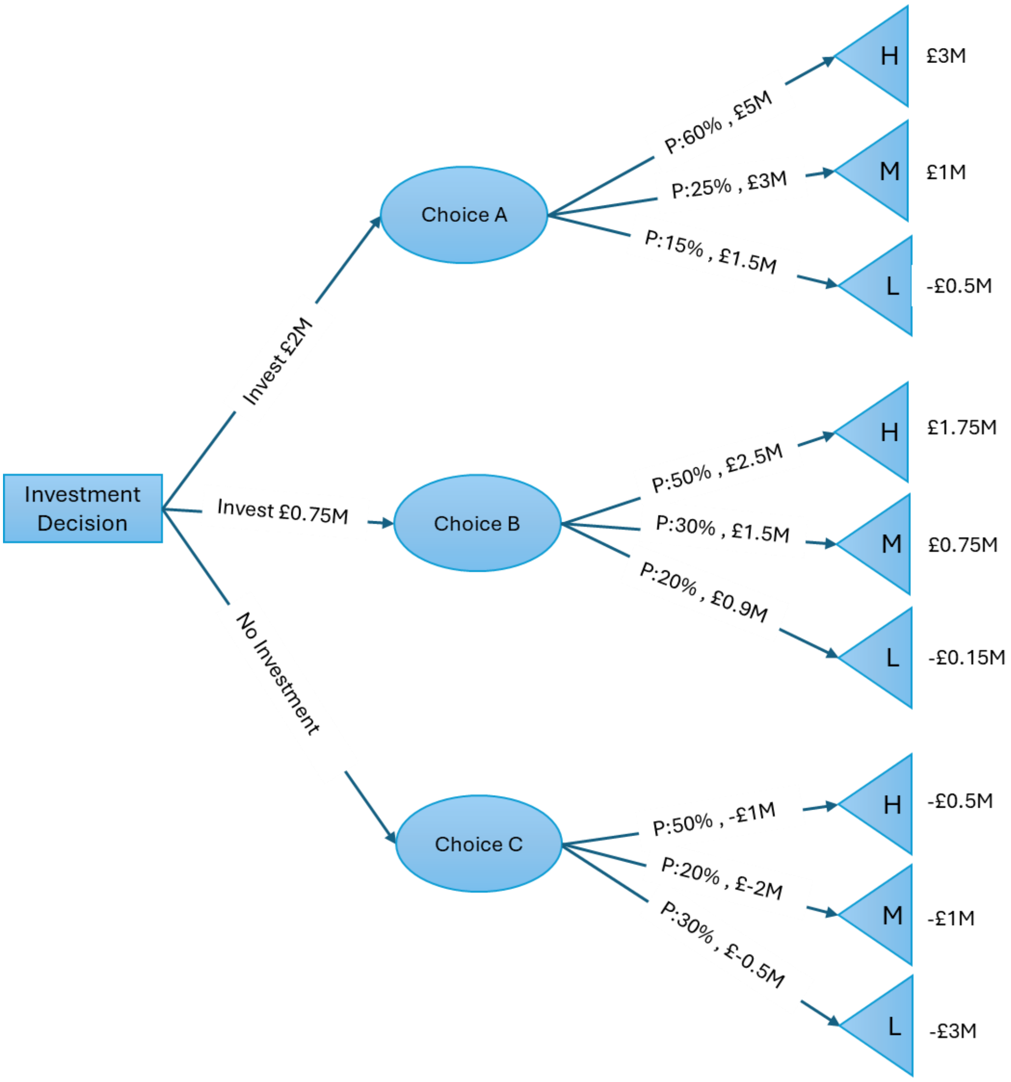
EGT2
ENGINEERING TRIPOS PART IIA

Module 3E3

MODELLING RISK - CRIB

Version EG/3

1 (a) (i)



(ii)

EMV$_{(A)}$ = (0.60 × (£5,000,000-£2,000,000) + (0.25 × (£3,000,000-£2,000,000)) +(0.15 x (£1,500,000-£2,000,000) = £1,975,000

(ii)

EMV$_{(B)}$ = (0.50 × (£2,500,000-£750,000) + (0.30 × (£1,500,000-£750,000) + (0.20 x (£900,00 – 750,000)) = £1,130,000

EMV$_{(C)}$ = 0 (no investment, no gain) but potential losses;

EMV$_{(C)}$=(0.30×−500,000)+(0.50×−1,000,000)+(0.20×−2,000,000) = −£1,050,000

(iii)

Since Option A has the highest expected monetary value, it is the most financially viable option.

