Version EG/4

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 8 May 2025 14.00 to 15.40

Module 3E3

MODELLING RISK

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***<u>not</u>*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3E3 Modelling Risk Related Formulas and Data Sheet (5 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 You have been appointed as the chief financial officer of a growing technology firm called SmartCookie Ltd. The company is considering multiple investment options to enhance its AI capabilities in order to remain competitive. However, each option involves financial risk, market uncertainty, and potential operational constraints. Additionally, SmartCookie Ltd. maintains an inventory of computing resources (GPUs, cloud processing credits, etc.) that are used to support existing customers. Investment in AI infrastructure could improve inventory efficiency, but failing to invest might lead to shortages and lost customers. The company's investment options are:

<u>Choice A:</u> Invest in a large-scale AI infrastructure upgrade, which requires an initial investment of £2,000,000. Market research suggests a 60% chance of achieving a high return of £5,000,000 through improved efficiency and market expansion. There is 25% chance that partial adoption with improved inventory turnover may yield £3,000,000. There is a 15% chance of achieving only £1,500,000 in returns due to market uncertainty and adoption challenges.

<u>Choice B:</u> Choose a moderate AI system upgrade with an investment of £750,000, which is a more cost-effective option than Choice A but may yield lower returns. There is a 50% chance of achieving £2,500,000, a 30% chance of moderate success with a return of £1,500,000, and a 20% chance of earning only £900,000.

<u>Choice C:</u> Continue operations without investment in AI. This option has 30% chance of causing £500,000 revenue drop due to market stagnation, 50% chance of causing £1,000,000 revenue drop due to competition, and 20% chance that market competition drives a severe revenue drop of £2,000,000.

(a) (i) Draw a decision tree that outlines all three choices and their corresponding investment costs, expected returns, and probabilities. Label each choice with costs, pay-offs, and probabilities. [20%]

(ii) Calculate the Expected Monetary Values (EMVs) for each investment choice. [10%]

(iii) Based on the EMVs recommend the most financially viable option to the owner of SmartCookie Ltd. [10%]

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(b) (i) Identify how the EMV of Choice A changes if the probability of high returns (currently 60%) varies between 50% and 70%. (Hint: assume that the other two outcomes adjust proportionally and maintain their ratio.) [20%]

After you submit your recommendations, the owner informs you that they have spoken to an AI strategy consultant, who suggests launching a targeted marketing campaign to boost the adoption and success of AI solutions. The consultant estimates a 70% chance that the campaign will ensure high returns for both Choice A and B. (Hint: If the campaign fails, probabilities revert to original conditions.)

(c) Assess the impact of the proposed AI marketing campaign:

(i) Draw a revised decision tree incorporating the additional probability of the marketing campaign's success. [15%]

(ii) Calculate the new expected values for Choices A ($EMV_{(A)Campaign}$) and B ($EMV_{(B)Campaign}$) considering the new probability of the marketing campaign ensuring high returns. [15%]

(iii) Determine the maximum amount SmartCookie Ltd. should be willing to pay for the AI marketing campaign to ensure profitability. [10%]

2 Dream Catcher is a fishing company operating in the Black Sea Region in Turkey. Dream Catcher operates a single fishing fleet in an open-access fishery and directly sells to customers at their local store. Customers arrive at the store, use online dashboards to place their orders, take a seat at the waiting area while their fresh orders are prepared, and pick up their orders when their number is called.

It is known that customers arrive with an average of 20 customers per hour (0.33 per minute). The coefficient of variation of arrival times is 1, following a Poisson arrival process. There is 1 fishmonger preparing orders, taking an average of 6 minutes per order and service times follow an exponential distribution. Customers cannot leave the store until their orders are ready.

