Version EG/3

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 7 May 2026 14:00 to 15:40

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3E3 Modelling Risk data sheet (5 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Sansberry is a national supermarket chain operating in the UK and providing a wide range of products to its customers. Their management team has decided to launch an in-store sushi product line in selected stores. Sushi has a very short shelf life and must be sold on the same day it is prepared. Any unsold sushi is discarded at the end of the day, creating both financial losses and waste.

As part of the launch decision, store managers at Sansberry are reviewing their operational setup to understand the risks associated with inventory availability, staffing capacity, and waste. In particular, they want to assess whether expected demand can be met reliably under current operating conditions and whether existing inventory policies are appropriate for the new product line.

Sansberry estimates that the mean demand μ for sushi trays for the new product line will be approximately 7,500 boxes per month with a standard deviation (σ) of 500. The fixed cost of placing an order with the supplier is £45 per order (regardless of order size). The cost of holding inventory is estimated at £0.05 per box per month.

(a) (i) Calculate the economic order quantity (EOQ). [5%]

(ii) Calculate the expected number of orders per month and the average cycle inventory. [5%]

(iii) Calculate the reorder point (ROP), assuming constant daily demand and a lead time of 6 days. [5%]

(b) Each sushi box is sold at a price of £5.50. The (fixed) cost of producing one sushi box (including fish, rice, labour, and packaging) is £3.20. Any leftover sushi boxes need to be discarded and have a waste handling cost of £0.80 per sushi box.

(i) Calculate the optimal order quantity using the Critical Fractile (F). [10%]

(ii) Calculate the expected profit from the new sushi line. [10%]

(iii) Explain how yield losses and supply–demand mismatch create operational, financial, and strategic risks for a business. [15%]

(c) Sansberry prepares fresh sushi boxes in-store using a manual, multi-step process. Each finished sushi box must contain 800 g of ingredients to meet quality standards. Production runs for one 10-hour shift per day and follows five sequential stages: preparation, chopping, wrapping, packaging, and labelling. Each stage is staffed by a dedicated team, and each employee has a fixed processing capacity, measured in kilograms of ingredients per hour. Ingredients move through the stages in order, and some material is lost at each stage due to trimming, handling errors, and quality rejections.

Yield losses occur throughout the process for quality and operational reasons. In preparation, around 10% of ingredients are lost during washing, trimming, and removal of unusable raw material. During chopping, a further 5% is lost through offcuts and small fragments discarded when cutting to size. The wrapping stage has the highest loss (around 20%) due to misaligned rolls, overfilling, or damaged seaweed sheets that fail quality checks. In packaging, approximately 15% of output is lost because of damaged containers, incorrect portioning, or sealing errors. Finally, in labelling, about 5% of products are discarded due to incorrect labels, missing allergen information, or scanning errors. A summary table below provides the staffing allocations and processing capacities for each stage.

Table 1 – Summary of the Sushi Making Process at Sansberry

Each sushi box requires 800 grams of ingredients					
Process Step	**Preparation** (kg ingredient per hour)	**Chopping** (kg ingredient per hour)	**Wrapping** (kg ingredient per hour)	**Packaging** (kg ingredient per hour)	**Labelling** (kg ingredient per hour)
Capacity per Employee	9	8	8	6	10
Number of Employees	4	4	4	6	3
Total Hours Worked	10	10	10	10	10
	-	-	-	-	-
Capacity (boxes / day)					
Mass/Yield Loss					
Remaining Yield					
Effective Capacity					
Bottleneck					
Inflow					

cont. (TURN OVER

(i) Define and calculate the effective capacity for the sushi production system at Sansberry (Please draw your own table in your answer sheet). [10%]

(ii) Which step is the system bottleneck? Why? What is the throughput of the above process in the number sushi boxes per day? [10%]

