Version AGiF/2

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 29 April 2026 9.30 to 11.10

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Engineering Data Book.

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Determining a convolutional encoder from an input/output sequence pair is a useful skill in cryptanalysis ("codebreaking"). Consider the input and output sequences of an unknown convolutional encoder of rate $R = 1/2$ and constraint length 3 (two delay elements), where the encoder starts in state $(0, 0)$:

Input:	1	0	1	1	0	1	0	0
Output:	10	11	01	01	00	01	11	11

Describe the convolutional encoder in octal form. [25%]

(b) What length input sequence is needed to determine a convolutional encoder of constraint length ℓ from an input/output sequence pair? [5%]

(c) Draw a state diagram for the encoder in Part (a). [10%]

(d) Is the encoder catastrophic? Justify your answer. [10%]

(e) A 3 bit sequence $\mathbf{U} = [U_1, U_2, U_3]$ has been encoded, followed by two termination bits, resulting in a code sequence $\mathbf{X} = [X_1, X_2, \ldots, X_{10}]$. This has been transmitted over a Binary Symmetric Channel (BSC) with transition probability $p = 0.1$ and received as $\mathbf{y} = [1, 1, 1, 1, 0, 0, 1, 1, 1, 1]$. Compute $\max_{\mathbf{u}} P_{\mathbf{Y}|\mathbf{U}}(\mathbf{y}|\mathbf{u})$ and specify the full set of vectors $\mathbf{u}$ that maximise this expression. [20%]

(f) Express the detour enumerator transfer function $T(J, N, D)$ for the encoder and hence determine the free distance d_{free} of the encoder. [30%]

2 One of nine equally likely messages is transmitted over an additive white Gaussian noise channel where the noise process $n(t)$ is assumed to have mean $\mathbb{E}[n(t)] = 0$ and autocorrelation function

$$\mathbb{E}[n(t)n(t+\tau)^*] = \frac{N_0}{2}\delta(\tau), \qquad \text{for all } t, \tau.$$

The waveforms are given below for $0 \leq t \leq 2$

$$\begin{aligned}
s_1(t) &= 0\\
s_2(t) &= 4\cos(2\pi t)\\
s_3(t) &= 8\cos(2\pi t)\\
s_4(t) &= 4\sin(2\pi t)\\
s_5(t) &= 4\cos(2\pi t) + 4\sin(2\pi t)\\
s_6(t) &= 8\cos(2\pi t) + 4\sin(2\pi t)\\
s_7(t) &= 8\sin(2\pi t)\\
s_8(t) &= 4\cos(2\pi t) + 8\sin(2\pi t)\\
s_9(t) &= 8\cos(2\pi t) + 8\sin(2\pi t)
\end{aligned}$$

The waveforms are zero for $t < 0$ and for $t > 2$.

(a) What is the transmission rate in bits per second? [10%]

(b) Determine an orthonormal basis for the signal space spanned by the nine waveforms. Justify your answer. [20%]

(c) What is the dimension of the signal space? [10%]

(d) Find the vector representation for each of the nine waveforms. [10%]

(e) Draw the signal constellation and decision regions for maximum likelihood detection. [20%]

(f) Calculate the exact error probability. [30%]

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3 (a) Consider the signal space defined by the three waveforms $f_n(t), n = 1, 2, 3$, shown in Fig. 1 below.

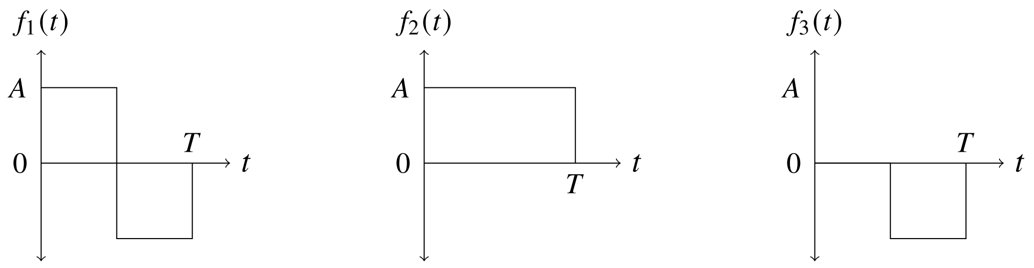

Fig. 1

(i) Give two orthonormal bases for this space. [10%]

(ii) What is the dimension of the signal space? [5%]

(iii) Provide the vector representation of the waveforms $f_n(t)$ according to each of the bases provided in Part (a)(i). [10%]

(iv) Draw the corresponding signal constellations. [10%]

(v) Calculate the energy of each waveform as well as the average energy. [10%]

(b) Consider an additive noise channel

$$y_k = x_k + z_k$$

where the noise samples are i.i.d. Laplacian distributed, i.e., with p.d.f.

$$p_Z(z) = \frac{1}{\sqrt{2\sigma^2}} e^{-\frac{\sqrt{2}|z|}{\sqrt{\sigma^2}}}$$

where σ^2 is the noise variance. The corresponding c.d.f. is

$$F_Z(z) = \begin{cases} \frac{1}{2} e^{\frac{\sqrt{2}z}{\sqrt{\sigma^2}}} & z \leq 0 \\ 1 - \frac{1}{2} e^{-\frac{\sqrt{2}z}{\sqrt{\sigma^2}}} & z > 0 \end{cases}$$

(i) Assuming non-equiprobable transmission, show that the optimal detector output symbol $\hat{x}$ can be expressed as

$$\hat{x} = \arg\min_{x \in \mathcal{X}} \left\{ |y - x| - \sqrt{\frac{\sigma^2}{2}} \log P_X(x) \right\}$$

where $\mathcal{X}$ is the signal constellation and $P_X(x)$ is the probability of transmitting symbol x. [20%]

