Version EG/1

EGT2
ENGINEERING TRIPOS PART IIA

Module 3E3

MODELLING RISK - CRIB

1 (a)

(i) Monthly Demand (D_m) = 7,500 boxes.

Annual Demand (D) = 7,500 x 12 = 90,000 boxes.

Ordering Cost (S) = £45 per order.

Holding Cost (H) = £0.05 per box per month = £0.60 per box per year.

$$EOQ = \sqrt{\frac{2DS}{H}}$$

Using Annual Demand (D) = 90,000, Order Cost (S) = £45, Holding Cost (H) = £0.60/year.
EOQ = sqrt((2 * 90,000 * 45) / 0.60) = 3,674 boxes

(ii) Orders/month = 7,500 / 3,674 ≈ 2.04 orders. Average Cycle Inventory = Q/2 = 1,837 boxes

(iii) Reorder Point (ROP)

Daily demand = 7,500 / 30 = 250 boxes/day

Lead time = 6 days

ROP = 250 × 6 = 1,500 boxes → Order when inventory reaches 1,500 boxes.

(b) (i) Calculate the optimal order quantity using the Critical Fractile (F). This is a Newsvendor Problem.

Data:

- Selling Price (p) = £5.50
- Cost (c) = £3.20
- Disposal Cost (g) = £0.80
- Mean (μ) = 7,500; Std Dev (σ) = 500.

Underage Cost (C_u): Profit lost per lost sale = p - c = £5.50 - £3.20 = £2.30.

Overage Cost (C_o): Cost incurred per unsold unit = Cost to make + Disposal cost = £3.20 + £0.80 = £4.00.

Version EG/1

Critical fractile:

$$F^* = \alpha = Pr[D \le Q] = \frac{c_u}{c_o + c_u} = \frac{£2.30}{£4 + £2.30} = 0.365$$

From normal tables, Z ≈ −0.35, F < 0.5, then Z is **negative**

Mean demand μ = 7,500
Std deviation σ = 500

Optimal quantity:

Q* = μ + Zσ = 7,500 − 175 ≈ **7,325 boxes**

Since Q < μ, we stock less than mean.

(ii) Expected Profit = (p-c) x (Expected Sales) – (c-s) x (Expected Leftover Inventory)

From the table $L(z_Q) = 0.598$

We can calculate the expected lost sales (ELS) as:

$$ELS = \sigma L(z_Q) = (500)(0.598) = 299$$

We can calculate expected sales (ES) as:

ES= $\mu - ELS = 7{,}500 - 299 = 7{,}201$

We can calculate the expected leftover inventory (ELOI) as:

$$ELOI = Q - ES = 7{,}325 - 7{,}201 = 124$$

Expected Profit = (p-c)(ES) – (c-s)(ELOI)

Expected Profit = (£5.50-£3.20)x(7,201) – (3.20+0.8)x (124) = 16,562– 496 = **£16,066**

(iii) Yield losses and supply–demand mismatch create operational, financial, and strategic risks for the business. From an operational perspective, high demand variability can result in either stockouts, leading to lost sales, or excess production that must be discarded, increasing waste and inefficiency. Financially, the asymmetry between underage and overage costs means that over-ordering is more severely penalised than under-ordering, which can significantly reduce profit margins. Strategically, persistent stockouts arising from conservative ordering decisions (where the optimal quantity is below mean demand) may undermine customer satisfaction and damage the firm's reputation for reliability, increasing the risk of long-term customer loss.

(c)

Each sushi box requires 800 grams of ingredients						
Process Step	**Preparation** (kg ingredient per hour)	**Chopping** (kg ingredient per hour)	**Wrapping** (kg ingredient per hour)	**Packaging** (kg ingredient per hour)	**Labelling** (kg ingredient per hour)	
Capacity per Employee	9	8	8	6	10	
Number of Employees	4	4	4	6	3	
Total Hours Worked	10	10	10	10	10	
Capacity (sushi boxes / day)	450	400	400	450	375	
Mass/Yield Loss	0.10	0.05	0.20	0.15	0.05	
Remaining Yield	0.90	0.95	0.80	0.85	0.95	
Effective Capacity	249	245	258	363	356	
Bottleneck		X				**Final Output**
Inflow	444.44	400.00	380.00	304.00	258.40	245.48

(i) Effective capacity is the maximum output a process step can achieve after accounting for yield losses and inefficiencies, expressed in terms of usable final output rather than nominal processing capability

Capacity Calculations

Preparation Capacity = (9 kg ingredient / hour) x (4 employees) x (10 hours) = 360 kg ingredients. Each box requires 800 grams → (360kg x 1000 grams/kg) /800 grams = 450 sushi boxes a day.

