4B19 Renewable Electrical Power - Exam Solutions (2026)

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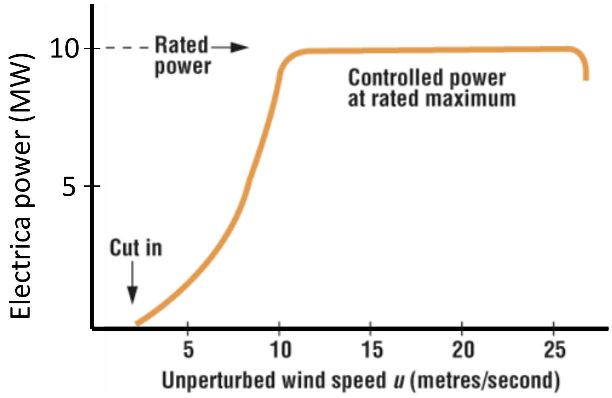
1(a)

- (i) Kinetic energy of wind proportional to ½ $m\, v^2$ = ½ $\rho\, A\, v\, v^2$ = ½ $\rho\, A\, v^3$ with the speed v, the wind cross section A, mass m, and density ρ.
- (ii)



- (iii) Cut-in speed: speed at which the turbine can start generating net power
- Rated wind speed: turbine reaches the rate power; above, the power is constant
- Stall speed: maximum speed, above which the power has to be curtailed to not damage the turbine
- (iv) Ways to reduce power: (1) change pitch, (2) rotate turbine axis out of the wind

1(b)

(i) No power is produced at the cut-in and the stall speeds. At 12 ms^{-1}, the turbine produces 10 MW.
At 6 ms^{-1}, P = $(6/12)^3 \times 10$ MW = 1.25 MW, and at 10 ms^{-1}, P = $(10/12)^3 \times 10$ MW = 5.8 MW.

2 ms^{-1}, 11%, 0 MW, 0 MWh
6 ms^{-1}, 41%, 1.25 MW, 4493 MWh
10 ms^{-1}, 22%, 5.8 MW, 11185 MWh
14 ms^{-1}, 14%, 10 MW, 12272 MWh
18 ms^{-1}, 8%, 10 MW, 7013 MWh
22 ms^{-1}, 4%, 0 MW, 0 MWh

Annual total energy supplied = 34963 MWh ≈ 35 GWh
(ii) Capacity factor = 34963 / (365 x 24 x 10) = 39.9% (approx. 40%)

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1(c)

(i) Same turbine but different generator
=> Same power for all points below the rated power, but power increases a bit beyond 12 ms^{-1}

13 MW = $(v/12\ ms^{-1})^3 \times 10$ MW => v = 12 ms^{-1} $\sqrt[3]{(13/10)}$ = 13 ms^{-1}
=> Higher power for 14 and 18 ms^{-1}

2 ms^{-1}, 11%, 0 MW, 0 MWh
6 ms^{-1}, 41%, 1.25 MW, 4493 MWh
10 ms^{-1}, 22%, 5.8 MW, 11185 MWh
14 ms^{-1}, 14%, 13 MW, 15954 MWh
18 ms^{-1}, 8%, 13 MW, 9117 MWh
22 ms^{-1}, 4%, 0 MW, 0 MWh

Annual total energy supplied = 40749 MWh ≈ 41 GWh
Capacity factor = 40749 / (365 x 24 x 13) = 35.8% (approx. 36%)

(ii) Ways to increase the power density:

- Power density of generator is product of rotational speed, electrical loading, magnetic loading, and excitation
  => increase any of those
- Increase rotating speed of turbine (in case of transmission already installed so that only gear ratio needs to be adjusted)
- More permanent magnet material or higher-quality permanent magnet material (potentially in combination with better magnetic steel)

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2(a)

Grid structure consisting the high-voltage transmission grid for longer-distance transport of electrical energy (National Grid, mostly 275 kV and 400 kV; other countries in Europe use 220 kV and 400 kV), the distribution grid (mostly 11 kV and 33 kV, in other countries in Europe also various levels in that range, such as 10 kV, 20 kV, 30 kV), which is mostly on medium voltage, and the last mile low-voltage grid for most end customers (230 V single phase, 400 V three-phase). There is one more level, 132 kV, i.e., a high voltage level, which is either considered as a distribution level or called the sub-transmission level. For longer distances, higher voltages allow lower current and therefore lower loss, lower use of materials (copper/aluminium as conductors and wood/steel/concrete for structure) and lower problems with reactive power and voltage issues due to high reactive currents. Low voltage for safe(r) use by end customers. Various structures solve the question:

Structure 1
Transmission grid (275 kV, 400 kV)
Distribution grid (11 kV, 33 kV, 132 kV)
(optional sub-transmission grid with 132 kV if not considered part of the distribution anyway)
Last mile in low voltage (400 V)

Alternative structure 2
High voltage
Medium voltage
Low voltage

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2(a)

Solar power is mostly connected to low voltage (residential and similar size, e.g., kilowatts), larger farms (megawatt level), e.g., those, typically medium voltage due to the higher power. Wind power is mostly connected to medium voltage (rarer to 132 kV). Offshore wind (not expected in the answer, although it is in the lecture notes) is rather different and typically connects to the high-voltage grid. The reasons are the power to keep currents under control (feeders on all levels typically several hundred amperes for all connected units, e.g., the residential buildings along a road) and to keep the voltage stable enough.

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2(b)

Globally mostly loss of inertia/frequency stability; temporal and spatial discrepancy between generation and consumption

2(c)

Locally primarily problems with conventional fusing concept; fault-current supply; voltage band violation, uncontrolled power flow, particularly in meshed grids; dynamic voltage instability (parasitic feedback between feed-back-controlled inverters, problems of current-limited constant-power control)

2(d)

An increasing share of wind and solar power tend to lead to a temporal and spatial divide between supply (generation) and demand (load). Temporally: Supply is when there is wind (stochastic with some temporal patterns) and sun (mostly spring/summer/autumn around noon for the UK). Spatially: Wind preferably in the North (and off-shore) and particularly less inhabited areas (less obstruction due to high buildings), solar mostly residential but most load is around the industrial hubs in the South.

Solar but also most wind (if not doubly-fed induction generators) convert their power with power electronics, which is feed-back controlled to the available power and does not involve frequency (or the voltage amplitude, which is a local problem instead of a global one): loss of inertia as the power is not linked to the actual frequency as in conventional generators (although could be linked to it by feed-back control, so-called virtual inertia) and therefore loss of frequency stability.

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2(d)

Power electronic converters do at present also not include the voltage amplitude into their fast, inner control loops: voltage instability as power and/or current control of wind and solar (inject so much current that the available power of the source is fully injected) without caring about the grid voltage at the point of injection.

Solar and wind do often (still) complicate protection. Conventional protection is based on sufficiently large fault currents (large fault currents of multiple times, e.g., 5-10 times, the rate current are required for AC grids) to make sure that faults can be detected and trip fuses (thermal fuses, which need a conductor to glow and finally burn, or protection contactors with over-current threshold). Solar and wind that constantly inject their rated current do not increase their current as a conventional generator does, which feeds with overload into a fault that reduces the voltage. Thus, they do not contribute to the fault current but only the rated current and the ratio of fault current versus rated current decreases to levels where protection may not trip any more.

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2(d)

Residential solar power, which typically injects into the low-voltage grid, furthermore complicates the protection of the line. The sum of the fuses of all connected houses is typically above the maximum current of the line as historically grid operators assumed that homes would not need the peak power at the same time but might need it from time to time, e.g., when pumps or other devices with a high start-up current are turned on, but assumed that not all homes would need such extra power at the same time. They instead protected the feeder at the medium-voltage-to-low-voltage transformer as they assumed that all power would flow through there anyway so that the rare case that all houses reach their limit and together overload the line can be avoided. However, with sources and sinks in the grid, not all power has to flow through this pivotal fuse anymore and residents can indeed overload and damage the cables.

Solar and wind power injected into a line locally change the voltage amplitude due to the voltage drop across the (complex) grid impedance. In the low-voltage grid, the grid impedance is typically more resistive, in higher voltages typically more inductive. Injected power typically increases the voltage locally. As the generated power is not constant, the voltage amplitude fluctuates and may exceed the tolerated voltage band (typically +/- 10%, for some cases +/- 5%). The grid can adjust voltages as transformers have typically several taps so that the ratio can be changed (for medium voltage and higher in almost all transformers during operation, for low voltage manually and in some problematic places also now during operation), but that is mechanical and slow. They were meant for adjustments for controllable generation and not for rapidly fluctuating renewable power (example of a cloud in solar, which needs only seconds to let the power drop from 100% to almost 0).

