Module 4A3 - TURBOMACHINERY 1
2026

Q1. (a) Boundary layers grow sidewall increases blockage and therefore reduces the effective exit flow area. This increases the exit Mach number. This reduces the diffusion on the blade suction surface thus reducing mid-height loss. The sidewall blockage occurs because the inlet boundary layer of the cascade is diffused through the blade row.

[10%]

(b) Relating the exit corrected mass flow functions with and without blockage

$$\dot{m}_1 = \dot{m}_2$$

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hscos\alpha_1 p_{01}} = \frac{\dot{m}\sqrt{c_p T_{02}}}{0.94 \times hscos\alpha_2 p_{02}} \times \frac{p_{02}}{p_{01}} \times \frac{cos\alpha_2}{cos\alpha_1} \times 0.94$$

Equation 1 above

Where

$$F(M_2) = \frac{\dot{m}\sqrt{c_p T_{02}}}{0.94 \times hscos\alpha_2 p_{02}}$$

Mach 0.55

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hscos\alpha_1 p_{01}} = 1.0208$$

The stagnation pressure coefficient is defined as

$$Y_p = \frac{p_{01} - p_{02}}{p_{01} - p_1} = 0.054$$

$$Y_p = \frac{1 - \frac{p_{02}}{p_{01}}}{1 - \frac{p_1}{p_{01}}} = 0.054$$

For Mach 0.55 at inlet

$$\frac{p_1}{p_{01}} = 0.8142$$

$$\frac{1 - \frac{p_{02}}{p_{01}}}{1 - 0.8142} = 0.054$$

$$1 - \frac{p_{02}}{p_{01}} = 0.054 \times 0.1858 = 0.10$$

$$\frac{p_{02}}{p_{01}} = 0.99$$

Subbing into equation 1 gives

$$F(M_2) = 1.0208 \times \frac{1}{0.99} \times \frac{cos55°}{cos35°} \times \frac{1}{0.94} = 0.7681$$

$$M_2 = 0.377$$

[30%]

(c)

$$DF = 1 - \frac{V_2}{V_1} + \frac{|V_{\theta 2} - V_{\theta 1}|}{2V_1}\frac{s}{c}$$

$V_x = V_2 \cos\alpha_2$, $V_x = V_1 \cos\alpha_1$, $V_{\theta 2} = V_x \tan\alpha_2$ and $V_{\theta 1} = V_x \tan\alpha_1$

$$DF = 1 - \frac{\cos\alpha_1}{\cos\alpha_2} + \frac{|\tan\alpha_2 - \tan\alpha_1|}{2}\frac{s}{c}\cos\alpha_1$$

$$\frac{s}{c} = \left(DF - 1 + \frac{\cos\alpha_1}{\cos\alpha_2}\right)\frac{2}{|\tan\alpha_2 - \tan\alpha_1|\cos\alpha_1}$$

DF=0.45 so

$$\frac{s}{c} = \left(0.48 - 1 + \frac{\cos 55^\circ}{\cos 35^\circ}\right)\frac{2}{|\tan 35^\circ - \tan 55^\circ|\cos 55^\circ} = 0.86$$

The answer above was accepted but it doesn't include the endwall blockage.

$$DF = 1 - \frac{\cos\alpha_1}{\cos\alpha_2} \times \frac{1}{0.94} + \frac{\left|\tan\alpha_2 \times \frac{1}{0.94} - \tan\alpha_1\right|}{2}\frac{s}{c}\cos\alpha_1$$

$$\frac{s}{c} = \left(0.48 - 1 + \frac{\cos 55^\circ}{0.94 \times \cos 35^\circ}\right)\frac{2}{|\tan 35^\circ / 0.94 - \tan 55^\circ|\cos 55^\circ} = 1.15$$

This is the value is with endwall blockage.

[20%]

(d) Working from the inlet to the throat plane of the cascade

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hscos\alpha_1 p_{01}} = \frac{\dot{m}\sqrt{c_p T_{01}}}{hop_o^*} \times \frac{p_o^*}{p_{01}} \times \frac{o}{scos\alpha_1}$$