Chopping capacity = (8 kg ingredients/hour) × (4 employees) × (10 hours) = 320 kg of ingredients per day. Each sushi box requires 800 grams of ingredients, therefore (320 kg × 1,000 grams/kg) / 800 grams = 400 sushi boxes per day.

Wrapping capacity = (8 kg ingredients/hour) × (4 employees) × (10 hours) = 320 kg of ingredients per day. Each sushi box requires 800 grams of ingredients, therefore (320 kg × 1,000 grams/kg) / 800 grams = 400 sushi boxes per day.

Packaging capacity = (6 kg ingredients/hour) × (6 employees) × (10 hours) = 360 kg of ingredients per day. Each sushi box requires 800 grams of ingredients, therefore (360 kg × 1,000 grams/kg) / 800 grams = 450 sushi boxes per day.

Labelling capacity = (10 kg ingredients/hour) × (3 employees) × (10 hours) = 300 kg of ingredients per day. Each sushi box requires 800 grams of ingredients, therefore (300 kg × 1,000 grams/kg) / 800 grams = 375 sushi boxes per day.

<u>Effective Capacity Calculations</u>

Preparation effective capacity = 450 × 0.90 × 0.95 × 0.80 × 0.85 × 0.95 ≈ 249 sushi boxes per day.

Chopping effective capacity = 400 × 0.95 × 0.80 × 0.85 × 0.95 ≈ 245 sushi boxes per day.

Wrapping effective capacity = 400 × 0.80 × 0.85 × 0.95 ≈ 258 sushi boxes per day.

Packaging effective capacity = 450 × 0.85 × 0.95 ≈ 363 sushi boxes per day.

Labelling effective capacity = 375 × 0.95 ≈ 356 sushi boxes per day.

The busiest step with highest utilisation is the bottleneck or the step that limits the capacity of the whole step is the bottleneck

ii) Chopping is the bottleneck with 245 boxes per day as it is the step with the lowest effective capacity.

Throughput of the system is equal to the bottleneck capacity which is 245 boxed per day.

(d) (i)

Oct (8,000 boxes): \$1301.7(8000) - 5M = 10,413,600 - 5,000,000 = £5,413,600.

Nov (8,200 boxes): \$1301.7(8200) - 5M = 10,673,940 - 5,000,000 = £5,673,940.

Dec (8,400 boxes): \$1301.7(8400) - 5M = 10,934,280 - 5,000,000 = £5,934,280.

Given the number of sushi boxes for October:8000 boxes, November:8200 boxes, December:8400 boxes, are outside of the extrapolation range and likely to be inaccurate.

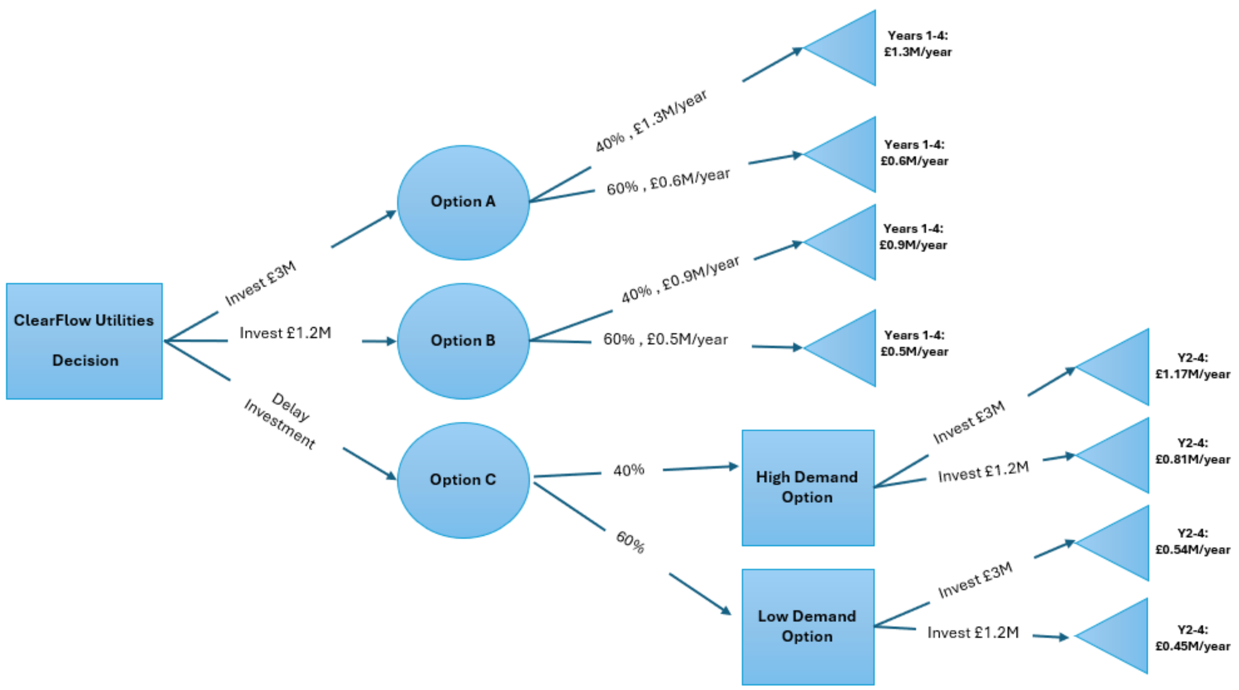
ii) Limitations**:** The linear model has a large negative intercept (-£5M), suggesting huge fixed costs or poor model fit at low volumes. Extrapolating to higher volumes (Oct-Dec) assumes the linear relationship (constant price/mix) holds, which may not be case. So, the reliability of the forecast is lower within the extrapolation range.

