Version KAS/4

EGT2
ENGINEERING TRIPOS PART IIA

Monday 4 May 2026   14.00 to 15.40

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Engineering Data Book
CUED approved calculator allowed

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Figure 1(a) shows the cross-section of an elastic beam, where $t \ll b$.

(i) Show that the $(x, y) = (0, 0)$ is the centroid of the cross-section. [10%]

(ii) Show that the magnitude of shear stress carried by the section due to a shear force F acting in the y-direction is given by

$$\tau = \frac{3(4b^2 - y^2)}{32b^3 t} F$$ [20%]

(iii) Find the position of the shear centre on the cross-section. [30%]

(b) Figure 1(b) shows a horizontal cantilever beam of length $L = 100$ mm that uses the cross-section as shown in Figure 1(a), rotated by 45°. Diaphragms (not shown) ensure that the cross-section does not distort in plane. The cantilever is loaded with a vertical force $W = 20$ N at the point C shown. The cantilever is made of Aluminium alloy, and has dimensions $b = 10$ mm and $t = 1$ mm. The support at the base of the cantilever, and the diaphragms, do not constrain warping.

Find the vertical and horizontal displacement of C due to the load applied. [40%]

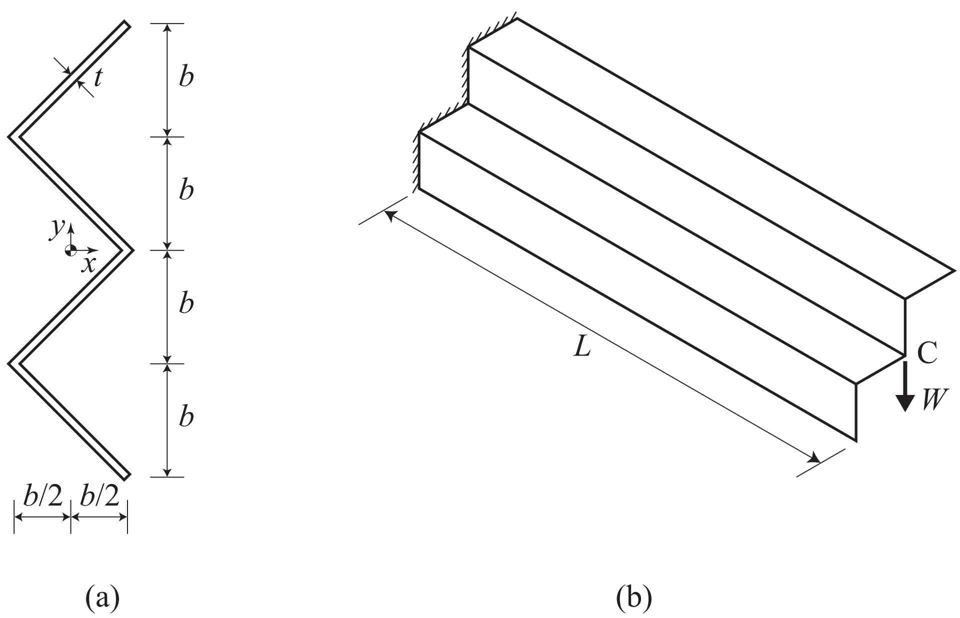

Fig. 1

2 (a) Figure 2(a) shows a frame in 2D. Each member is of length L and bending stiffness EI, and the members are rigidly joined together at joints B and C. All members can be considered effectively to have infinite axial stiffness.

Show that the relationship between the couples applied at B and C, Q_B and Q_C respectively, and the corresponding rotations θ_B and θ_C, can be written as the stiffness relationship

$$\begin{bmatrix} Q_B \\ Q_C \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} 12 & 2 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}$$ [20%]

(b) Figure 2(b) shows the same frame in 3D, with additional horizontal members perpendicular to the main frame, also rigidly connected at B and C. All members have length $L = 4\,\text{m}$ and are made of the steel Square Hollow Section listed in the Structures Databook of size $80\,\text{mm} \times 80\,\text{mm}$ with a wall thickness $5\,\text{mm}$. This section has $I = 137\,\text{cm}^4$ and a ratio $GJ/EI = 0.611$. A vertical load $W = 2\,\text{kN}$ is applied to the frame halfway between B and C.