(d) (i) Sansberry provided you with the following two graphs. Figure 1 illustrates how the sales trend for the past months and the following sales estimates for Oct-26: 8000, Nov-26:8200, Dec-26: 8400 for sushi boxes. Figure 2 plots the relationship between the sushi box sales and the total revenue. Estimate the total revenue Sansberry should expect to achieve in October 2026, November 2026 and December 2026, and comment on the limitations of using linear regression for this exercise. [10%]

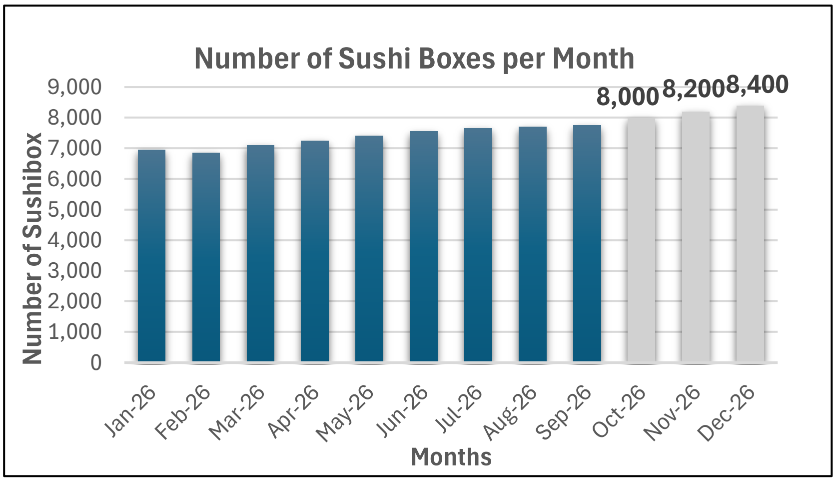


Figure 1 – Sales Trends for Sushi Boxes in 2026

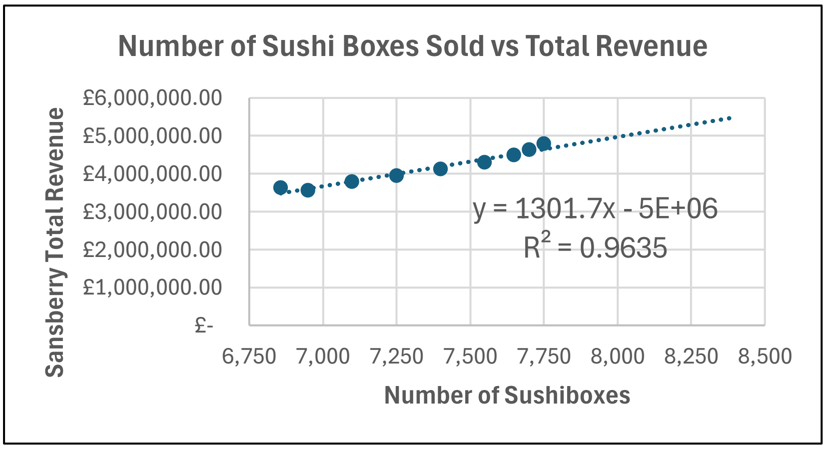


Figure 2 – Number of Sushi Boxes Sold vs Total Revenue

(ii) Using Figure 2, explain the limitations of using regression in forecasting and evaluate Sansberry's expectations. [10%]

(iii) What actions Sansberry should consider, to increase its capacity to the estimated rates October through December (Hint: Consider your throughput and bottleneck calculations in (c(ii)) [10%]

2 You have been appointed as an operations analyst for a regional water utility company, ClearFlow Utilities. The company is considering alternative strategies to manage increasing variability in water demand caused by climate change and consumption patterns. ClearFlow evaluates investments over a 4-year planning horizon and uses a discount rate of 8% per year. All cash flows occur at the end of each year unless stated otherwise. ClearFlow is considering the following mutually exclusive options. The company predicts a 0.4 chance of high demand growth and 0.6 probability that the demand growth will be low.