At Mach 1

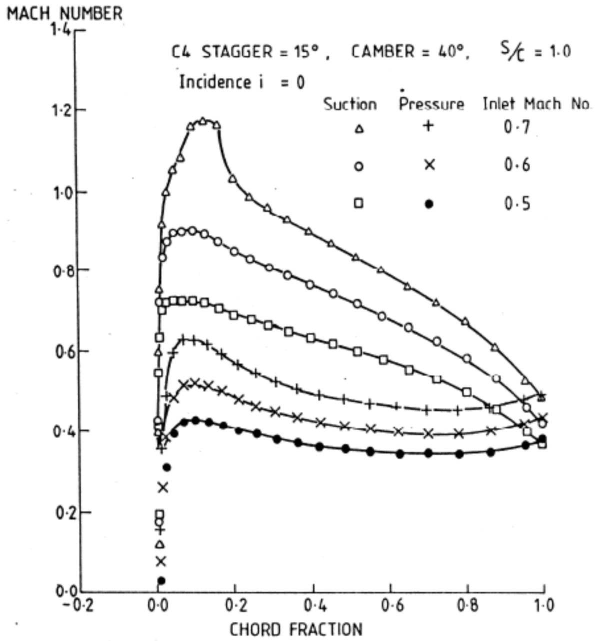
$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hop_o^*} = 1.281$$

$$\alpha_{1\,choke} = cos^{-1}\left(\frac{F(1)}{F(0.55)}\frac{o}{s}\right) = cos^{-1}\left(\frac{1.281}{1.0208} \times 0.6\right) = 41.15°$$

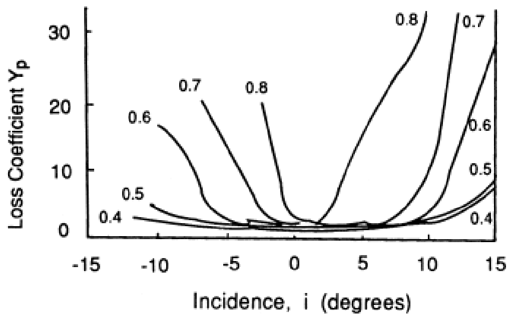
So the incidence is 41.15°-55°=-13.85°

[15%]

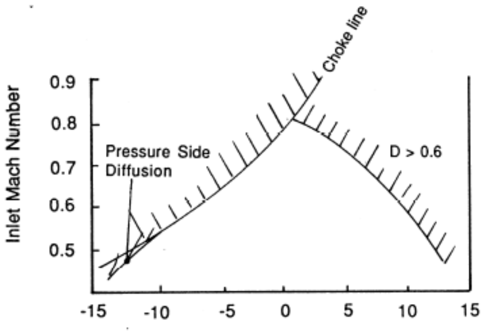
(e) As the Mach number rises, the diffusion factor on the blade rises and at Mach numbers greater than 0.7 a sonic bubble occurs.



At positive incidence this means that the diffusion factor rises at a lower incidence. It also means that the sonic bubble occurs at a lower incidence. The shock which ends the sonic bubble can separate the boundary layer causing loss to rise.



At negative incidence the blade is limited by choking. As the Mach number rises the negative incidence at which choking occurs becomes closer to the design incidence.



Choke line
0.9
0.8
0.7
0.6
0.5
Inlet Mach Number
Pressure Side
Diffusion
D > 0.6
-15
-10
-5
0
5
10
15

[25%]

Q2. (a) Euler's work equation

$$\Delta h_0 = U(V_{\theta 2} - V_{\theta 3})$$

For a repeating stage $V_{x1} = V_{x2} = V_{x3}$

$$\Delta h_0 = UV_x(tan\alpha_2 - tan\alpha_3)$$

$$\psi = \frac{\Delta h_0}{U^2} = \phi(tan\alpha_2 - tan\alpha_3) \quad \text{or} \quad tan\alpha_2 = tan\alpha_1 + \frac{\psi}{\phi}$$

$$\Lambda = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = 1 - \frac{\Delta h_{stator}}{\Delta h_{stage}} = 1 - \frac{1}{2}\frac{(V_2^2 - V_1^2)}{\Delta h_{stage}} = 1 - \frac{1}{2}\frac{\left(V_{\theta 2}^2 - V_{\theta 1}^2\right)}{\Delta h_{stage}} = 1 - \frac{1}{2}\frac{V_x^2(tan\alpha_2{}^2 - tan\alpha_1{}^2)}{\Delta h_{stage}}$$

NB $h_{01} = h_{02}$ and so $h_2 - h_1 = V_1^2 - V_2^2$

$$\Lambda = 1 - \frac{1}{2}\frac{\phi^2(tan\alpha_2{}^2 - tan\alpha_1{}^2)}{\psi}$$

Now because it is a repeating stage $\alpha_3 = \alpha_1$. Subbing in from above

$$\Lambda = 1 - \frac{1}{2}\frac{\phi^2\left(tan\alpha_1{}^2 + \left(\frac{\psi}{\phi}\right)^2 + 2\frac{\psi}{\phi}tan\alpha_1 - tan\alpha_1{}^2\right)}{\psi} = 1 - \frac{1}{2}\psi - \phi tan\alpha_1$$

$$\Lambda = 1 - \frac{1}{2}\psi - \phi tan\alpha_1$$

$$\psi = 2(1 - \Lambda - \phi tan\alpha_1)$$

(b) (i) Blade speed U = 220 m s^{-1}, a reaction Λ = 0.5, and an interstage swirl α_1 =0°.

$$\psi = 2(1 - 0.5) = 1$$

$$\Delta h_{stage} = \psi U^2 = 1 \times 220^2 = 48.4\ kJ/kg$$

So the number of stages is

$$N_{stage} = \frac{300}{48.4} = 6.2$$

So 7 stages are required.

