2024/2025 IIA 3D5 - Water Engin1eering Prof. D. Liang

1.

(a.i) The infiltration rate *f* is

$$f(t) = f_c + (f_0 - f_c)e^{-K_f t} = 10 + 20e^{-0.5t}$$

$f(0)$ = 30 mm/hr

$f(2)$ = 17.36 mm/hr

$f(4)$ = 12.71 mm/hr

Not all the rain falling in the second two hours infiltrates into the ground, so the rain falling in the second two hours also contributes to the outflow.

The amount of water infiltrated is

$$\int_{t_0}^{t_1}\left[f_c + (f_0 - f_c)e^{-K_f t}\right]dt = f_c(t_1 - t_0) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_1} - e^{-K_f t_0}\right)$$

1st two-hour period: $\int_0^2 \left(10 + 20e^{-0.5t}\right)dt = 10 \times 2 - \frac{1}{0.5} \times 20 \times \left(e^{-1} - 1\right) = 45.28$ mm

2nd two-hour period: $\int_2^4 \left(10 + 20e^{-0.5t}\right)dt = 10 \times 2 - \frac{1}{0.5} \times 20 \times \left(e^{-2} - e^{-1}\right) = 29.30$ mm

Total volume of runoff is

1st two-hour period: (2×40-45.28)×0.001 × 20×1000000 = 694400 m^3

2nd two-hour period: (2×20-29.30)×0.001 × 20×1000000 = 214000 m^3

Sum the corresponding volumes in each two hours

	time (hour)	1	3	5	7	9	11	13
	1st two hours: 694400 m^3×	3%	16%	35%	27%	15%	4%	
+	2nd two hours: 214000 m^3×	0	3%	16%	35%	27%	15%	4%
=	total volume (m^3)	20832	117524	277280	262388	161940	59876	8560

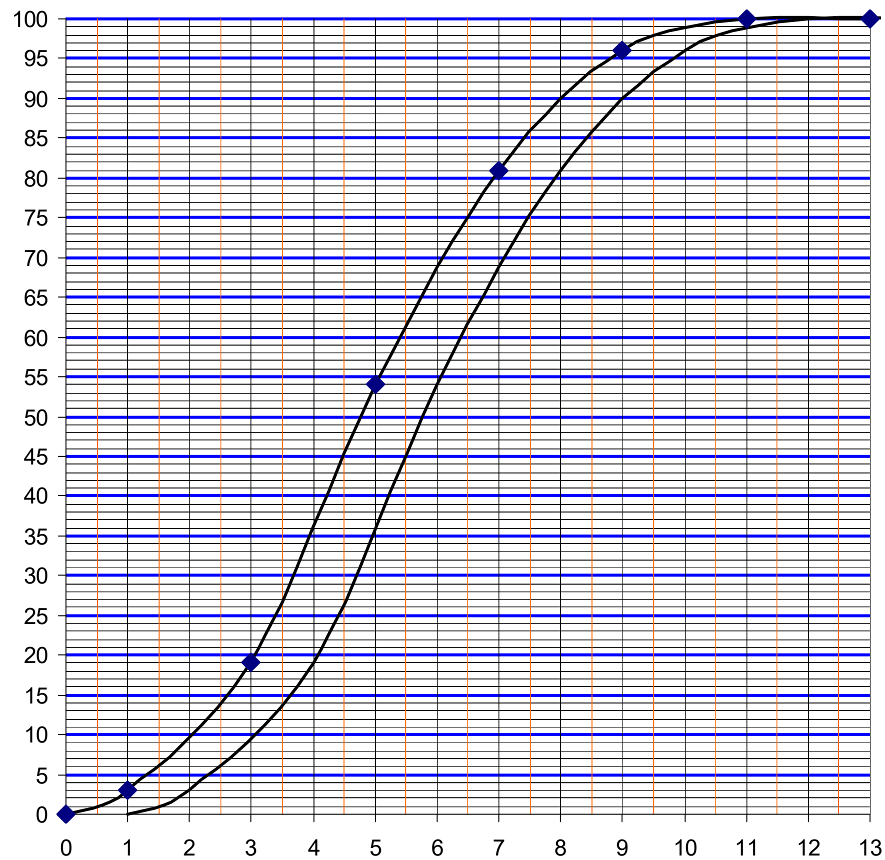
The peak discharge occurs at the third two-hour period. The discharge is

$$\frac{277280}{7200} = 38.51 \text{ m}^3/\text{s}$$

(a. ii) Answering this part needs to construct the hydrograph on the basis of a unit time of 1 hour.