(a) What is the queuing system description of Dream Catcher? Explain it in terms of arrivals, departures, and queues, considering the fish stock renewal rate as a key parameter. [10%]

(b) (i) How many fishmongers should Dream Catcher schedule to ensure that the average waiting time is no more than 3 minutes? [15%]

(ii) Dream Catcher management decides to add a separate counter with a separate queue for bulk orders (e.g., restaurants), employing 3 experienced fishmongers. Bulk order customers arrive with an average of 8 customers per hour, but their orders take 12 minutes to prepare on average. The coefficient of variation of arrival times of bulk order customers is also 1. Regular customers still experience the same preparation time as before. What is the expected waiting time in queue for regular and bulk order customers under the new system? (Hint: use the minimum number of fishmongers you identified in (i) for the regular customers) [15%]

(c) The historical monthly sales data (January to June) for Dream Catcher's store are 200, 250, 230, 270, 260, and 310 units.

(i) Using the 3-month simple moving average, forecast the sales for July. Show your calculations. [15%]

(ii) Using exponential smoothing, forecast the sales for July. Use a smoothing factor of $\alpha = 0.3$. Show your calculations. Interpret the smoothing factor. [15%]

(iii) Dream Catcher receives fresh fish deliveries at a rate of 50 kg per hour. Customers arrive at the store at an average rate of 20 per hour, each ordering

(cont.

an average of 3 kg of fish. The fish replenishment follows a Poisson process, and the service time of a single fishmonger follows an exponential distribution with a mean of 4 minutes per order. Assuming the system follows an M/M/1 queue, what is the probability that the store will run out of stock? [15%]

(d) Besides customer flow, Dream Catcher also needs to manage its fish inventory. Assume that the fish stock is replenished periodically, and the number of fish available fluctuates based on customer demand. How can you model this inventory system using a Markov chain. [15%]

3 Raymour Foods (RF), a successful sandwich manufacturer based in the East of England, is currently experiencing rapid business growth. This growth is driven by a recent hygiene-related incident involving one of its competitors. As a result, RF has begun exploring ways to enhance its production capacity to meet the increasing market demand. The Continuous Improvement team at RF has analysed their process and reached out to you for recommendations. To simplify the process, RF has focused on sandwich bread and other ingredients as two critical resource groups. Each sandwich requires 2 slices of bread and 200 grams of ingredients to pass the quality control tests. RF currently runs on a single shift which takes 12 hours. (For RF, 1 day = 1 shift = 12 hours). Their basic process flow includes:

Goods-In: Employees in this section are responsible for the arrival and storage of bread loaves. RF receives 1,500 loaves of bread per employee per shift and 5 tonnes of ingredients per employee per shift. In goods-in area, 10 people work with bread loaves and 6 employees manage other ingredients. Every loaf has 20 slices of bread. Bread loaves arrive from a single supplier and a review of the past deliveries illustrated that 10% of the bread loaves received are crushed during transport and are unsuitable for sandwich making. Furthermore, due to challenges with correct-stock-rotation 5% of other ingredients are discarded without being used (excluding bread loss). Bread and ingredient operations run in parallel and do not affect each other in this step.

Work-in-Progress (WIP): 26 Employees are working for the preparation of ingredients in WIP. These ingredients undergo a series of preparation steps such as chopping, slicing, mixing, and whipping. Prepared ingredients are placed on to the trolleys and sent to production lines to be used in sandwiches. Each WIP employee can prepare 80 kilograms of ingredients per hour. During the preparation process, other ingredients are subject to 25% manufacturing loss (the industry standard is 10%, which is unavoidable with current available technologies) and another 5% of the ingredients are wasted due to production planning and miscommunication problems.

Production Lines: RF has 5 production lines, and each line works for 12 hours every day. All lines have the same capacity and can produce up to 2,500 sandwiches per hour. During the production bread slices fall onto the floor and some ingredients are overused. RF estimated these losses to be around 15% of the produced sandwiches from each line.