Option A: Expand reservoir capacity

ClearFlow can invest £3,000,000 at Year 0 (t = 0) to expand an existing reservoir (the upfront cost is not discounted). The project then generates net benefits at the end of each year in Years 1–4 (t = 1, 2, 3, 4), which are discounted back to Year 0 at 8% per year. If demand growth is high (probability 0.4), net benefits are £1,300,000 per year in Years 1–4 due to reduced water shortages and emergency purchases. If demand growth is low (probability 0.6), net benefits are £600,000 per year in Years 1–4.

Option B: Invest in demand management programmes

Alternatively, ClearFlow can invest £1,200,000 at Year 0 (t = 0) in water efficiency programmes (smart meters, pricing incentives, and public awareness), with the upfront cost not discounted. The programme generates net benefits at the end of each year in Years 1–4 (t = 1, 2, 3, 4), which are discounted back to Year 0 at 8% per year. If demand growth is high (probability 0.4), net benefits are £900,000 per year in Years 1–4. If demand growth is low (probability 0.6), net benefits are £500,000 per year in Years 1–4.

Option C: Delay investment

ClearFlow may choose not to invest at Year 0 and instead wait one year to observe early demand trends. At the end of Year 1 (t = 1), demand growth (high or low) is revealed with certainty, and ClearFlow then chooses either Option A or Option B.

However, delaying investment has two consequences:

1. Any chosen investment cost is incurred at the end of Year 1 (t = 1) and is discounted once back to Year 0 at 8% per year. No benefits are received in Year 1; benefits

are only received at the end of Years 2–4 (t = 2, 3, 4) and are reduced by 10% (i.e., multiplied by 0.9) due to early capacity constraints and customer dissatisfaction.

(a) Draw a decision tree representing Options A, B, and C. The tree should clearly show decision nodes, chance nodes, timing of information, probabilities, and cash flows. [20%]

(b) Using discounted cash flow analysis:

(i) Calculate the expected net present values for all options (Hint: For Option C assume that at the end of Year 1 ClearFlow chooses the option with the higher NPV given the realised demand state). [20%]

(ii) Based on your results, recommend the preferred strategy. Justify your recommendation. [10%]

(c) After presenting your recommendation, senior management considers commissioning a detailed demand forecast study.

<u>Demand forecast study</u>

Before choosing any option, ClearFlow can commission a forecasting study at a cost of £400,000 paid immediately (Year 0). The study does not change demand outcomes, but it provides imperfect information:

If demand growth is actually high, the study predicts “high demand” with probability 0.8.

If demand growth is actually low, the study predicts “low demand” with probability 0.7.

ClearFlow must decide whether to commission the study before making its investment decision.

(i) Calculate the expected value of the information from the forecast study. [15%]

(ii) Discuss whether ClearFlow should commission the study. Clearly state your decision rule. [15%]

(iii) Explain the role of decision trees in risk-adjusted decision making. How do these models ensure that resource allocation remains consistent with an organization’s purpose during periods of high volatility? Discuss the technical and practical limitations of decision trees and how these can affect corporate strategy. [20%]

3 A regional transport authority is planning to install a public electric vehicle (EV) charging hub located near M11 motorway. The hub will provide fast-charging services for passenger cars and larger commercial vehicles. Vehicles will arrive at the hub, join the queue if chargers are occupied, and leave once charging is complete. Drivers must wait until their vehicle is fully charged before departing. It is known that passenger vehicles arrive at the charging hub at an average rate of 12 vehicles per hour. The coefficient of variation of inter-arrival times is 1. The charging time at a fast charger is exponentially distributed with a mean of 18 minutes per vehicle.