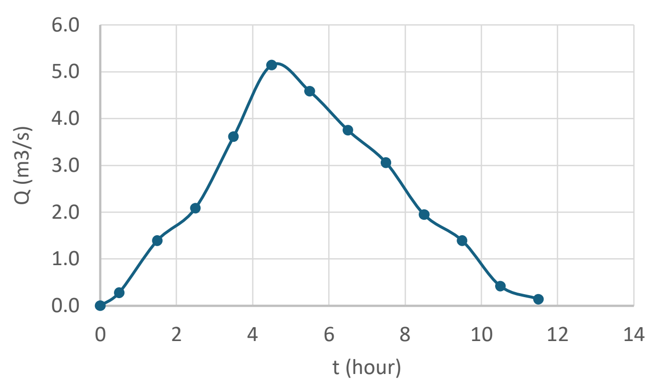
Plot S curve:

t:	0	1	3	5	7	9	11
%:	0	3	19	54	81	96	100



Shift by one hour and subtract:

t (hr)	0	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5
%:	0	1	5	7.5	13	18.5	16.5	13.5	11	7	5	1.5	0.5
Q(m^3/s)	0.0	0.3	1.4	2.1	3.6	5.1	4.6	3.8	3.1	1.9	1.4	0.4	0.1



(b) Generally, the characteristic lines are curved, but it can be proved that one family of characteristic lines are straight for subcritical flows in a semi-infinitely long channel with uniform initial condition and negligible $g(S_b - S_f)$.

2. (a)

$$U = \frac{1}{n} \cdot R_h^{\frac{2}{3}} S_b^{\frac{1}{2}}$$

$$q = uh = \frac{1}{n} \cdot h^{\frac{5}{3}} S_b^{\frac{1}{2}}$$

$$0.4 = \frac{1}{0.03} \times h^{\frac{5}{3}} \times 0.0001^{\frac{1}{2}}$$

h = 1.1 m

From $C = 7.8 \ln\left(\frac{12.0 \cdot R_h}{k_s}\right)$, $C = \frac{1}{n} \cdot R_h^{1/6}$

We have $7.8\, ln\left(\frac{12.0 \cdot R_h}{k_s}\right) = \frac{1}{n} \cdot R_h^{1/6}$

$$7.8\, ln\left(\frac{12.0 \times 1.1}{k_s}\right) = \frac{1}{0.03} \times 1.1^{1/6} = 33.87$$

k_s = 0.17 m

(b) Gradually varied flow: $\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b - \frac{n^2 \cdot U^2}{R_h^{4/3}}}{1 - Fr^2}$

When h = 0.457 m: $U = \frac{4.65}{0.457} \approx 10.17$ m/s

$$S_f = \frac{n^2 \cdot U^2}{R_h^{4/3}} = \frac{0.02^2 \cdot 10.17^2}{0.457^{4/3}} \approx 0.1175$$

$$Fr = \frac{U}{\sqrt{gh}} = \frac{10.17}{\sqrt{9.81 \times 0.457}} \approx 4.803$$

$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{0.0003 - 0.1175}{1 - 4.803^2} \approx 0.0053$$

When h = 0.5 m: $U = \frac{4.65}{0.5} = 9.23$ m/s

$$S_f = \frac{n^2 \cdot U^2}{R_h^{4/3}} = \frac{0.02^2 \cdot 9.3^2}{0.5^{4/3}} \approx 0.0870$$

$$Fr = \frac{U}{\sqrt{gh}} = \frac{9.3}{\sqrt{9.81 \times 0.5}} \approx 4.196$$

$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{0.0003 - 0.0870}{1 - 4.196^2} \approx 0.0052$$

So $\frac{0.5 - 0.457}{\Delta x} = \frac{0.0053 + 0.0052}{2}$

$\Delta x = 8.19$ m

Or $$\frac{0.5-0.457}{\Delta x}=\frac{0.0003-\frac{0.1175+0.0870}{2}}{1-\left(\frac{4.803+4.196}{2}\right)^2}=0.00530$$

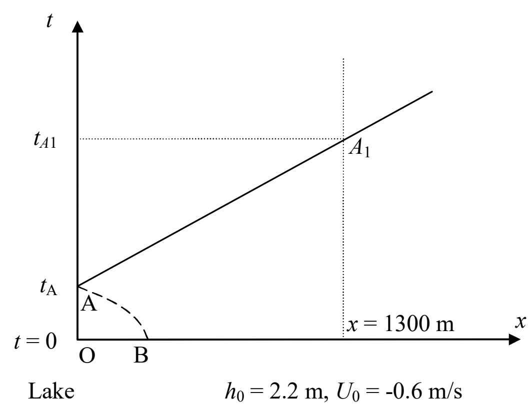
$\Delta x = 8.11$ m

Or using $$\frac{d}{dx}\left(h+\frac{U^2}{2g}\right)=S_b-S_f$$

$$\frac{\left(0.5+\frac{9.292^2}{2\times 9.81}\right)-\left(0.457+\frac{10.17^2}{2\times 9.81}\right)}{\Delta x}=0.0003-\frac{0.1175+0.0870}{2}$$

$\Delta x = 8.12$ m

(c)



Line AA_1 is straight. The water depth is 2.2 + 0.3 = 2.5 m.

