4B19 2025 - Crib

1. a) Variable speed wind turbines can operate at optimum power coefficient over a range of wind speeds, thereby extracting more energy than fixed speed ones. However, additional engineering, hence cost, is involved.

Tip speed ratio: $\lambda = \frac{\omega R}{v}$ so ratio of speed of blade tip : wind speed.

Power coefficient: C_p, factor by which turbine output power differs from power contained in the wind that passes the swept area.

i.e. $P = \frac{1}{2} C_p \rho A v^3$ where $\frac{1}{2}\rho A v^3$ is the total wind power.

[10%]

b) i) $C_p = 0.15 \lambda e^{-\lambda/8}$

$$\frac{dC_p}{d\lambda} = 0.15\left(e^{-\lambda/8} - \frac{\lambda}{8} e^{-\lambda/8}\right) = 0 \text{ at maximum}$$

$\therefore$ $\underline{\lambda_{opt} = 8}$ and $C_{p\,max} = 0.15 \times 8 \times e^{-1} = \underline{0.441}$ [10%]

ii) $P = 5\text{ MW}$ at $v = 12\text{ ms}^{-1}$ $\Rightarrow$ $5 \times 10^6 = \frac{1}{2} \times 0.441 \times 1.23 A \times 12^3$

giving swept area $A = 10669\text{ m}^3$ so $\underline{R = 58.3\text{ m}}$ [5%]

iii) Output power will be 0 for $v < 3\ ms^{-1}$ and $v > 20\ ms^{-1}$, so discard first and last columns of the table.

For wind speeds of $8\ ms^{-1}$ and $11\ ms^{-1}$, $C_p = 0.441$ and $P \propto v^3$

$$\therefore P \text{ at } 8\ ms^{-1} = \left(\frac{8}{12}\right)^3 \times 5 = \underline{1.48\ MW}$$

$$P \text{ at } 11\ ms^{-1} = \left(\frac{11}{12}\right)^3 \times 5 = \underline{3.85\ MW}$$

For remaining wind speeds need to find λ and C_p. To do this, need to find ω at $v = 6\ ms^{-1}$ and $v = 12\ ms^{-1}$

$$6\ ms^{-1}:\ 8 = \frac{\omega R}{v} = \frac{\omega \times 58.3}{6} \quad \text{so } \omega = 0.823\ rad\ s^{-1}$$

$$12\ ms^{-1}:\ 8 = \frac{\omega R}{v} = \frac{\omega \times 58.3}{12} \quad \text{so } \omega = 1.65\ rad s^{-1}$$

$$v = 5\ ms^{-1}:\ \lambda = \frac{0.823 \times 58.3}{5} = 9.60 \text{ giving } C_p = 0.434$$

$$v = 14\ ms^{-1}:\ \lambda = \frac{1.65 \times 58.3}{14} = 6.87 \text{ giving } C_p = 0.436$$

$v = 17\ ms^{-1}$: $\lambda = 5.66$ giving $C_p = 0.419$

For $v = 14\ ms^{-1}$ and $17\ ms^{-1}$, P will be limited to $P_{rated} = \underline{5\ MW}$ by furling the blades. For $v = 5\ ms^{-1}$, $P = \frac{1}{2} \times 0.434 \times 1.23 \times \pi \times 58.3^2 \times 5^3$

$$= \underline{0.356\ MW.}$$

Total energy in MWhr $= (0.356\times40 + 1.48\times120 + 3.85\times90 + 5\times(60+35)\times24$

$= \underline{24320 \text{ MWhr}}$

Capacity factor $= \dfrac{24320}{5\times365\times24} = \underline{0.555}$ [30%]