[20%]

(ii) Velocity triangles and flow angles

Axial velocity:

$$V_x = \phi U = 0.35 \times 220 = 77 m/s$$

Since α_1 =0° absolute inlet swirl $V_{\theta 1} = V_{\theta 3} = 0$

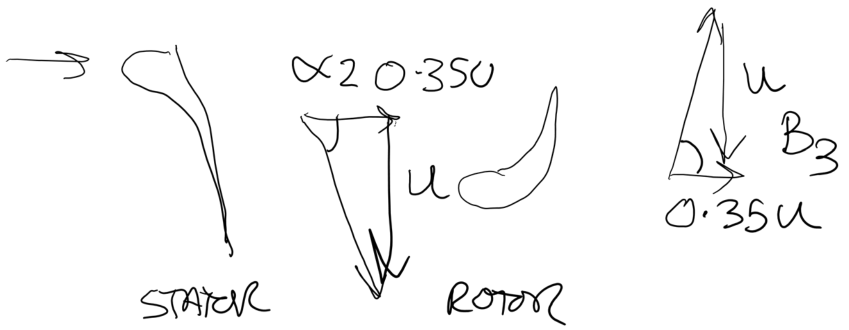
From Euler work:

$$\frac{\Delta h_0}{U^2} = 1 = \frac{(V_{\theta 2} - V_{\theta 3})}{U} = \frac{(V_{\theta 2})}{U}$$

$V_{\theta 2}$ =220 m s^{-1}

Thus absolute flow angle

$$\alpha_2 = \arctan\,(V_{\theta 2}/V_x) = \arctan\,(220/77) = 70.7°$$



[20%]

(iii) The correlation is for a 50% reaction stage, so the loss coefficient of the stator and rotor are the same. The loss coefficient of each blade row is referenced to the exit velocity of that blade row.

$$\eta_{tt} = 1 - 0.05\left(V_2^2 + W_3^2\right) / \Delta h_0$$

$$W_3^2 = V_2^2 = U^2 + \left(0.35U\right)^2 = 1.1225U^2$$

$$\eta_{tt} = 1 - 0.05 \times 1.1225 \times 2 = 0.8878$$

Stage is 88.8% efficient

[20%]

(iv) The polytropic efficiency is known as the small-stage efficiency because, in its definition, the turbine is divided into infinitely many small stages, each referenced to its own local exit temperature. In contrast, the isentropic efficiency uses the exit temperature of the whole turbine, accounting for the reheat effect whereby irreversibilities locally reheat the flow within the turbine.

If the turbine is split into 7 stages, each with the isentropic efficiency given by the formula in the question, then — because each stage operates across a small pressure ratio relative to that of the whole turbine — the isentropic efficiency of each stage approaches its polytropic efficiency.

$$\frac{T_{out}}{T_{in}} = \left(\frac{p_{out}}{p_{in}}\right)^{\frac{(\gamma-1)}{\eta\gamma}}$$

T_{in} =1400K $\quad T_{out} = T_{in} - \frac{\Delta h_0}{c_p} = 1400 - \frac{300}{1.15} = 1139.1K$

$$\frac{p_{out}}{p_{in}} = \left(\frac{T_{out}}{T_{in}}\right)^{\frac{\gamma}{\eta(\gamma-1)}} = \left(\frac{1139.1}{1400}\right)^{\frac{1.33}{0.888\times(0.33)}} = 0.392$$