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Goods-Out: Once the sandwiches are packed into the boxes, they reach the Goods-Out section through the conveyor belt. In this section, employees prepare customer orders based on order sheets. Goods-Out can pack and ship 215 sandwiches per hour per employee at each production line. Records show that 5% of the time customers are sent the wrong sandwiches which are later discarded as waste.

Table 1 – Summary of Operations at Raymour Foods

Each sandwich requires 2 slices of bread and 200 grams of ingredients (1 Bread Loaf = 20 Slices of Bread, Number of Lines = 5)					
Process Step ➔	**Goods-In** (Bread Loaves/shift)	**Goods-In** (tonnes Ingredients/shift)	**Work-in-Progress** (kg Ingredients/hour)	**Production Lines** (Sandwiches/ Hour/Line)	**Goods-Out** (Sandwiches/ Hour/Line)
Capacity per Employee	1,500	5	80	45	215
Process Time (1 Shift = 12 hours)	1.00	1.00	0.083	0.083	0.083
Number of Employees	10	6	26	50	10
Capacity (Sandwiches/ day)					
Mass/Yield Loss					
Remaining Yield					
Effective Capacity					
Bottleneck					
Inflow					

(a) Calculate the effective capacity of the production system at RF. (You can use the table above for your calculations.) [20%]

(b) Which step is the system bottleneck? Why? What is the throughput of the above process in the number of sandwiches per day? [10%]

(c) (i) Explain the business risks associated with yield losses. [15%]

(ii) RF is considering investing in new technologies to increase its capacity. Technology A eliminates the losses in the Goods-In process, while Technology B improves picking accuracy and eliminates Goods-Out losses. Evaluate the potential of these technologies and explain how they would affect the overall system capacity. [20%]

(cont.

(d) (i) If the cost of each sandwich is calculated as £1, what is the annual cost (1 year =240 working days) of yield losses at RF? [15%]

(ii) RF is expecting to produce Jan-25: 79,900; Feb-25: 69,741, and Mar-25: 95,600 packs of sandwiches as shown in Figure 1. Estimate the total waste amount for these months using Figure 2. [10%]

(iii) Using Figure 2, explain the limitations of using regression in forecasting. Which of the forecasts for Jan-25, Feb-25 and Mar-25 has a greater chance of being <u>inaccurate</u> and why? [10%]