(i) Calculate θ_B and θ_C. [40%]

(ii) Calculate the maximum longitudinal stress carried in the beam BC. [40%]

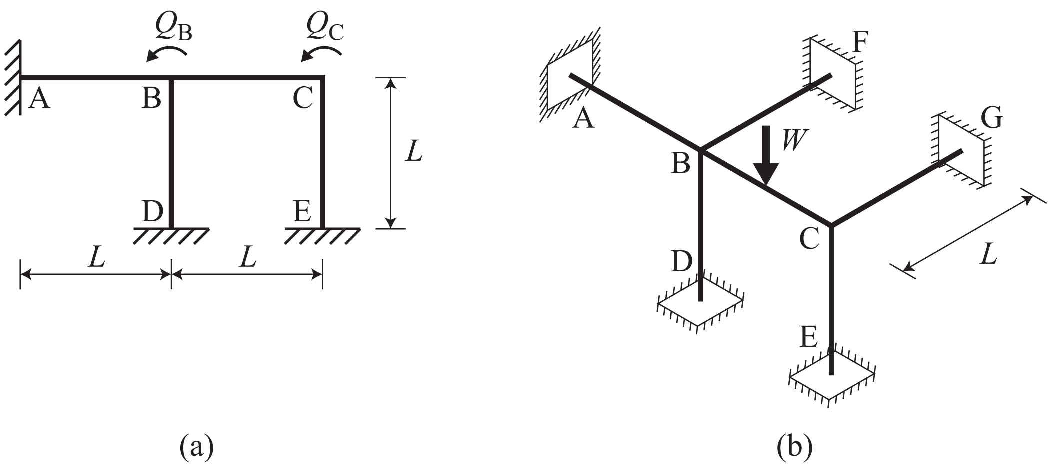

Fig. 2

3 (a) Figure 3(a) shows a rigid, horizontal cantilever of length, L, which is connected to ground by a frictionless pin-joint. A torsional spring of linear stiffness, c [Nm rad^{-1}], furnishes a restoring couple when the cantilever rotates. An axial force, P, is applied initially to the tip of the cantilever.

Show that the cantilever buckles when P is equal to c/L: you may ignore self-weight. Sketch the post-buckling variation of P with cantilever rotation. [30%]

(b) Figure 3(b) shows the same cantilever initially, where a transverse force, Q, is applied to the tip in addition to, and orthogonal to, P.

(i) Sketch the variation of P with cantilever rotation when $Q = P/10$, and when Q acts in the same direction as the rotation. [20%]

(ii) When Q acts opposite to the rotation, show that the variation of P is asymptotic to a particular angle of rotation, and calculate this angle when Q is equal in magnitude to $P/10$. Sketch the variation of P. [30%]

(iii) Suggest a physical reason for the asymptotic behaviour. [20%]

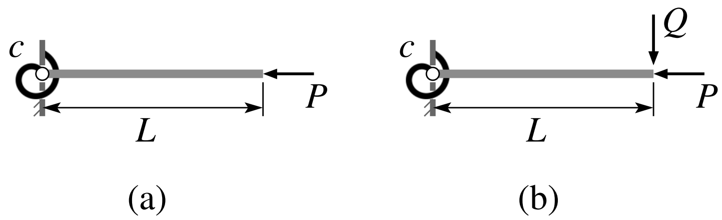

Fig. 3

4 Figure 4 shows a discrete cantilever beam, AC, made from two rigid bars, AB and BC, interconnected by frictionless pin-joints: joint A is connected to ground. There are torsional springs of linear stiffness, c [Nm rad^{-1}], at joints A and B. An axial horizontal force, P, is applied to joint C, in order to cause buckling.

(a) For the indicated deformed shape, calculate the total energy potential for relatively small angles of rotation, θ_1 and θ_2. [30%]

(b) Determine the total tangent stiffness matrix of the system. [30%]

(c) Compute the eigenvalues and sketch their variation with P. [30%]

(d) Comment on the relationship between the loading and the eigenvalues. [10%]

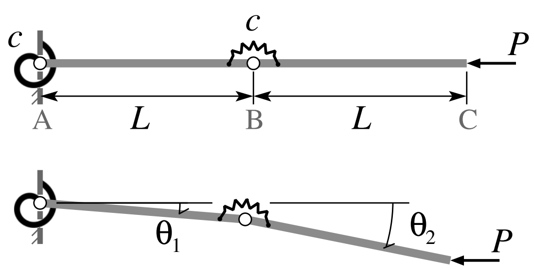

Fig. 4

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