Recommendation: Invest in Option A.

(b) When the probability of high returns (p) changes between 50% and 70%, the remaining probability (1−p) is split between partial returns and low returns in their original ratio of 25%:15% (simplified to 5:3)

(i) p = Probability of **High Return** (varies from 0.50 to 0.70)

If p=50%;

- Remaining probability = 1−0.5=0.5 (50%).
- Split into **5:3 ratio**:
  - Partial Return = 5/8×0.5=31.25%
  - Low Return = 3/8×0.5=18.75%

If p=70%;

- Remaining probability = 1−0.7=0.3 (30%).
- Split into **5:3 ratio**:
  - Partial Return = 5/8×0.3=18.75%
  - Low Return = 3/8×0.3=11.25%

EMV_A= (p×High Return)+(Partial Probability×Partial Return)+(Low Probability×Low Return)

$EMV_A=(p\times £3M) + (5/8(1-p)\times £1M)+(3/8\ (1-p)\times(-£0.5M))$

$EMV_A=2.5625p+0.4375$ (in £M);

***p*=50% ➔** EMV_A= (2.5625×0.5) +0.4375=£1.718M

- High Return Contribution: 0.5×£3M=£1.5M
- Partial Return Contribution: 0.3125×£1M=£0.3125M
- Low Return Contribution: 0.1875×(−£0.5M) =−£0.09375M

***p*=70% ➔** EMV= (2.5625×0.7) +0.4375= £2.231M

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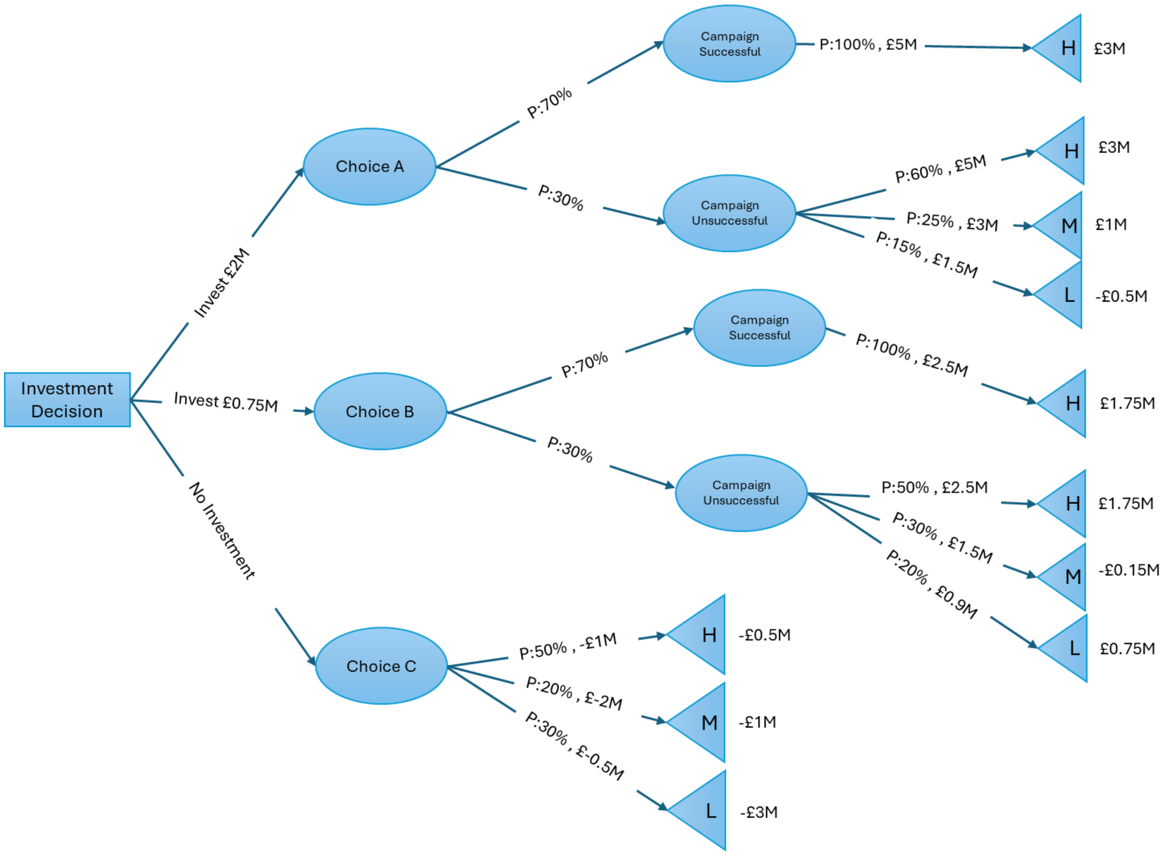
- High Return Contribution: 0.7×£3M= £2.1M
- Partial Return Contribution: 0.1875×£1M=£0.1875M
- Low Return Contribution: 0.1125×(−£0.5M) = −£0.05625M