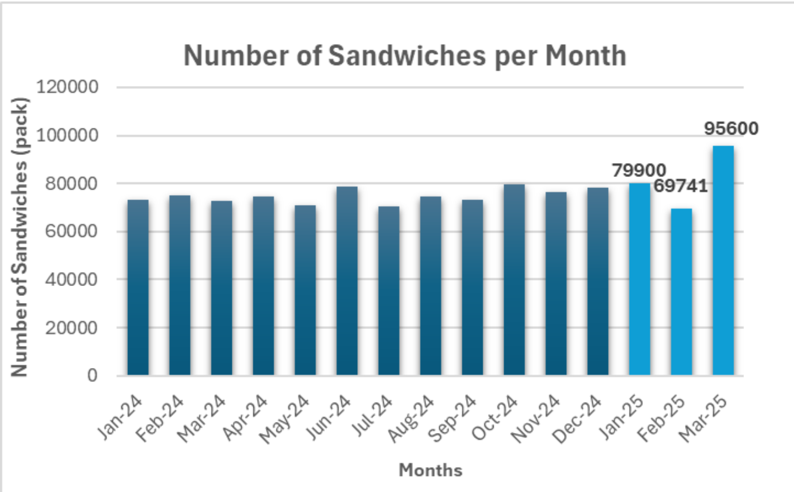

Figure 1 –Total Number of Sandwiches Produced and Planned

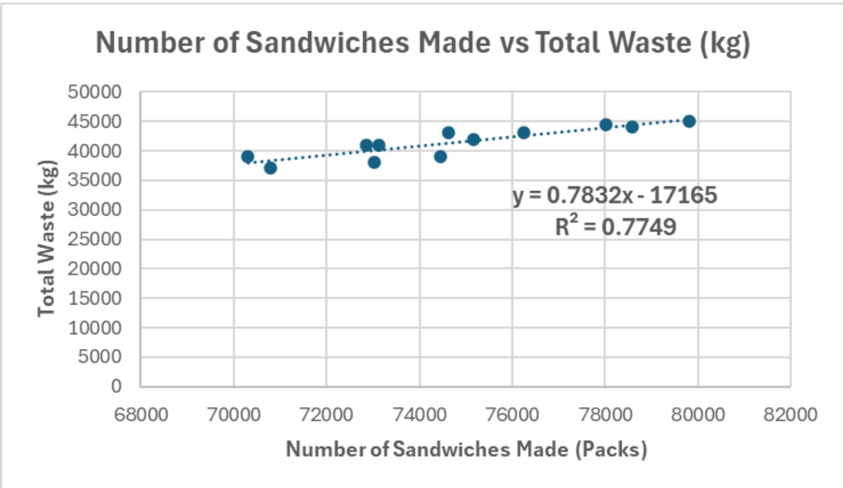

Figure 2 –Number of Sandwiches Made vs Total Waste Produced

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SPECIAL DATA SHEET

Queuing Theory Formulas

Little's Law

$$L_q = \lambda W_q$$

Utilisation Factor

$$\rho = \frac{\lambda}{\mu}, \quad (\text{where } \rho < 1 \text{ for stability})$$

Expected Number of Customers in the Queue

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Probability of Having *n* Customers in the System

$$P_n = (1 - \rho)\rho^n, \quad n \geq 0$$

Expected Waiting Time in the Queue

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

Expected Time a Customer Spends in the System

$$W = \frac{1}{\mu - \lambda}$$

Expected Number of Customers in the System

$$L = \frac{\lambda}{\mu - \lambda}$$

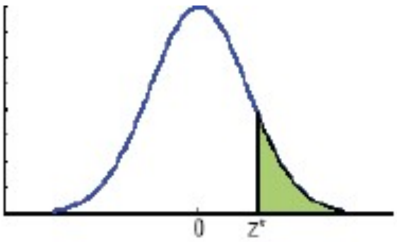
Exponential Smoothing Formulas

$$F_t = \alpha\, Y_{t-1} + (1 - \alpha)\, F_{t-1}$$

Decision Trees Formulas

$$EMV = \sum_{i=1}^{n} P_i \times V_i$$

Standard Normal distribution table: *Areas under the standard normal curve beyond z*, i.e., shaded area.*



z*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

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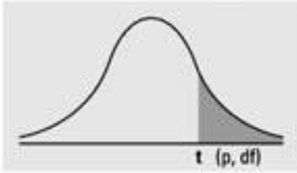
t-distribution table:

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



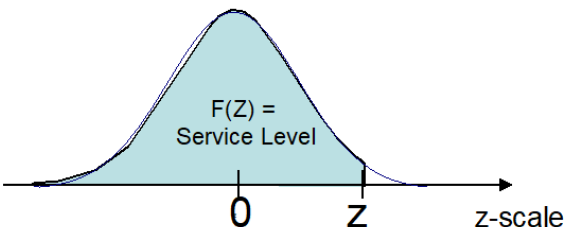
df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

Z-Chart & Loss Function

F(Z) is the probability that a variable from a standard normal distribution will be less than or equal to Z, or alternately, the service level for a quantity ordered with a z-value of Z.