So pressure ratio = 1/0.392 =2.55

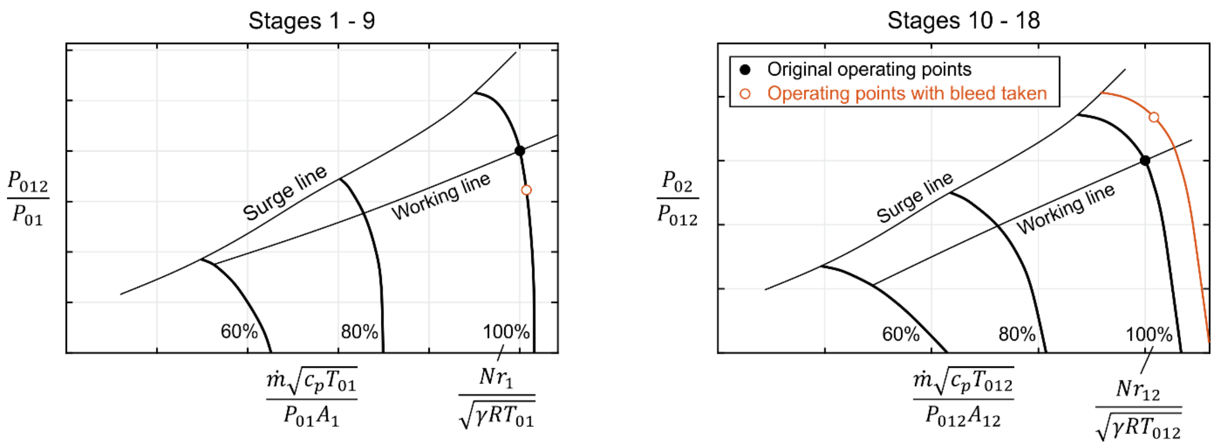
Repeating stage design makes the blade velocity triangles similar so it is easier to manufacture. However, the choice of reaction and interstage swirl has results in a loading of one which is very low and this means that it take 7 stages to achieve a pressure ratio of 2.55. If the reaction was reduced or negative interstage swirl used then loadings of two or higher could have been achieved significantly reducing the number of stages.

[20%]

Q3. (a) Sketch of front half of compressor, the front half is at a high Mach number and so steep characteristics are expected but not a necessity. Station 1 is at inlet to the 1st stage, station 12 is at inlet to the 9th.

The absolute rotational speed is the same, as are the inlet conditions, and therefore the first compressor is always constrained to operate on the 100% corrected speed characteristic. Taking bleed at the exit of stage 9 reduces the local pressure and therefore the operating point drops down the characteristic.

[20%]



(b) Sketch of rear half of compressor, station 2 is at outlet from the 18th stage. As stages 1-9 are operating at a reduced pressure ratio the temperature at stage 9 inlet will be reduced relative to the original value, this means that the non-dimensional shaft speed for the second half of the compressor will be increased. The effect is slight given the exponent relating the pressure ratio to the temperature ratio for efficient machines.

$$\sqrt{\frac{T_{012}}{T_{01}}} = \left(\frac{P_{012}}{P_{01}}\right)^{\frac{\gamma-1}{2\gamma\eta}} \approx \left(\frac{P_{012}}{P_{01}}\right)^{0.16}$$

As the overall pressure ratio is constant the operating point of the second half of the compressor is throttled up this higher non-dimensional speed characteristic to compensate for the reduced pressure ratio of the front half.

[20%]

(c) The second half of the compressor is throttled from the working line up towards stall by the bleed, the shaft speed is also increased relative to the design value. Above the working line reductions in mass flow function are amplified through the stages of the machine, this means that stage 18 will be pushed closest to its stall boundary. At high shaft speed the front of a compressor mismatches towards choke while the rear mismatches towards stall, again stage 18 will be pushed closest to its stall boundary.

[20%]

(d) Stages 1-9 will all be operating at a higher flow coefficient than design, this causes negative incidence and excessive Mach numbers, both push the stages towards choke, possible pressure side separation and reduced efficiency.

Stages 10-18 all operate at a lower flow coefficient than design, this causes positive incidence and likely suction side separation in the final stages, again reducing efficiency.

[10%]

(e) (i) Compressor exit flow can be related with bleed flow and inlet flow with and without bleed

$$\dot{m}_2 = \dot{m}_1 = \dot{m}_1' + \dot{m}_{bl}$$

Inlet non-dimensional mass flow rate is constant

$$\frac{\dot{m}_1\sqrt{c_p T_{01}}}{P_{01}A_1} = const$$

Inlet conditions are constant and so it reduces to

$$\frac{\dot{m}_1}{A_1} = const$$

Rearranging

$$\frac{A_1{}'}{A_1} = \frac{\dot{m}_1{}'}{\dot{m}_1}$$

Putting in bleed rate

$$\dot{m}_{bl} = 0.15\dot{m}_1'$$

Gives

$$\dot{m}_1 = 0.85\dot{m}_1'$$

Yielding the solution

$$\frac{\dot{m}_1{}'}{\dot{m}_1} = \frac{1}{0.85} = \frac{A_1{}'}{A_1} = 1.176$$

A 17.6% increase in inlet area.

[20%]

(e) (ii) The reduction in hub-tip radius ratio will increase the incidence swing at the casing section relative to the meanline incidences seen by the first stage at part speed. This will push the first stage further towards localised part span stall and will reduce the performance of the stage. This could degrade the matching at part speed and make the compressor more difficult to start.

[10%]