(c)

(i)



(ii)

EMV(A)=£1,975,000

EMV(B)=£1,130,000

EMV$_{(A)}$ If the campaign succeeds (70%) and delivers High Return;

=5,000,000−2,000,000=£3,000,000

If it fails (30%) and reverts to original 60-25-15 distribution ➔ EMV$_{(A)}$=£1,975,000

EMV$_{(A)Campaign}$= (0.70×3,000,000) + (0.30×1,975,000) =£2,692,500

$EMV_{(B)}$ If the campaign succeeds (70%) and delivers High Return;

$=2,500,000-750,000=£1,750,000$

If it fails (30%) and reverts to original 50-30-20 distribution ➔ $EMV_{(B)}$ =£1,975,000

$EMV_{(B)Campaign}= (0.70\times1,750,000) + (0.30\times1,130,000) =£1,564,000$

(iii) The maximum amount SmartCookie Ltd. should be willing to pay for the marketing campaign is equal to the increase in the highest expected monetary value (Choice A):

Max Campaign Cost;

Increase in $EMV_{A(Campaign)}$ $2,692,500 – X > 1,975,000$

$X=2,692,500-1,975,000=£717,500$

2

(a) Arrivals: Customers arrive at a rate of 20 customers per hour (0.33 customers per minute), with a coefficient of variation of 1.

Service: There is a single fishmonger serving customers, taking an average of 6 minutes per order (service rate of 15 orders per hour).

Queue: Customers must wait in the waiting area until their orders are completed.

Departure: Customers pick up their orders and leave.

(b)

(i) To ensure an average waiting time of no more than 3 minutes, we apply the queuing formula for an M/M/1 system:

Arrival rate (λ) = 0.33 customers per minute

Service rate (μ) = 1/6= 0.167 customers per minute

To ensure that service rate, the number of minimum fishmongers needed we can analyse the queuing system using Little's Law: L= λW, where:

L is the average number of customers in the system (including those being served)

λ is the average arrival rate of customers

W_q Average customer waiting time

s Number of Fishmongers

We similarly define L_q and W_q for the above parameters related to the queue rather than the system.

<u>Case For Fishmonger = 1</u>

μ = 0.167, utilisation rate (ρ) is $\lambda/(\mu$ s) = 0.33/(0.167 *1)= 1.98

Since utilisation exceeds 0.99, a single fishmonger cannot handle the workload, and the system would be unstable.

<u>Case For Fishmonger = 2</u>

ρ = (0.33)/(0.167*2) = 0.99, Looking up in annexed table we have that L_q = 97.51

Then $W_q = L_q / \lambda$

W_q = 97.51/0.33 = 295.5 minutes , does not satisfy <3 minutes condition

Case for Fishmonger =3

ρ = (0.33)/(0.167*3) = 0.66, Looking up in annexed table we have that L_q = 0.8446

then $W_q = L_q / \lambda$; W_q = 0.8446/0.33 = 2.56 minutes < 3, (Lq= 0.9847 ➔ 2.98 minutes)

Therefore, we need at least 3 Fishmongers to keep the waiting time under 3 minutes.

(ii) Regular customers:

Arrival rate: λ=0.33

Service time: 6 minutes, 0.167 customers per minute

Number of fishmongers: 3

Bulk customers

Arrival Rate: λ= 8/60 = 0.133

Service time: 12 minutes, 1/12 = 0.0833 customers per minute

We have two independent queues;

Regular Customers

ρ = (0.33)/3*0.167 = 0.66, Looking up in annexed table we have that L_q = 0.8446

then $W_q = L_q / \lambda$; W_q = 0.8446/0.33 = 2.56 minutes

Bulk customers

ρ = (0.133)/(3*0.0833) = 0.532, Looking up in annexed table we have that L_q = 0.2368

$W_q= L_q/ \lambda$, W_q = 0.2368/0.1333 = 1.78 minutes, (For L_q = 0.3583; W_q= 2.68 minutes or 2.32 with interpolation)

Expected waiting time for bulk customers is 1.78 minutes (2.68 and 2.32 also accepted).