(ii) Calculate the error probability for a constellation $\mathcal{X} = \{-A, +A\}$ with equiprobable symbols. [20%]

(iii) Explain how the error probability in Part (b)(ii) depends on the signal-to-noise ratio $\frac{A^2}{\sigma^2}$. Compare this dependence to the error probability of the same constellation over an AWGN. [15%]

4 (a) Explain the concept of intersymbol interference (ISI). [10%]

(b) Explain the steps to find the coefficients of a zero-forcing linear equaliser for an ISI channel where the sampled output of the low-pass filter is given by [15%]

$$r_m = g_0 X_m + g_1 X_{m-1} + g_2 X_{m-2} + n_m$$

where r_m is the m-th received sample, g_0, g_1, g_2, g_3 are the coefficients of the overall filter after sampling, X_m is the m-th transmitted symbol and n_m is the m-th noise sample.

(c) The pulse shown in Fig. 2 is used for binary transmission. When a zero is sent, $x(t) = p(t)$ and when a 1 is sent $x(t) = -p(t)$.

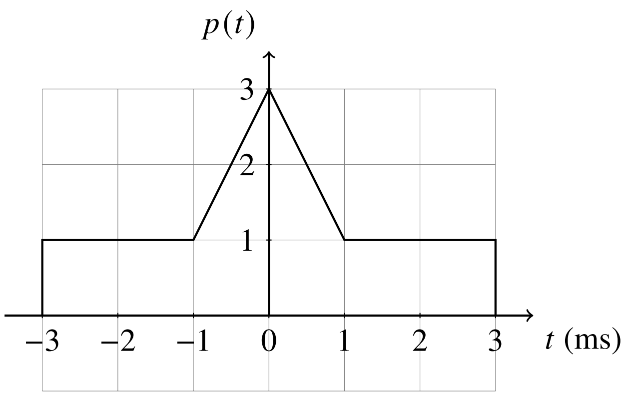

Fig. 2

(i) Sketch the received signal $y(t)$ when the transmitted bits are 0 1 1 0 in the absence of noise when the signalling period is $T = 6$ ms. [10%]

(ii) For what range of signalling periods T does the pulse $p(t)$ induce ISI? Justify your answer. [15%]

(d) Consider a fading channel.

(i) Explain the condition that the signalling period T needs to fulfill in order for the channel to have a single tap, i.e., a flat fading channel. [10%]

(ii) Using $Q(x) \leq e^{-\frac{x^2}{2}}$, find a bound on the probability of error as a function of the signal-to-noise ratio for BPSK transmission over a flat fading channel with one transmit and L receive antennas. Assume that the channels between the transmit antenna and each receive antenna experience independent Rayleigh fading. [20%]

(iii) How does the result in Part (d)(ii) change if we now employ a binary linear block code with minimum distance $d_{\min}$? [10%]

(iv) The probability of error of a wireless communication system over flat Rayleigh fading as a function of the signal-to-noise ratio is shown in Fig. 3. What is the diversity order of the system? Justify your answer. [10%]

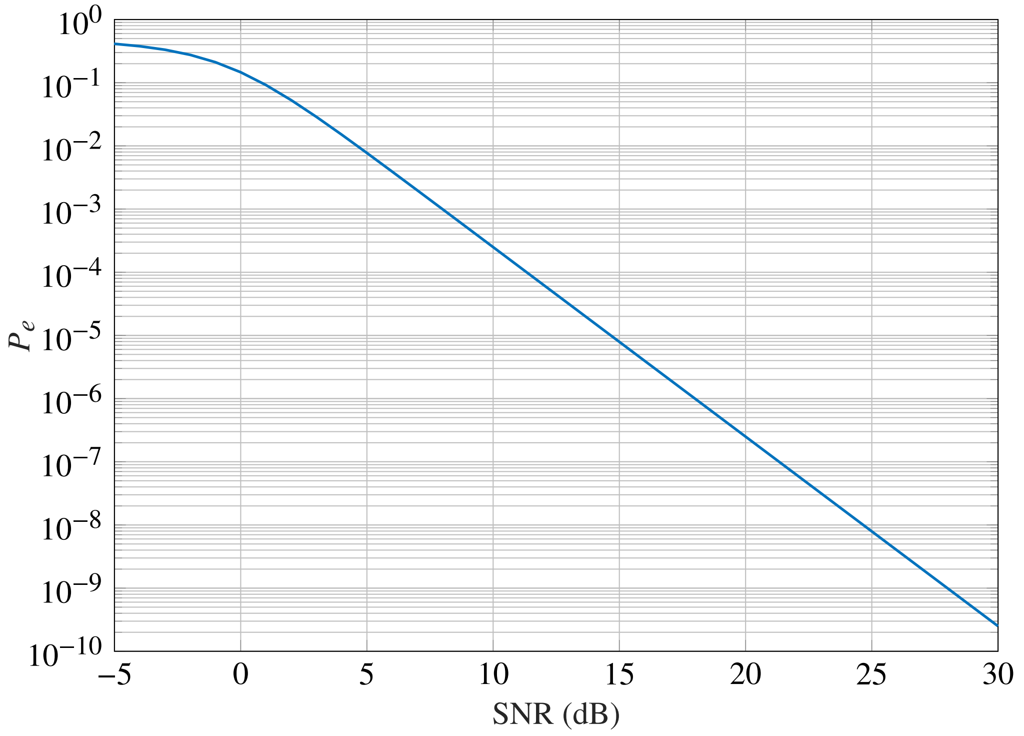

Fig. 3

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