2 = 0$

$$\lambda = \frac{(d+a) \pm [(d+a)^2 - 4ad + 4b^2]^{1/2}}{2}$$

$\underline{d+a = 3c - 2PL}$ $\qquad [\ldots]^{1/2} = [d^2 + 2ad + a^2 - 4ad + 4b^2]^{1/2}$

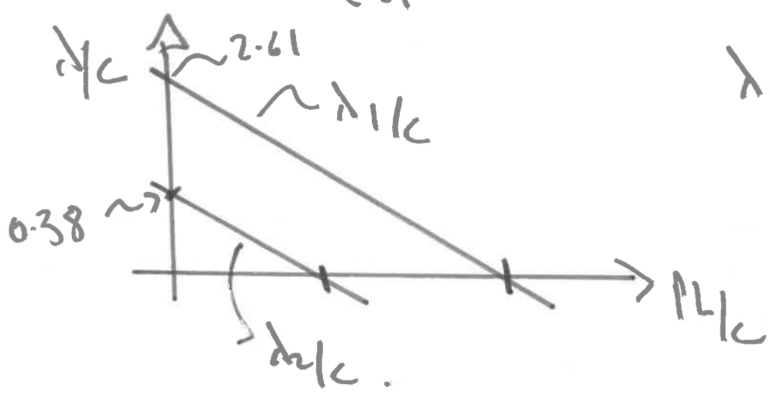
$= [d^2 - 2ad + a^2 + 4b^2]^{1/2}$

$= [(d-a)^2 + 4b^2]^{1/2}$

$d - a = c - PL - 2c + PL = -c$: $\rightarrow [c^2 + 4c^2]^{1/2} = \underline{\sqrt{5}c}$

$$\lambda_{1,2} = \frac{3c - 2PL \pm \sqrt{5}c}{2}$$

$\lambda_1 = c\Big[\underbrace{\tfrac{3}{2} + \tfrac{\sqrt{5}}{2}}_{2.61}\Big] - PL$: $\lambda_2 = c\Big[\underbrace{\tfrac{3}{2} - \tfrac{\sqrt{5}}{2}}_{0.382}\Big] - PL$



$\lambda_1 = 0 \Rightarrow \frac{PL}{c} = 2.61$

$\underbrace{\lambda_2 = 0 \Rightarrow PL/c = 0.382}_{\text{lowest} \rightarrow \text{primary buckling mode}}$

Need eigenmodes: $\begin{bmatrix} a-\lambda_i & -b \\ -b & d-\lambda_i \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = 0$

P.T.O.

$$\begin{bmatrix} 2c-\rho L-\lambda_i & -c \\ -c & c-\rho L-\lambda_i \end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = 0.$$

$$(2c-\rho L-\lambda_i)\theta_1 - c\theta_2 = 0.$$

For λ_1: $\left[2c - \cancel{\rho L} + \cancel{\rho L} - c\left[\frac{3}{2}+\frac{\sqrt{5}}{2}\right]\right]\theta_1 - c\theta_2 = 0$

$$\cancel{c}\left[2 - 3/2 - \frac{\sqrt{5}}{2}\right]\theta_1 = \cancel{c}\,\theta_2$$

$$(1/2 - \sqrt{5}/2)\,\theta_1 = \theta_2$$

$$-0.618\,\theta_1 = \theta_2$$

For λ_2 $\theta_1\left\{2c - \cancel{\rho L} + \cancel{\rho L} - c\left[3/2 - \sqrt{5}/2\right]\right\} - c\theta_2 = 0$

$$\theta_1\left[2\cancel{c} - \frac{3\cancel{c}}{2} + \frac{\sqrt{5}\cancel{c}}{2}\right] = \cancel{c}\,\theta_2$$

$$\theta_1\left[1/2 + \sqrt{5}/2\right] = \theta_1$$

$$1.618\,\theta_1 = \theta_2.$$

λ_2: (fundamental)

λ_1: (second)