iii) Expectations**:** The R-squared (0.96) is high, indicating good fit within the data range. However, the forecasted volume (8,400) exceeds the operational capacity calculated in part (c) (~184/day * 30 = 5,520/month). Sansberry cannot achieve these revenue targets because it is beyond their current daily capacity. Moreover, because the forecasted values extend beyond the interpolation range they are less likely to be accurate. Sansberry can consider increasing capacity at the Chopping station (the bottleneck). Add 1 employee or extend shift hours. Investigating yield improvements at Wrapping (20% loss is high) to reduce upstream waste and load on Chopping could also help improve throughput.

Version EG/1

2

a) decision tree



b) (i)

We use the Net Present Value (NPV) approach combined with Expected Value (EV) under demand uncertainty. All cash flows are discounted at 8% per year.

Step 1: Discount factors at 8%

The value at Year 0 of £1 received at the end of each future year is:

Year 1: 1 / (1.08) ^1 = 0.92593
Year 2: 1 / (1.08) ^2 = 0.85734
Year 3: 1 / (1.08) ^3 = 0.79383
Year 4: 1 / (1.08) ^4 = 0.73503

From these we derive two useful annuity (cumulative discount factor) factors:

4-year annuity factor (Years 1–4) — used for Options A and B:

NPV (8%, 4) = 0.92593 + 0.85734 + 0.79383 + 0.73503 = 3.31213

Years 2–4 deferred factor — used for Option C (no benefits in Year 1):

DF (Years 2–4) = 0.85734 + 0.79383 + 0.73503 = 2.38620

Note: For Option C, benefits start at the end of Year 2 (not Year 1), so we sum the discount factors for Years 2, 3 and 4 directly. This brings those cash flows to Year 0 in one step. You can choose to discount later.

Option A: expand reservoir (invest at Year 0)

Invest £3.0m at Year 0; benefits in Years 1–4.

High demand (p = 0.4): benefits = £1.3m per year

NPV = $-3.0 + 1.3 \times 3.31213$

$= -3.0 + 4.30577$

$=$ £1.30577 m

Low demand (p = 0.6): benefits = £0.6m per year

NPV = $-3.0 + 0.6 \times 3.31213$

$= -3.0 + 1.98728$

$= -$£1.01272 m

Expected NPV of Option A

ENPV(A) = $0.4 \times 1.30577 + 0.6 \times (-1.01272)$

$= 0.52231 - 0.60763$

$= -$£0.08533 m

Option B: demand management (invest at Year 0)

Invest £1.2m at Year 0; benefits in Years 1–4.