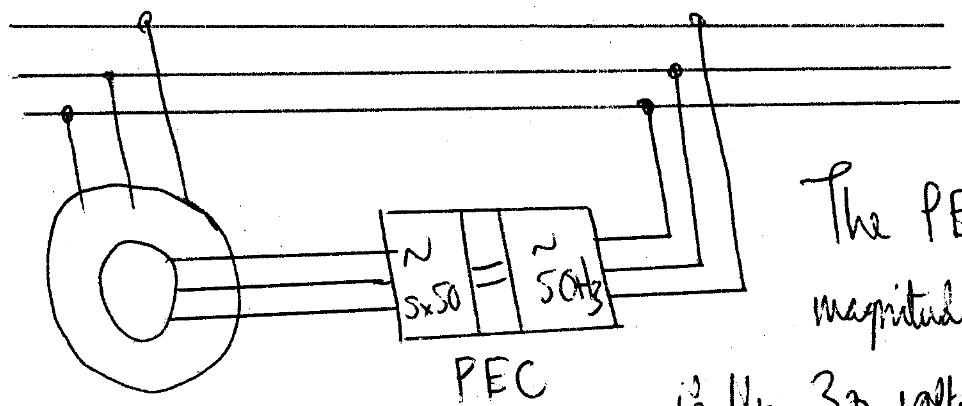
c) i) Since $v = 10\ ms^{-1}$ is in the range when $\lambda = \lambda_{opt} = 8$:

$8 = \dfrac{\omega_t R}{10}$ giving $\omega_t = 1.372$ rad s^{-1}

$\omega_g = (1-s)\omega_s = (1-(-0.01))\times\underbrace{\dfrac{2\pi\times50}{8}}_{\omega_s = 39.27} = 39.66$ rad s^{-1}

$\therefore N_g = \omega_g/\omega_t = \underline{28.9}$ [15%]

ii)



The PEC can vary the magnitude, phase and frequency of the 3ϕ voltage injected into the rotor in order to vary its speed. [10%]

At $v = 12\ ms^{-1}$, $\omega_t = 1.65$ rad s^{-1}; $\omega_g = 28.9\times1.65 = 47.7$ rad s^{-1}

and $s = \dfrac{39.27-47.7}{39.27} = -0.215$ so $f = -0.215\times50 = \underline{-10.7\ Hz}$

i.e. reverse sequence [20%]

At $v = 6\ ms^{-1}$, $\omega_t = 0.823$ rad s^{-1} so $\omega_g = 23.8$ rad s^{-1}

$s = \dfrac{39.27-23.8}{39.27} = 0.394$ so $f = 0.394\times50 = \underline{19.7\ Hz}$

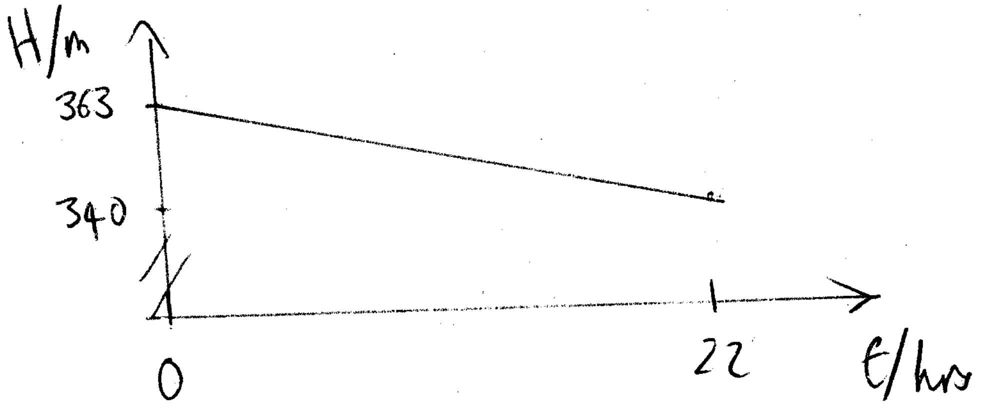
2 (a) PE of water at height H, volume V and density ρ is $\rho V g H$. Assuming constant head H, $P = \frac{d(PE)}{dt} = \rho g H \frac{dV}{dt}$ $= \rho g H Q$ where Q is the volumetric flow rate. η accounts for losses and thus represents the overall efficiency, giving $P = \eta \rho g H Q$

Pumped storage schemes are a form of hydroelectic scheme in which the turbines/generators can operate as pumps/motors. Thus, at periods of low demand surplus power is used to pump water uphill to the high level reservoir. At periods of high demand the stored energy in the water is released and converted into electrical energy.