(a) (i) Describe the queuing system at the EV charging hub in terms of arrivals, service process, and queue structure. State your assumptions. [15%]

(ii) How many fast chargers should be installed at the hub to ensure that the average waiting time in the queue is no more than 4 minutes? [20%]

(iii)The local authority introduces a separate charging area for commercial fleet vehicles, equipped with 3 chargers reserved for fleet use. Fleet vehicles arrive at an average rate of 4 vehicles per hour, and their charging times are exponentially distributed with a mean of 25 minutes per vehicle. The coefficient of variation of fleet vehicle inter-arrival times is also 1. Passenger vehicles continue to use the original chargers with unchanged charging times. Estimate the expected waiting times in queues for private vehicles and fleet vehicles under this new system. [20%]

(b) The transport authority has collected monthly data on the number of EV charging sessions over the past six months, January to June: 1,200, 1,350, 1,300, 1,420, 1,390, and 1,520 charging sessions.

(i) Using a 3-month simple moving average, forecast the number of charging sessions for July. Show your calculations. [15%]

(ii) Using exponential smoothing with a smoothing factor $\alpha = 0.3$, forecast the number of charging sessions for July. Show your calculations and explain the role of the smoothing factor. [15%]

(c) Using the EV Charging example explain the concept of a Markov chain and the Markov property. Discuss how Markov chains can be used to model state transitions in a service system specifically focusing on the transition between idle, occupied, and maintenance states. Explain how this can help identify risks related to congestion, reliability, or system performance over time. [15%]

END OF PAPER

Version EG/3

EGT2
ENGINEERING TRIPOS PART IIA

SPECIAL DATA SHEET

Queuing Theory Formulas

Little's Law

$$L_q = \lambda W_q$$

Utilisation Factor

$$\rho = \frac{\lambda}{\mu}, \quad (\text{where } \rho < 1 \text{ for stability})$$

Expected Number of Customers in the Queue

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Probability of Having *n* Customers in the System

$$P_n = (1 - \rho)\rho^n, \quad n \geq 0$$

Expected Waiting Time in the Queue

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

Expected Time a Customer Spends in the System

$$W = \frac{1}{\mu - \lambda}$$

Expected Number of Customers in the System

$$L = \frac{\lambda}{\mu - \lambda}$$

Exponential Smoothing Formulas

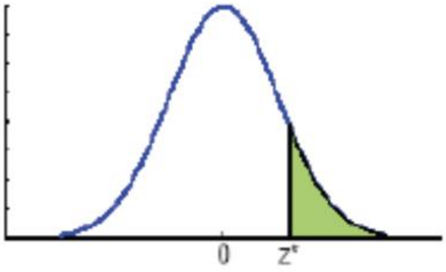
$$F_t = \alpha\, Y_{t-1} + (1 - \alpha)\, F_{t-1}$$

Decision Trees Formulas

$$EMV = \sum_{i=1}^{n} P_i \times V_i$$

Standard Normal distribution table:

Areas under the standard normal curve beyond z, i.e., shaded area.*

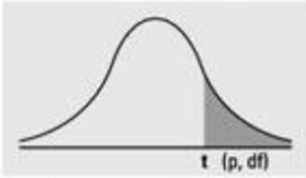


z*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

Version EG/3

t-distribution table:

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



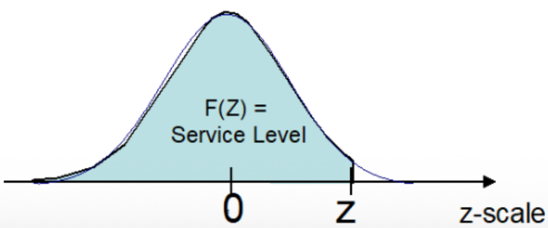
df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

Z-Chart & Loss Function

F(Z) is the probability that a variable from a standard normal distribution will be less than or equal to Z, or alternately, the service level for a quantity ordered with a z-value of Z.