L(Z) is the standard loss function, i.e. the expected number of lost sales as a fraction of the standard deviation. Hence, the lost sales = L(Z) x σ_{DEMAND}

Z	F(Z)	L(Z)	Z	F(Z)	L(Z)	Z	F(Z)	L(Z)	Z	F(Z)	L(Z)
-3.00	0.0013	3.000	-1.48	0.0694	1.511	0.04	0.5160	0.379	1.56	0.9406	0.026
-2.96	0.0015	2.960	-1.44	0.0749	1.474	0.08	0.5319	0.360	1.60	0.9452	0.023
-2.92	0.0018	2.921	-1.40	0.0808	1.437	0.12	0.5478	0.342	1.64	0.9495	0.021
-2.88	0.0020	2.881	-1.36	0.0869	1.400	0.16	0.5636	0.324	1.68	0.9535	0.019
-2.84	0.0023	2.841	-1.32	0.0934	1.364	0.20	0.5793	0.307	1.72	0.9573	0.017
-2.80	0.0026	2.801	-1.28	0.1003	1.327	0.24	0.5948	0.290	1.76	0.9608	0.016
-2.76	0.0029	2.761	-1.24	0.1075	1.292	0.28	0.6103	0.274	1.80	0.9641	0.014
-2.72	0.0033	2.721	-1.20	0.1151	1.256	0.32	0.6255	0.259	1.84	0.9671	0.013
-2.68	0.0037	2.681	-1.16	0.1230	1.221	0.36	0.6406	0.245	1.88	0.9699	0.012
-2.64	0.0041	2.641	-1.12	0.1314	1.186	0.40	0.6554	0.230	1.92	0.9726	0.010
-2.60	0.0047	2.601	-1.08	0.1401	1.151	0.44	0.6700	0.217	1.96	0.9750	0.009
-2.56	0.0052	2.562	-1.04	0.1492	1.117	0.48	0.6844	0.204	2.00	0.9772	0.008
-2.52	0.0059	2.522	-1.00	0.1587	1.083	0.52	0.6985	0.192	2.04	0.9793	0.008
-2.48	0.0066	2.482	-0.96	0.1685	1.050	0.56	0.7123	0.180	2.08	0.9812	0.007
-2.44	0.0073	2.442	-0.92	0.1788	1.017	0.60	0.7257	0.169	2.12	0.9830	0.006
-2.40	0.0082	2.403	-0.88	0.1894	0.984	0.64	0.7389	0.158	2.16	0.9846	0.005
-2.36	0.0091	2.363	-0.84	0.2005	0.952	0.68	0.7517	0.148	2.20	0.9861	0.005
-2.32	0.0102	2.323	-0.80	0.2119	0.920	0.72	0.7642	0.138	2.24	0.9875	0.004
-2.28	0.0113	2.284	-0.76	0.2236	0.889	0.76	0.7764	0.129	2.28	0.9887	0.004
-2.24	0.0125	2.244	-0.72	0.2358	0.858	0.80	0.7881	0.120	2.32	0.9898	0.003
-2.20	0.0139	2.205	-0.68	0.2483	0.828	0.84	0.7995	0.112	2.36	0.9909	0.003
-2.16	0.0154	2.165	-0.64	0.2611	0.798	0.88	0.8106	0.104	2.40	0.9918	0.003
-2.12	0.0170	2.126	-0.60	0.2743	0.769	0.92	0.8212	0.097	2.44	0.9927	0.002
-2.08	0.0188	2.087	-0.56	0.2877	0.740	0.96	0.8315	0.090	2.48	0.9934	0.002
-2.04	0.0207	2.048	-0.52	0.3015	0.712	1.00	0.8413	0.083	2.52	0.9941	0.002
-2.00	0.0228	2.008	-0.48	0.3156	0.684	1.04	0.8508	0.077	2.56	0.9948	0.002
-1.96	0.0250	1.969	-0.44	0.3300	0.657	1.08	0.8599	0.071	2.60	0.9953	0.001
-1.92	0.0274	1.930	-0.40	0.3446	0.630	1.12	0.8686	0.066	2.64	0.9959	0.001
-1.88	0.0301	1.892	-0.36	0.3594	0.605	1.16	0.8770	0.061	2.68	0.9963	0.001
-1.84	0.0329	1.853	-0.32	0.3745	0.579	1.20	0.8849	0.056	2.72	0.9967	0.001
-1.80	0.0359	1.814	-0.28	0.3897	0.554	1.24	0.8925	0.052	2.76	0.9971	0.001
-1.76	0.0392	1.776	-0.24	0.4052	0.530	1.28	0.8997	0.047	2.80	0.9974	0.001
-1.72	0.0427	1.737	-0.20	0.4207	0.507	1.32	0.9066	0.044	2.84	0.9977	0.001
-1.68	0.0465	1.699	-0.16	0.4364	0.484	1.36	0.9131	0.040	2.88	0.9980	0.001
-1.64	0.0505	1.661	-0.12	0.4522	0.462	1.40	0.9192	0.037	2.92	0.9982	0.001
-1.60	0.0548	1.623	-0.08	0.4681	0.440	1.44	0.9251	0.034	2.96	0.9985	0.000
-1.56	0.0594	1.586	-0.04	0.4840	0.419	1.48	0.9306	0.031	3.00	0.9987	0.000
-1.52	0.0643	1.548	0.00	0.5000	0.399	1.52	0.9357	0.028			