- More renewable means more unpredictability of generating resources and so more storage will be needed, as provided by these schemes.
- They can absorb excess energy at times of high output and low demand, hence avoiding constraint payments.
- They can provide additional inertia, fast acting reserve capacity and help with reactive power demand.

[20%]

b) i) $Q = \dfrac{10\times10^6}{22\times60\times60} = \underline{126\ m^3 s^{-1}}$



[10%]

ii) P_{max} when $H = 363m \Rightarrow P_{max} = 0.73\times1000\times9.81\times363\times126$

$= \underline{328\ MW}$

P_{min} when $H = 340m$ giving $\underline{307\ MW}$

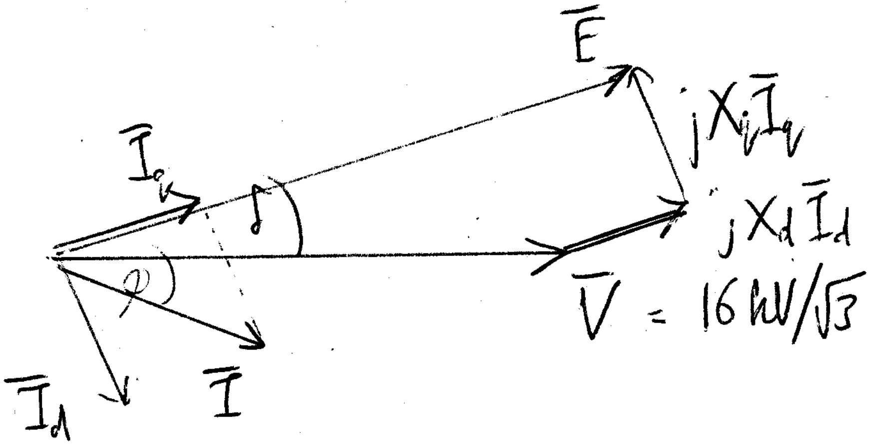
Thus over 22 hours, average power is 318 MW

∴ Total energy supplied = 318×22 MWhr = $\underline{7.00\ GWhr}$ [10%]

c) i) $S = (P^2+Q^2)^{1/2} = (100^2+30^2)^{1/2} = \underline{104.4\ MVA}$

$\omega_s = 2\pi f/p = 2\pi\times50/5 = \underline{62.83\ rad\,s^{-1}}$ [10%]

ii)



$S = 104.4\times10^6 = \sqrt{3}\times16000\times I \Rightarrow I = \underline{3767\ A}$

$\cos\phi = P/S = 100/104.4 = 0.958 \Rightarrow \phi = 16.7^\circ$ lagging

$V\sin\delta = X_q I_q = X_q I\cos(\phi+\delta) = X_q I(\cos\phi\cos\delta - \sin\phi\sin\delta)$

$\div\cos\delta$ $\quad V\tan\delta + X_q I\sin\phi\tan\delta = X_q I\cos\phi$

$$\tan\delta = \frac{X_q I\cos\phi}{V + X_q I\sin\phi} = \frac{1.8\times3767\times0.958}{16000/\sqrt{3} + 1.8\times3767\times0.287}$$

$$= 0.5807 \Rightarrow \underline{\delta = 30.1^\circ}$$

$E = V\cos\delta + X_d I_d$, $I_d = I\sin(\phi+\delta) = 3767\sin(16.7^\circ + 30.1^\circ)$
$= 2748\ A$

$$= \frac{16000}{\sqrt{3}}\cos30.1^\circ + 2.5\times2748$$

$= 14.9\ kV$ $\underline{(25.7\ kV\ \text{line})}$ [20%]