(c) The historical monthly sales data (January to June) for Dream Catcher's store are 200, 250, 230, 270, 260, and 310 units.

(i) To calculate the 3-month simple moving average (M_A), we'll take the average of sales for every consecutive group of 3 months. Then, forecasting the period t will depend on observations in t-1, t-2, and t-3. The solutions can only be computed from April onward.

M_A sales in April = (200 + 250 + 230) / 3 = 680 / 3 = 226.67,

M_A sales in May = (250 + 230 + 270) / 3 = 750 / 3 = 250,

M_A sales in June = (230 + 270 + 260) / 3 = 760 / 3 = 253.33,

M_A sales in July = (270 + 260 + 310) / 3 = 840 / 3 = 280

(ii) The smoothing factor allows us to control the weighting between past and new data in calculations like exponential smoothing. A higher smoothing factor gives more weight to recent data, making the output more responsive to change. A lower smoothing factor gives more weight to past data

$F_t = \alpha Y_{t-1} + (1- \alpha) F_{t-1}$ Considering the forecast for February is observed in January,

F_2=0.3(250) +0.7(200) =75+140=215

F_3=0.3(230) + 0.7(215) = 69+150.5=219.5

F_4=0.3(270) + 0.7(219.5) = 81+153.65=234.65

F_5=0.3(260) + 0.7(234.65) = 78+164.255= 242.26

F_6=0.3(310) + 0.7(242.26) = 93+169.58=262.58

(iii)

Customer arrival rate: 20 customers per hour

Fish demand per customer: 3 kg per customer

Fish replenishment rate: 50 kg per hour

Service time per customer: 4 minutes (i.e., 15 customers per hour)

Total Demand= Customer Arrival Rate * Fish Demand Per Customer,

Then, Total Demand 20*3=60 kg per hour

$\rho = 20 / 1*15 = 1.33 > 1$ ➔This means the queue is unstable: customers arrive faster than they can be served, and the queue will infinitely grow. Since the goal of a business is to increase earnings and maximise revenue. Revenue = Min {Production (or Service in this case), Demand}. Dream Catcher is expected to stabilise its system by making the queue efficient. If we assume that the replenishment rate is constant and cannot be increased (which is true for most eco-system services), for the system to be stable in queueing theory, the replenishment rate must be greater than or equal to the consumption rate. Since: 50<60, The system is unstable and will run out of stock 100% of the time. This is how fisheries collapse and become extinct all around the world.

(d) Dream Catcher's fish inventory fluctuates based on demand and supply, making it possible to model it as a Markov chain.

Defining the States:

Let I_t represent the number of fish in stock at time *t*. The system can be in the following states: **State 0:** No fish left in stock (stockout).

- **State 1:** Few fish in stock.
- **State 2:** Moderate fish stock.
- **State N:** Maximum fish inventory.

Decreasing Stock (Demand): Transition from state I to I−1 happens when a customer buys fish. This occurs with probability $P(I \rightarrow I-1) = \lambda$

Increasing Stock (Replenishment): Transition from state I to I+1 when new fish are added to stock. This occurs with probability $P(I \rightarrow I+1) = \mu$.

3 (a)

Goods In Capacity for Bread Loaves = ((1,500 loaves/day/employee) * (20 Slices/ Loaf) *10 employees) / 2 slices per sandwich = 150,000 sandwiches/day

Goods In Capacity for Ingredients = ((5 tonnes ingredients/day) * (10^6 grams ingredient /tonnes ingredient)) * (6 employees)/ (200 grams ingredients/sandwich) = 150,000 sandwiches / day. Losses in Goods-in; Bread and Ingredient losses do not influence each other, as bread slices and other ingredients are given independently. Joint losses occur after WIP.

For Work-in-Progress, Production Lines and Goods Out, a shift is given as 12 hours.

Work-in-Progress Capacity for Ingredients = ((80 kg ingredients/ hour) * 0.833(12 hours/day)) * (26 employees) / 200 grams of ingredients / sandwich)) = 124,800 sandwiches / day. Only the yield loss beyond the best available technology should be taken into the account here.