From the boundary condition: $t_A=\frac{0.3}{0.5}=0.6$ h

Along –ve line AB

$$U_A-2\sqrt{9.81\times 2.5}=-0.6-2\sqrt{9.81\times 2.2} \Rightarrow U_A=0.013 \text{ m/s}$$

Note that U_A and U_0 have opposite signs.

Positive line AA_1 is straight: $\frac{dx}{dt}=U_A+\sqrt{gh_A}$

$$\frac{1300-0}{t_{A1}-0.6\times 3600}=0.013+\sqrt{9.81\times 2.5}$$

$t_b = 2160+261.8 = 2421.8$ s

3

(a.i)

$$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z} \cdot \frac{a}{h-a} \right)^{\frac{w_s}{\kappa u_*}}$$

$$\frac{1}{10} = \left(\frac{4-0.04}{0.04} \times \frac{0.004}{4-0.004}\right)^{\frac{w_s}{\kappa u_*}} = (0.099)^{\frac{w_s}{\kappa u_*}}$$

$$\frac{w_s}{\kappa u_*} = 1.0$$

$$u_* = \sqrt{gR_hS_b} = \sqrt{9.81 \times 4 \times 0.001} = 0.198 \text{ m/s}$$

w_s = 0.0792 m/s

(a.ii) $$w_s = \sqrt{\frac{4g}{3C_D}} \cdot \sqrt{d \cdot \frac{\rho_s - \rho}{\rho}} \approx 2.86 \times \sqrt{d \cdot (2.65 - 1)}$$

d = 4.65 X 10^{-4} m = 0.465 mm

(a.iii) $$q_s = \int_a^h \bar{c}(z)\bar{u}(z)dz = 11.6 \cdot u_* \cdot \bar{c}(a) \cdot a \cdot \left[I_1 \ln\left(\frac{30h}{k_s}\right) + I_2 \right]$$

Set *a* = 0.004 m, and h = 4 m. a/h=0.001.

I_1 = 1.277, I_2 = - 4.944

Sediment transport rate between 0.004 m above bed to water surface is:

$$q_{s1} = 11.6 \cdot u_* \cdot \bar{c}(0.004) \cdot 0.004 \cdot \left[1.277 \times ln\left(\frac{30\times4}{0.004}\right) - 4.944\right] = 0.38u_* \cdot \bar{c}(0.004)$$

Set *a* = 0.04 m, and h = 4 m. a/h=0.01.

I_1 = 0.788, I_2 = - 2.107

Sediment transport rate between 0.04 m above bed to water surface is:

$$q_{s2} = 11.6 \cdot u_* \cdot \bar{c}(0.04) \cdot 0.04 \cdot \left[0.788 \times ln\left(\frac{30\times4}{0.004}\right) - 2.107\right]$$

$$= 2.79u_* \cdot \bar{c}(0.04) = 0.279u_* \cdot \bar{c}(0.004)$$

So, the sediment transport rate between 0.004 m to 0.04 m above bed is:

$$q_{s1} - q_{s2} = (0.38 - 0.28)u_* \cdot \bar{c}(0.004) = 0.1\, u_* \cdot \bar{c}(0.004)$$

Hence, the required ratio is:

0.1/0.28 = 0.36

(b) Change framework of reference. Fix the framework on the boat.

This is a 3D continuous release problem. Considering the water surface as a mirror:

$$\bar{c}(x, y, z) = \frac{2\dot{M}}{4\pi x\sqrt{D_y D_z}} exp\left(-\frac{y^2}{4D_y x/U} - \frac{z^2}{4D_z x/U}\right)$$

$$\bar{c}(x = 100, y = 0, z = 0) = \frac{2\times 10}{4\pi\times 100\times D} = 0.1$$

$D = 0.16\ m^2/s$

(c) $\sqrt{2Dt}$ is the second moment of the concentration distribution; it is a measure of the size of the pollutant cloud or the transported distance due to diffusion.

4.

(a)

The advective transport moves pollutants in the direction of the bulk flow.

The diffusive transport moves pollutants from a high concentration region to a low concentration region.

The Peclet number is defined to be the ratio of the transport rate due to advection to the transport rate due to diffusion.

(b) The molecular diffusion arises from the random motion of molecules, which can be regarded as a property of the fluid.