Z & *L(z)* for special service levels

Service Level F(z)	*z*	*L(z)*
75%	0.67	0.150
90%	1.28	0.047
95%	1.64	0.021
99%	2.33	0.003

Inventory management:

(Q,R) model - optimal solution:

$$Q = \sqrt{\frac{2\lambda[K+pn(R)]}{h}}; \qquad F(R) = 1 - \frac{Qh}{p\lambda}$$

Newsvendor model:

$$F(Q^*) = \frac{c_u}{c_u + c_o}$$

Queuing theory:

Values of L_q for s servers, with mean utilisation rate ρ, assuming Poisson arrivals and exponential service times (known as a *M/M/s* queue).

Utilisation rate (ρ)*	*Number of servers (s)* 1	2	3	4	5
.10	.0111	.0020	.0004	.0001	.0000
.20	.0500	.0167	.0062	.0024	.0010
.30	.1286	.0593	.0300	.0159	.0086
.35	.1885	.0977	.0552	.0325	.0196
.40	.2667	.1524	.0941	.0605	.0398
.45	.3682	.2285	.1522	.1052	.0743
.50	.5000	.3333	.2368	.1739	.1304
.55	.6722	.4771	.3583	.2772	.2185
.60	.9000	.6750	.5321	.4306	.3542
.62	1.0116	.7743	.6213	.5109	.4269
.64	1.1378	.8880	.7246	.6051	.5130
.66	1.2812	1.0188	.8446	.7158	.6152
.68	1.4450	1.1698	.9847	.8461	.7367
.70	1.6333	1.3451	1.1488	1.0002	.8816
.72	1.8514	1.5500	1.3423	1.1834	1.0553
.74	2.1062	1.7914	1.5721	1.4025	1.2646
.76	2.4067	2.0785	1.8472	1.6668	1.5187
.78	2.7655	2.4237	2.1803	1.9887	1.8302
.80	3.2000	2.8444	2.5888	2.3857	2.2165
.82	3.7356	3.3661	3.0979	2.8832	2.7029
.84	4.4100	4.0265	3.7456	3.5190	3.3273
.86	5.2829	4.8852	4.5914	4.3526	4.1493
.88	6.4533	6.0414	5.7345	5.3834	5.2682
.90	8.1000	7.6737	7.3535	7.0898	6.8624
.92	10.5800	10.1392	9.8056	9.5290	9.2893
.94	14.7267	14.2712	13.9240	13.6344	13.3821
.96	23.0400	22.5698	22.2088	21.9060	21.6408
.98	48.0200	47.5350	47.1602	46.8439	46.5656
.99	98.0101	97.5176	97.1357	96.8127	96.4274

Multi-server waiting in queue

$$W_q = \frac{1}{\mu s} \frac{\rho^{\sqrt{2(s+1)}-1}}{1-\rho} \frac{CV_a^2 + CV_s^2}{2}$$