Production Lines Capacity = ((45 sandwiches/ hour/line) * (12 hours/day) * 50 Employees * 5 lines) = 135,000 sandwiches / day

Goods Out Capacity = (215 sandwiches/hour/line/employee) * (12 hours/day) * 5 lines*10 employees) = 129,000 sandwiches /day

Each sandwich requires 2 slices of bread and 200 grams of ingredients (1 Bread Loaf= 20 Slices of Bread, Number of Lines=5)						
Process Step	Goods-In (Bread Loaves/shift)	Goods-In (Ingredients in tonnes/shift)	Work-in-Progress (kg Ingredients/hour)	Production (Sandwiches/Hour/Line)	Goods-Out (Sandwiches/ Hour/Employee /Line)	
Capacity per Employee	1,500	5	80	45	215	
Process Time (1 Shift = 12 hours)	1.00	1.00	0.083	0.083	0.083	
Number of Employees	10	6	26	50	10	
Capacity (sandwich/ day)	150,000	150,000	124,800	135,000	129,000	
Mass/Yield Loss	0.10	0.05	0.20	0.15	0.05	
Remaining Yield	0.90	0.95	0.80	0.85	0.95	
Effective Capacity	87,210	92,055	80,621	109,013	122,550	
Bottleneck			X			**Final Output**
Inflow	138,666.67	131,368.42	124,800.00	99,840.00	84,864.00	80620.8

Effective capacity = Capacity x remaining yield of all subsequent steps. (except the first step in which bread and ingredient run in parallel and losses does not affect each other. (**Remaining yield in WIP should be calculated as 20% instead of 30% as the 10% of that loss is given as industry standard)*

Goods In Effective Capacity $_{(Bread)}$ = 150,000 * 0.90*0.80*0.85*0.95 = 87,210

Goods In Effective Capacity $_{(Ingredients)}$ = 150,000 * 0.95*0.80*0.85*0.95 = 92,055

WIP Effective Capacity = 124,800 * 0.80*0.85*0.95 = 80,621

Production Effective Capacity = 135,000*0.85*0.95 = 109,013

Goods-Out Effective Capacity =129,000*0.95 = 122,550

Inflow Rate Goods-In $_{(Bread)}$ = 124,800/0.90 = 138,666.67

Inflow Rate Goods-In $_{(Ingredients}$) =124,800/0.95 = 131,368.42

Inflow Rate WIP = 124,800 (bottleneck capacity)

Inflow Rate Production = 124,800*0.80 = 99,840

Inflow Rate Goods-Out = 99,840*0.85 = 84,864

Final System Output = 84,864 * 0.95 = 80,620.8

(b) Lowest effective capacity is equal to the system bottleneck. In this case, preparation of ingredients within the work-in-progress step is the bottleneck, as it has the lowest effective capacity. Based on that bottleneck, this system's throughput rate is 80,621 Sandwiches /day.

(c)

(i) In RF explained yield losses are a result of system-wide practice related inefficiencies. These inefficiencies cause valuable input materials and production time to be wasted. This also results in reduced market competitiveness as it increases the input costs to produce same level of outputs. Furthermore, increased material dependency also reduces business resilience in the face of disruptions. If a company needs 10% more materials to produce the same quantity due to higher yield losses, any resource shortage on resources will have a greater impact on business.

(ii) Technology A seem to have greater advantage as the yield losses in Goods-in process are higher than goods-out process. Technology A would improve the throughput rate from 80,621 sandwiches/day to 96,900 sandwiches/day while Technology B would increase the capacity from 122,550 sandwiches/day to 129,000 sandwiches/day. However, as Technology B delivers improvements after the bottleneck, it would positively influence the final output, while Technology A would not have a positive impact on the system capacity.

(d)

(i) If there were no yield losses, the system capacity would be 124,800 (lowest capacity rate) sandwiches per day. With yield losses, the effective system capacity is 80,621 sandwiches per day. At £1 per sandwich, yield losses account for £44,179 per day. For 240 days, £10,620,960 per year.

(ii) January 25 = (79,900 sandwiches x 0.7882) -17165 = 45,412 kg Waste

February 25 = (69,741 sandwiches x 0.7882) -17165 = 37,456 kg Waste

March 25 = (95,600 sandwiches x 0.7882) -17165 = 57,708 kg Waste

(iii) While regression can be used for forecasting, it works better within the interpolation range and extrapolation tends to be less reliable. Therefore, February and March estimates fall inside the extrapolation range and are less reliable.

END OF PAPER