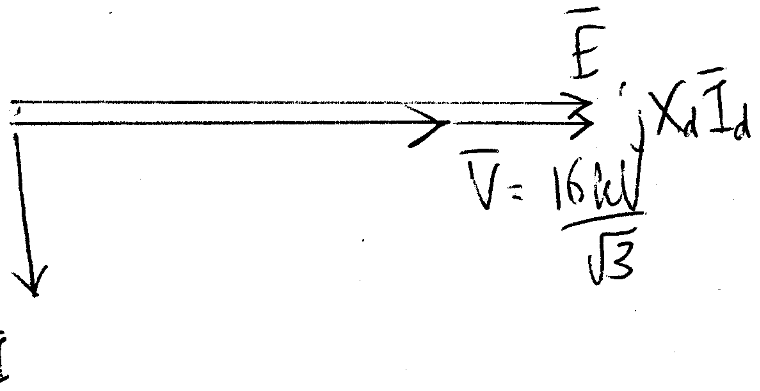
d) i) $W_{KE} \propto f^2 = Kf^2$ and $P_{gen} - P_{dem} = \Delta P = \frac{dW_{KE}}{dt} = 2Kf\frac{df}{dt}$

$\therefore$ Large K due to large inertia reduces df/dt so more stable. Grid-connected battery energy storage can respond in 0.25 secs, fast response increasingly needed as grid inertia steadily reducing. [10%]

ii) $\delta W_{KE} = 3.2\times10^6 = \frac{1}{2}J\left(\omega_s^2 - (\omega_s - \delta\omega_s)^2\right) \approx \frac{1}{2}J.2\omega_s\delta\omega_s = J\omega_s\delta\omega_s$

$\delta\omega_s = \frac{2\pi\Delta f}{p} = \frac{2\pi\times0.2}{5}$, $\omega_s = \frac{2\pi\times50}{5}$ giving $J = \underline{203\times10^3\ kg\,m^2}$ [10%]

iii)



Note. $\bar{I}_d = \bar{I}$, $\bar{I}_q = 0$ and $\sqrt{3}\,V I = Q = 50\times10^6$

so $I = 1804$ A

$E = \frac{16000}{\sqrt{3}} + 1804 \times 2.5 = 13.75$ kV (23.8 kV line)

[10%]

Examiner's comments

Q1 Wind turbine: 13 IIB attempts, mean 66.9%

There were many good attempts at this question with candidates scoring heavily on parts (a) and (b). A few candidates didn't cap the output power at rated in (b)(iii), or didn't account for the altered power coefficient. Part (c) proved to be more challenging, although most candidates correctly calculated the gearbox ratio in (c)(i). Some candidates confused the system diagram with the equivalent circuit in (c)(ii), and only one or two candidate were able to calculate the range of converter frequencies needed in (c)(iii). A common mistake was to calculate the slip s but then forget that the required frequency will be sf.

Q2 Hydroelectricity: 13 IIB attempts, mean 58.9%

Another popular question that received some good answers. Most candidates did well on parts (a) and (b), although a few confused the head of water with the depth of the resevoir leading to errors. Most candidates did well on (c)(i), but (c)(ii) proved more challenging, requiring careful calculation.(d)(i) was well answered, but (ii) and (iii) were somewhat novel and very few candidates were able to succeed here. In (c)(ii) a common error was to confuse the grid constant with moment of inertia.

Question 3

Examiner's comment:

This was the least popular question, only answered by 25% of IIB students. It was generally answered very well, with a high average mark (17.6/20).

In 3(b), all IIB students correctly identified that the n-type material should be on top, but many did not have the correct reasoning. The question states that the electrodes are totally transparent, so considerations of finger electrodes and associated series resistance are not relevant.

Part (a)(i)

We assume:

(1) The junction is abrupt and all acceptors are ionised, so that in the p-type region $p = N_A$. In the p-type region there is fixed charge due to ionised acceptors.
(2) The electric field exists only within the depletion region. There are no electric fields outside the depletion region, therefore:

$$\mu E \frac{\partial(\Delta n)}{\partial x} = 0$$

(3) We are in steady-state so the left hand side is zero.

$$\frac{\partial(\Delta n)}{\partial t} = 0$$

Therefore

$$D_e \frac{\partial^2(\Delta n)}{\partial x^2} = \frac{\Delta n}{\tau_e}$$

$$\therefore \frac{\partial^2(\Delta n)}{\partial x^2} = \frac{\Delta n}{{L_e}^2}$$