L(Z) is the standard loss function, i.e. the expected number of lost sales as a fraction of the standard deviation. Hence, the lost sales = L(Z) x σ_{DEMAND}

Z	F(Z)	L(Z)	Z	F(Z)	L(Z)	Z	F(Z)	L(Z)	Z	F(Z)	L(Z)
-3.00	0.0013	3.000	-1.48	0.0694	1.511	0.04	0.5160	0.379	1.56	0.9406	0.026
-2.96	0.0015	2.960	-1.44	0.0749	1.474	0.08	0.5319	0.360	1.60	0.9452	0.023
-2.92	0.0018	2.921	-1.40	0.0808	1.437	0.12	0.5478	0.342	1.64	0.9495	0.021
-2.88	0.0020	2.881	-1.36	0.0869	1.400	0.16	0.5636	0.324	1.68	0.9535	0.019
-2.84	0.0023	2.841	-1.32	0.0934	1.364	0.20	0.5793	0.307	1.72	0.9573	0.017
-2.80	0.0026	2.801	-1.28	0.1003	1.327	0.24	0.5948	0.290	1.76	0.9608	0.016
-2.76	0.0029	2.761	-1.24	0.1075	1.292	0.28	0.6103	0.274	1.80	0.9641	0.014
-2.72	0.0033	2.721	-1.20	0.1151	1.256	0.32	0.6255	0.259	1.84	0.9671	0.013
-2.68	0.0037	2.681	-1.16	0.1230	1.221	0.36	0.6406	0.245	1.88	0.9699	0.012
-2.64	0.0041	2.641	-1.12	0.1314	1.186	0.40	0.6554	0.230	1.92	0.9726	0.010
-2.60	0.0047	2.601	-1.08	0.1401	1.151	0.44	0.6700	0.217	1.96	0.9750	0.009
-2.56	0.0052	2.562	-1.04	0.1492	1.117	0.48	0.6844	0.204	2.00	0.9772	0.008
-2.52	0.0059	2.522	-1.00	0.1587	1.083	0.52	0.6985	0.192	2.04	0.9793	0.008
-2.48	0.0066	2.482	-0.96	0.1685	1.050	0.56	0.7123	0.180	2.08	0.9812	0.007
-2.44	0.0073	2.442	-0.92	0.1788	1.017	0.60	0.7257	0.169	2.12	0.9830	0.006
-2.40	0.0082	2.403	-0.88	0.1894	0.984	0.64	0.7389	0.158	2.16	0.9846	0.005
-2.36	0.0091	2.363	-0.84	0.2005	0.952	0.68	0.7517	0.148	2.20	0.9861	0.005
-2.32	0.0102	2.323	-0.80	0.2119	0.920	0.72	0.7642	0.138	2.24	0.9875	0.004
-2.28	0.0113	2.284	-0.76	0.2236	0.889	0.76	0.7764	0.129	2.28	0.9887	0.004
-2.24	0.0125	2.244	-0.72	0.2358	0.858	0.80	0.7881	0.120	2.32	0.9898	0.003
-2.20	0.0139	2.205	-0.68	0.2483	0.828	0.84	0.7995	0.112	2.36	0.9909	0.003
-2.16	0.0154	2.165	-0.64	0.2611	0.798	0.88	0.8106	0.104	2.40	0.9918	0.003
-2.12	0.0170	2.126	-0.60	0.2743	0.769	0.92	0.8212	0.097	2.44	0.9927	0.002
-2.08	0.0188	2.087	-0.56	0.2877	0.740	0.96	0.8315	0.090	2.48	0.9934	0.002
-2.04	0.0207	2.048	-0.52	0.3015	0.712	1.00	0.8413	0.083	2.52	0.9941	0.002
-2.00	0.0228	2.008	-0.48	0.3156	0.684	1.04	0.8508	0.077	2.56	0.9948	0.002
-1.96	0.0250	1.969	-0.44	0.3300	0.657	1.08	0.8599	0.071	2.60	0.9953	0.001
-1.92	0.0274	1.930	-0.40	0.3446	0.630	1.12	0.8686	0.066	2.64	0.9959	0.001
-1.88	0.0301	1.892	-0.36	0.3594	0.605	1.16	0.8770	0.061	2.68	0.9963	0.001
-1.84	0.0329	1.853	-0.32	0.3745	0.579	1.20	0.8849	0.056	2.72	0.9967	0.001
-1.80	0.0359	1.814	-0.28	0.3897	0.554	1.24	0.8925	0.052	2.76	0.9971	0.001
-1.76	0.0392	1.776	-0.24	0.4052	0.530	1.28	0.8997	0.047	2.80	0.9974	0.001
-1.72	0.0427	1.737	-0.20	0.4207	0.507	1.32	0.9066	0.044	2.84	0.9977	0.001
-1.68	0.0465	1.699	-0.16	0.4364	0.484	1.36	0.9131	0.040	2.88	0.9980	0.001
-1.64	0.0505	1.661	-0.12	0.4522	0.462	1.40	0.9192	0.037	2.92	0.9982	0.001
-1.60	0.0548	1.623	-0.08	0.4681	0.440	1.44	0.9251	0.034	2.96	0.9985	0.000
-1.56	0.0594	1.586	-0.04	0.4840	0.419	1.48	0.9306	0.031	3.00	0.9987	0.000
-1.52	0.0643	1.548	0.00	0.5000	0.399	1.52	0.9357	0.028			