High demand (p = 0.4): benefits = £0.9m per year

NPV = $-1.2 + 0.9 \times 3.31213$

$= -1.2 + 2.98091$

$=$ £1.78091 m

Low demand (p = 0.6): benefits = £0.5m per year

NPV = $-1.2 + 0.5 \times 3.31213$

$= -1.2 + 1.65606$

$= £0.45606$ m

Expected NPV of Option B

ENPV(B) = 0.4 × 1.78091 + 0.6 × 0.45606

= 0.71237 + 0.27364

= £0.98600 m

ENPV(B) ≈ £986,004

<u>Option C: delay 1 year, then choose the higher NPV at end of Year 1 (given the realised state)</u>

Key implications of delaying:

• The investment cost is incurred at the end of Year 1, so it is discounted once to Year 0 (factor 0.92593).

• No benefits are received in Year 1.

• Benefits are received in Years 2, 3 and 4 only, and each is reduced by 10% (multiplied by 0.9).

We discount each Option C cash flow directly to Year 0:

NPV at Year 0 = −(Investment) × 0.92593 + (0.9 × Annual benefit) × 2.38620

If High demand is revealed ($p = 0.4$)

Choose A (benefit £1.3m → reduced to £1.17m):

NPV = −3.0 × 0.92593 + 1.17 × 2.38620

= −2.77778 + 2.79185

= £0.01407 m

Choose B (benefit £0.9m → reduced to £0.81m):

NPV = −1.2 × 0.92593 + 0.81 × 2.38620

= −1.11111 + 1.93282

$= £0.82171$ m

→ Under High demand, choose Option B (£0.82171 m > £0.01407 m).

If Low demand is revealed (p = 0.6)

Choose A (benefit £0.6m → reduced to £0.54m):

$\text{NPV} = -3.0 \times 0.92593 + 0.54 \times 2.38620$

$= -2.77778 + 1.28855$

$= -£1.48923$ m

Choose B (benefit £0.5m → reduced to £0.45m):

$\text{NPV} = -1.2 \times 0.92593 + 0.45 \times 2.38620$

$= -1.11111 + 1.07379$

$= -£0.03732$ m

→Under Low demand, choose Option B (−£0.03732 m > −£1.48923 m).

Under both demand states, Option C leads to choosing Option B at the end of Year 1.

Expected NPV of Option C

$\text{ENPV(C)} = 0.4 \times 0.82171 + 0.6 \times (-0.03732)$

$= 0.32868 - 0.02239$

$= £0.30629$ m

ENPV(C) ≈ £306,292

Summary of expected NPVs

ENPV(A) ≈ −£85,328

ENPV(B) ≈ £986,004

ENPV(C) ≈ £306,292

ii) Justification: Option B has the highest expected net present value, about £0.986m, compared with about £0.306m for Option C and about −£0.085m for Option A. This indicates that Option B provides the best value for money over the 4-year horizon after accounting for uncertainty in demand growth and the time value of money.

Option C is not preferred even though it is positive, because delaying sacrifices all Year 1 benefits and reduces Years 2–4 benefits by 10%. The "value of waiting" is also limited here because, once demand is observed at the end of Year 1, Option B remains the better choice under both high and low demand states, meaning the extra information does not change the decision and therefore does not compensate for the delay penalty.

Option A is not recommended on expected value grounds because its expected NPV is slightly negative, largely due to the high upfront cost and the risk that demand growth is low (0.6 probability), where the benefits are insufficient to recover the investment within the planning horizon.

c) (i)

Expected value of information from the forecast study (EVSI)

Step 1: Prior probabilities

P(High) = 0.4

P(Low) = 0.6

Step 2: Study accuracy (given)

P(Predict High | High) = 0.8

P(Predict Low | Low) = 0.7

So, P(Predict High | Low) = 0.3 and P(Predict Low | High) = 0.2

Step 3: Probability of each study prediction

P(Predict High) = 0.4×0.8 + 0.6×0.3 = 0.32 + 0.18 = 0.50

P(Predict Low) = 0.4×0.2 + 0.6×0.7 = 0.08 + 0.42 = 0.50

Step 4: Posterior probabilities (Bayes' rule)

P(High | Predict High) = $(0.4\times0.8) / 0.5 = 0.64$

P(High | Predict Low) = $(0.4\times0.2) / 0.5 = 0.16$

Step 5: Use the NPVs from part (b) (in £m)

Option A: NPV if High = 1.305764892, NPV if Low = −1.012723896

Option B: NPV if High = 1.780914156, NPV if Low = 0.456063420

Option C: NPV if High = 0.821711631, NPV if Low = −0.037320699

Conditional expected NPVs given the study result

If the study predicts High (P(High)=0.64, P(Low)=0.36)