The general solution to this differential equation is

$$\Delta n = F \exp\left(\frac{-x}{L_e}\right) + G\exp\left(\frac{x}{L_e}\right)$$

To satisfy the boundary condition that the minority carrier concentration can only decay away from the depletion edge, $G = 0$, so

$$\Delta n = F\exp\left(\frac{-x}{L_e}\right)$$

To find the constant F, we use the boundary condition that at x=0,

$$n = n(0) = \frac{n_i^2}{N_A} \exp\left(\frac{eV}{kT}\right)$$

And also note that $\Delta n = n - \frac{n_i^2}{N_A}$.

So,

$$\Delta n(0) = \frac{n_i^2}{N_A} \exp\left(\frac{eV}{kT}\right) - \frac{n_i^2}{N_A}$$

This gives the solution

$$\Delta n = \frac{n_i^2}{N_A}\left[\exp\left(\frac{eV}{kT}\right) - 1\right] \exp\left(\frac{-x}{L_e}\right)$$

Also, we have assumed there is no net recombination or generation in the depletion region, and that the injected minority carrier concentration is much less than the majority carrier concentration (low injection). [20%]

Comparing to the expression

$$\Delta n = A\left[\exp\left(\frac{eV}{kT}\right) - 1\right] \exp(-Bx),$$

we see that $A = \frac{n_i^2}{N_A}$ and $B = \frac{1}{L_e} = \frac{1}{\sqrt{D_e \tau_e}}$

Part (a)(ii)

As the n-doping is much higher than the p-doping, we can assume that the reverse saturation current is dominated by minority carrier electrons on the p-type side.

$$\begin{aligned} J_e &= eD_e \frac{dn}{dx} \\ &= -eD_e AB\left[\exp\left(\frac{eV}{kT}\right) - 1\right] \exp(-Bx) \end{aligned}$$

Evaluating at x=0 gives

$$J_e = -eAB\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

And converting to net current gives

$$I \approx \text{area } \times eD_e AB\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

$$= I_s \left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

The reverse saturation current is therefore given by

$$I_s = \text{area } \times qD_eAB$$

$$= 1.0 \times 10^{-2} \times 1.602 \times 10^{-19} \times 0.01 \times 2 \times 10^{13}$$

$$=320 \text{ pA}$$

[20%]

Part (b)

The n-type side should be at the top, because it is the thinner side, and its placement at the top means the junction is nearer the surface, where most photons are absorbed.

Part (c)
(i)

$$I_{sc} = I_s \left(\exp\left(\frac{eV_{oc}}{kT}\right) - 1\right)$$
$$= 300 \times 10^{-12} \times \left(\exp\left(\frac{0.6}{0.025861}\right) - 1\right)$$
$$= 3.57 \text{ A}$$

[15%]

(ii)

$$g_{opt} = \frac{I_{sc}}{\text{area } \times q(L_e + L_h)}$$

Assume L_h is negligible, which is reasonable given the high doping level in the n-type region.

$$L_e = \frac{I_{sc}}{\text{area } \times e g_{opt}} = \frac{3.57}{10^{-2} \times 1.602 \times 10^{-19} \times (2 \times 10^{25})}$$

$$= 1.12 \times 10^{-4} \text{m}$$
$$=100 \text{ µm} \text{ to 1 significant figure}$$ [15%]

Question 4

Examiner's comment: This question was the most popular question, answered by almost 75% of IIB students, with an average mark of 13.6/20.

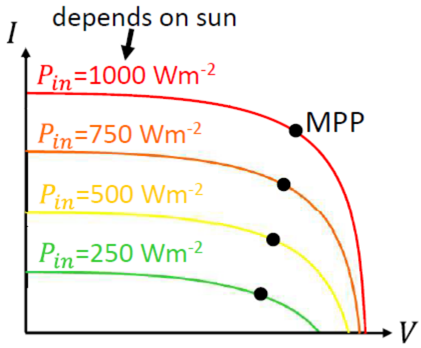
It was generally answered well. However, most students neglected some details in part 4(a).