The turbulent diffusion is caused by the fluctuation of fluid particles or parcels when the Reynolds number is sufficiently large.

Both transport pollutants from a high concentration region to a low concentration region.

The molecular diffusion is on a small scale compared with that of the turbulent diffusion. The turbulent fluctuation of the fluid particles is not actually random, and it depends on the flow situations (domain boundary, Reynolds number …). The turbulent diffusion coefficient may vary with time and space, and some researchers even question whether the turbulent diffusion can be modelled by diffusion coefficients.

(c.i) Static lift = 10 m.

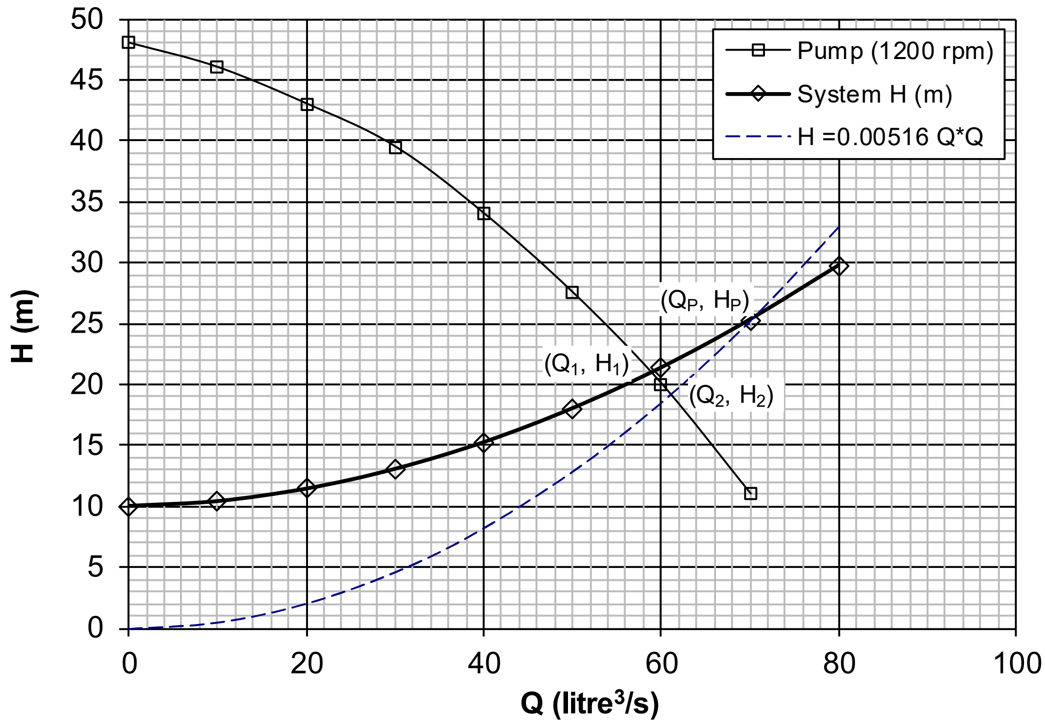
Roughness height: $\frac{k_s}{D} = 0.0005$

The head of the system is: $10 + \lambda \frac{L}{D}\frac{U^2}{2g} = 10 + \lambda \frac{5000}{0.3}\frac{U^2}{2\times 9.81}$

Q (litre/s)	0	10	20	30	40	50	60	70	80
Pump (1200 rpm)	48	46	43	39.5	34	27.5	20	11	
U (m/s)	0	0.14	0.28	0.42	0.57	0.71	0.85	0.99	1.13
Re	0	4.2E+4	8.5E+4	1.3E+5	1.7E+5	2.1E+5	2.5E+5	3.0E+5	3.4E+5
λ	0	0.0235	0.0210	0.0198	0.0192	0.0188	0.0185	0.0183	0.0181
System H (m)	10	10.4	11.4	13.0	15.2	18.0	21.3	25.3	29.7

Plot the pump curve and system curve, which cross at the duty point:

Q_1 = 59 l/s



(c.ii)

Plot the parabola through the origin and (Q_P = 70 l/s, H_P = 25.3 m). It crosses the pump curve (1200 rpm) at:

Q_2 = 61 l/s, H_2 = 19.2 m

According to either $\frac{Q_p}{N_p} = \frac{Q_2}{N_2}$ or $\frac{H_p}{N_p^{\;2}} = \frac{H_2}{N_2^{\;2}}$, where N_2 = 1200 rpm, we get N_p.

$$\frac{70}{N_p} = \frac{61}{1200}$$ => N_p = 1377 rpm

$$\frac{25.3}{N_p^{\;2}} = \frac{19.2}{1200^2}$$ => N_p = 1377.5 rpm