ENPV(A | Predict High) = $0.64\times1.305764892 + 0.36\times(-1.012723896) = 0.471108928$ (£m)

ENPV(B | Predict High) = $0.64\times1.780914156 + 0.36\times0.456063420 = 1.303967891$ (£m)

ENPV(C | Predict High) = $0.64\times0.821711631 + 0.36\times(-0.037320699) = 0.512459993$ (£m)

Best decision: choose Option B

If the study predicts Low (P(High)=0.16, P(Low)=0.84)

ENPV(A | Predict Low) = $0.16\times1.305764892 + 0.84\times(-1.012723896) =$ **−0.641765690** (£m)

ENPV(B | Predict Low) = $0.16\times1.780914156 + 0.84\times0.456063420 =$ **0.668039538 (£m)**

ENPV(C | Predict Low) = $0.16\times0.821711631 + 0.84\times(-0.037320699) =$ **0.100124474 (£m)**

Best decision: choose Option B

Step 6: Expected value with the study (before paying for it)

EV(with study) = $0.5\times1.303967891 + 0.5\times0.668039538$

= 0.986003714 (£m)

Step 7: Expected value without the study

From part (b), the best option without extra information is Option B with ENPV = 0.986003714 (£m)

Therefore, EVSI = EV(with study) − EV(without study)

EVSI = 0.986003714 − 0.986003714 = 0.000000000 (£m)

EVSI = £0

c)(ii) Should ClearFlow commission the study? Decision rule and decision

Decision rule: commission the study if EVSI is greater than the cost of the study (both measured in present value terms at Year 0).

Here, EVSI = £0 and the study costs £400,000 paid immediately (Year 0), so the net value of commissioning is:

Net value = EVSI − 0.400 (£m) = 0 − 0.400 = −0.400 (£m)

Decision: do not commission the study.

Reason: the study does not change the recommended action (Option B is optimal whether the study predicts high or low), so the information has no decision value, and the cost would reduce value by £400,000.

c)(iii) Role of decision trees in risk-adjusted decision making, and limitations

Decision trees support risk-adjusted decision making by mapping decisions, uncertainties, and consequences over time, then valuing each path using probabilities and discounted outcomes. This makes trade-offs explicit and highlights which uncertainties drive value.

In high volatility, they keep resource allocation aligned with organisational purpose by testing each option against multiple plausible futures and purpose-relevant outcomes (e.g., reliability, affordability, resilience, compliance), rather than a single forecast. They also capture flexibility (delay, stage, switch), enabling adaptive allocation as new information arrives.

When using decision trees, results are highly sensitive to inputs (probabilities, costs) that are often subjective; uncertainty must be simplified into discrete states; and trees can become complex and hard to audit, creating false precision. These issues can push strategy towards overconfidence in expected NPV, underweight tail risks, and underinvest in robustness or non-financial objectives.

Decision trees are most effective when used alongside sensitivity analysis, scenario stress testing, and clear governance over assumptions, so that the model informs judgement rather than replacing it.

3

(a) (i)

Arrivals: Passenger vehicles arrive at a rate of 12 vehicles per hour (0.2 vehicles per minute). Since the coefficient of variation (C_v) of inter-arrival times is 1, the arrival process follows a Poisson distribution (or exponential inter-arrival times).

Service: There are multiple fast chargers (servers) available. The service time follows an exponential distribution with a mean of 18 minutes per vehicle, resulting in a service rate (μ) of 3.33 vehicles per hour per charger.

Queue: Vehicles join a single or multiple-line queue if all chargers are occupied. The system operates on a First-Come, First-Served (FCFS) basis. The queue capacity is assumed to be infinite unless otherwise specified by the hub's physical footprint.

Departure: Drivers must wait for a full charge; once the service time is complete, the vehicle leaves the hub immediately, making the charger available for the next vehicle.

(ii)

Arrival Rate (λ): 12 vehicles per hour

Service Rate (μ): One vehicle per 18 minutes = 3.33 vehicles per hour per charger

We calculate the average waiting time in the queue to see which s value meets the target of < 4 minutes.