Z & *L(z)* for special service levels

Service Level F(z)	*z*	*L(z)*
75%	0.67	0.150
90%	1.28	0.047
95%	1.64	0.021
99%	2.33	0.003

Version EG/3

Inventory management:

(Q,R) model - optimal solution:

$$Q = \sqrt{\frac{2\lambda[K+pn(R)]}{h}}; \qquad F(R) = 1 - \frac{Qh}{p\lambda}$$

Newsvendor model:

$$F(Q^*) = \frac{c_u}{c_u + c_o}$$

Queuing theory:

Values of L_q for s servers, with mean utilisation rate ρ, assuming Poisson arrivals and exponential service times (known as a *M/M/s* queue).

Utilisation rate	*Number of servers (s)*				
(ρ)*	1	2	3	4	5
.10	.0111	.0020	.0004	.0001	.0000
.20	.0500	.0167	.0062	.0024	.0010
.30	.1286	.0593	.0300	.0159	.0086
.35	.1885	.0977	.0552	.0325	.0196
.40	.2667	.1524	.0941	.0605	.0398
.45	.3682	.2285	.1522	.1052	.0743
.50	.5000	.3333	.2368	.1739	.1304
.55	.6722	.4771	.3583	.2772	.2185
.60	.9000	.6750	.5321	.4306	.3542
.62	1.0116	.7743	.6213	.5109	.4269
.64	1.1378	.8880	.7246	.6051	.5130
.66	1.2812	1.0188	.8446	.7158	.6152
.68	1.4450	1.1698	.9847	.8461	.7367
.70	1.6333	1.3451	1.1488	1.0002	.8816
.72	1.8514	1.5500	1.3423	1.1834	1.0553
.74	2.1062	1.7914	1.5721	1.4025	1.2646
.76	2.4067	2.0785	1.8472	1.6668	1.5187
.78	2.7655	2.4237	2.1803	1.9887	1.8302
.80	3.2000	2.8444	2.5888	2.3857	2.2165
.82	3.7356	3.3661	3.0979	2.8832	2.7029
.84	4.4100	4.0265	3.7456	3.5190	3.3273
.86	5.2829	4.8852	4.5914	4.3526	4.1493
.88	6.4533	6.0414	5.7345	5.3834	5.2682
.90	8.1000	7.6737	7.3535	7.0898	6.8624
.92	10.5800	10.1392	9.8056	9.5290	9.2893
.94	14.7267	14.2712	13.9240	13.6344	13.3821
.96	23.0400	22.5698	22.2088	21.9060	21.6408
.98	48.0200	47.5350	47.1602	46.8439	46.5656
.99	98.0101	97.5176	97.1357	96.8127	96.4274

Multi-server waiting in queue

$$W_q = \frac{1}{\mu s} \frac{\rho^{\sqrt{2(s+1)}-1}}{1-\rho} \frac{CV_a^2 + CV_s^2}{2}$$