In part 4(b), some students did not answer the question about multijunction solar cells, or confused multijunction solar cells with p-i-n solar cells.

In part 4(c), some students misunderstood that anti-reflection coatings and concentrator photovoltaics do not increase optical path length.

Part (a)

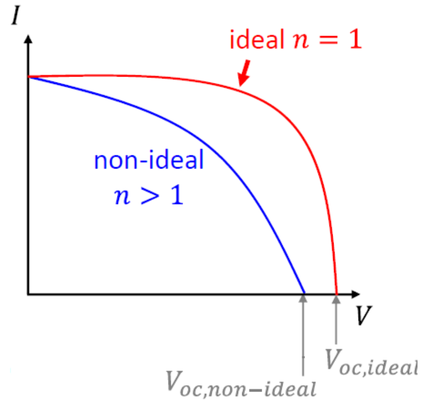
(i) I_{sc} is linearly proportional to the optical generation rate:



V_{oc} and FF also increase slightly with increasing optical generation rate, but not linearly.

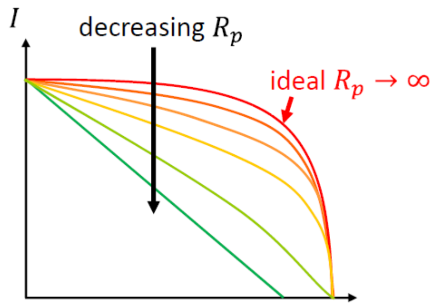
[10%]

(i) Recombination in the depletion region causes the curve to deviate from ideal. The ideality factor becomes greater than 1 which reduces the fill factor and V_{oc}. I_{sc} is less affected.



[10%]

(ii) Shunt pathways cause a parallel resistance R_p, which reduces the fill factor and V_{oc}. I_{sc} is less affected.



[10%]

(b) In a single-junction solar cell, the maximum V_{oc} is determined by the material's bandgap. Photons with energy greater than the bandgap will be absorbed, giving rise to electron-hole pairs of the same energy, but any excess energy above the bandgap is quickly lost as heat to the lattice as the electrons and holes cool to the band edges. This limits the V_{oc}. Photons with energy less than the bandgap are not absorbed at all, which limits the I_{sc}.

Multijunction solar cells consist of several sub-cells different bandgaps in series. The top cell has the highest bandgap and absorbs the highest energy photons. It generates a high open-circuit voltage, $V_{oc,\,top}$. It is transparent to the lower energy photons, which pass through to the lower cells. The lower cells have progressively smaller bandgaps to absorb the photons transmitted through the upper layers. The total V_{oc} is given by summing the V_{oc} of the individual sub-cells. This achieves a high $V_{oc,\,total}$ without sacrificing I_{sc}.

[30%]

(c) The optical path length can be doubled by adding a rear reflector, and increased further by roughening of the top and bottom surfaces. [10%]

(d) (i)

$$FF_o = \frac{\frac{eV_{OC}}{kT} - \ln\left(\frac{eV_a}{kT} + 0.72\right)}{\frac{eV_{OC}}{kT} + 1}$$

$$\frac{eV_{oc}}{kT} = \frac{1.602 \times 10^{-19} \times 0.6}{1.381 \times 10^{-23} \times 300} = 23.2$$

$$FF_o = \frac{23.2 - \ln(23.2 + 0.72)}{23.2}$$

=0.827

But there is also the series resistance of 0.02 Ω which alters the FF according to:

$$FF_1 = FF_0(1 - r_s)$$

where

$$r_s = \frac{0.02}{R_0}$$

and

$$R_0 = \frac{V_{oc}}{I_{sc}} = \frac{0.6}{3} = 0.2\Omega$$

so

$$r_s = \frac{0.02}{0.2} = 0.1$$

$$FF_1 = FF_0(1 - r_s) = 0.827(1 - 0.1) = 0.745$$

[20%]

(d)(ii)

Efficiency

$$\frac{V_{oc} I_{SC} FF_1}{P_{ch}}$$

$$= \frac{0.6 \times 3 \times 0.745}{1 \times 10^3 \times 10^{-2}} = 0.134$$

= 13.4%

[10%]