M/M/s steady state only exists if $\rho < 1$ (equivalently $\lambda < s\mu$). If $\rho \geq 1$, the average queue grows without bound and the average waiting time is infinite.

s = 1 charger

$\rho = 12 / (1\times3.3333) = 3.6 > 1$ ➔ System unstable, and queue grows infinitely

s = 2 chargers

$\rho = 12 / (2\times3.3333) = 1.8 > 1$ ➔ System unstable, and queue grows infinitely

s = 3 chargers

$\rho = 12 / (3\times3.3333) = 1.2 > 1$ ➔ System unstable, and queue grows infinitely

Version EG/1

$s = 4$ chargers

$\rho = 12 / (4 \times 3.3333) = 0.9$

Looking up in annexed table we have that $L_q = 7.08$

Applying Little's Law ($W_q = L_q / \lambda$):

- $W_q = 7.08 / 12 = 0.59$ hours
- $W_q = 0.59 * 60 = 35.4$ minutes

$s = 5$ Chargers

Utilization (ρ): $12 / (5 * 3.333) = 0.72$

Looking up in annexed table we have that $L_q = 1.055$

Applying Little's Law ($W_q = L_q / \lambda$):

- $W_q = 1.055 / 12 = 0.0879$ hours
- $W_q = 0.0879 * 60 = 5.28$ minutes

5.28 > 4, does not satisfy condition.

$s = 6$ Chargers

Utilization (ρ): $12 / 6 * 3.333 = 0.60$

Looking up in annexed table we have that $L_q = 0.2948$

Applying Little's Law ($W_q = L_q / \lambda$):

- $W_q = 0.2948 / 12 = 0.0245$ hours
- $W_q = 0.0245 * 60 = 1.47$ minutes

1.47< 4, satisfies the condition.

The regional transport authority needs at least **6 fast chargers** to keep the average waiting time under 4 minutes.

(iii)

<u>The passenger vehicles</u> still have the same arrival and service parameters, and these are two separate queues. So $W_q = 0.0245 * 60 = 1.47$ minutes for passenger vehicles.

<u>Commercial Fleet Vehicles</u>

Arrival Rate (λ): 4 vehicles per hour

Service Rate (μ): One vehicle per 25 minutes = 2.4 vehicles per hour per charger

Number of Chargers (s): 3

Utilization (ρ) = 4h / (3*2.4h) = 0.556

Looking up in annexed table we have that $L_q = 0.3583$

Applying Little's Law:

$Wq = Lq / \lambda_c$ =0.3583/4 = 0.0896 hours

Wq = 0.0896 * 60 =5.37 minutes

(b)(i)

M_A charging sessions in April = (1,200 + 1,350 + 1,300) / 3 = 3,850 / 3 = 1,283.33

M_A charging sessions in May = (1,350 + 1,300 + 1,420) / 3 = 4,070 / 3 = 1,356.67

M_A charging sessions in June = (1,300 + 1,420 + 1,390) / 3 = 4,110 / 3 = 1,370.00

M_A charging sessions in July = (1,420 + 1,390 + 1,520) / 3 = 4,330 / 3 = **1,443.33**

(ii)

(α= 0.3$)

Using the formula $F_{t+1} = \alpha A_t + (1 - \alpha) F_t$

Forecast for March = 0.3(1,350) + 0.7(1,200) = 405 + 840 = 1,245

Forecast for April = 0.3(1,300) + 0.7(1,245) = 390 + 871.5 = 1,261.5

Forecast for May = 0.3(1,420) + 0.7(1,261.5) = 426 + 883.05 = 1,309.05

Forecast for June = 0.3(1,390) + 0.7(1,309.05) = 417 + 916.34 = 1,333.34

Forecast for July = 0.3(1,520) + 0.7(1,333.34) = 456 + 933.34 = **1,389.34**

(c) A Markov chain is a model of a system that moves between a set of states over time with fixed transition probabilities. The Markov property means the next state depends only on the current state, not on how the system arrived there.

In an EV charging system, each charger can be modelled as being in one of three states: idle (available), occupied (charging), or maintenance (out of service). Transitions occur as vehicles arrive and start charging (idle → occupied), sessions finish (occupied → idle), faults occur (idle/occupied → maintenance), and repairs are completed (maintenance → idle). With these transition probabilities, the Markov chain can be used to estimate long-run state proportions, expected time spent in each state, and how often the system becomes capacity-constrained.

This helps identify risks over time. Higher probability of being occupied or in maintenance reduces the fraction of available chargers, increasing congestion and expected waiting. Frequent or prolonged maintenance lowers reliability and can create sudden drops in capacity, making queues more likely even at moderate demand. Tracking state probabilities over time supports decisions on adding chargers, improving maintenance response, or managing demand to protect service performance.