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2(d)

Methods to reduce problems

Loss of inertia:
- Keep rotating mass in the grid (dismantled generators that do not generate power anymore but stay connected and rotate for their rotating kinetic energy)
- Virtual inertia: design control loops for power electronic converters that respond to frequency changes and inject more or less power dependent on $\mathrm{d}f/\mathrm{d}t$.
- Increase grid, e.g., more connections to the continental European grid to share available inertia

Temporal discrepancy of supply and demand:
- Storage (battery, hydrostorage)
- Demand-side control (problem that both supply and demand are now stochastic and fluctuating: do not allow the demand to freely control devices, but adjust them to availability, either through forced curtailment or by exposing the demand, which so far had fixed rates to the actual cost of energy at each specific time)
- Have controllable power supplies (gas but can also be renewable, e.g., biomass) that are mostly backup and can ramp up and down to; potentially generate economic case for operators that they are paid for the service of providing capacity too (capacity market) and not only for power delivered (which would be the intra-day or spot market)
- Increase connections to neighbors with better storage potential (e.g., Norway with notably larger hydrostorage potential than the UK)

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2(d)

Voltage band violation

- Reinforce the grid: build lines with lower impedance (particularly lower resistance in low voltage, i.e., larger cross section)
- Let power electronics, particularly solar inverters, adjust their reactive power according to the actual grid voltage (now required for new devices) => inductive voltage drop allows stabilising the voltage, but not so effective in the low-voltage grid due to mostly resistive behaviour
- Medium-to-low-voltage transformers with tap changers that can adjust the transformer ratio during operation (relatively new technology)
- Mesh grids: connect a problematic branch to other nearby feeders and transformers too (though at the risk of circulating currents and unwanted power flow due to the voltage differences)

Fusing problem and dynamic voltage problem

- Let power electronic converters provide over-load current when voltage drops and force generation to not turn themselves off when the grid is unstable (so-called fault-ride-through capability)
- Reinforce the grid: make sure that nearby transformers alone have enough short-circuit capacity (the short-circuit voltage ratio of the transformer should be small for that, e.g., <<10%, which typically means low loss in the transformer too)

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2(e)

Larger grids per se tend to be more robust to many problems as the available reserve of the entire interconnection can be available to local problems (as long as there are no bottle necks inside that would disconnect an area in case of a problem, e.g., Spain in 2025). The absolute reserve, also frequency reserve increases. Redundancies in case of a local failures are higher.
Furthermore, the larger grid spans several time and climate zones => typical temporal demand patterns could average out but also supply (solar peak at noon spreads out over a longer period). Different climate zones and renewable energy potential can complement each other (large solar potential further in the South of Europe, coastal and offshore wind in the UK, hydrostorage potential in Nordic countries

Technology required to connect the UK to surrounding grid is high-voltage direct current (also called HVDC) in cables on the ground of the sea. The high voltage is required to reduce the current for the same power and thus the cross-section of the required cable. DC instead of AC (similar to offshore wind) does not cause reactive power need. The long cable has substantial capacitance, which AC would need to charge and discharge in every cycle. Furthermore, the DC conversion also avoids frequency mismatch with neighboring countries. For background: The UK would have to decide to whom to be closest and connect in synchronous AC. The UK grid could for instance be run synchronously to the French grid through a tight AC connection through the channel. As France is part of the synchronous continental interconnection (which reaches from Ukraine and Turkey to Portugal and from Estonia to Algeria), the reserve would increase manifold and the UK would automatically be synchronous to most of continental Europe – but not Ireland or the Nordic countries, which run their own 50 Hz systems (Ireland and Norther Ireland Synchronous Area, Nordic Synchronous Area).

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2(f)

Advantages: Large offshore wind potential, little obstruction by buildings, vegetation, topology, few complaints by nearby residents, in the UK closer to the large industrial hubs than for instance sites in the North of Scottland

Transport on land:
- AC connection
    - synchronise all wind power already to mains ac and transport it on land
    - Collect power locally at lower voltage and transform it up to a higher voltage for transport
    - Challenge of reactive power in cable (dilemma: water has high dielectric constant, and inductance increases if conductors are spaced out too much and large intrinsic capacitance if not spaced out)
    - Works well if distance to shore is short enough; then similar to on-land wind power
- DC connection
    - For off-shore power far outside, reactive power in cable would be too problematic (current mostly charging up and down cable capacitance)
    - Power collected from wind generators in AC or DC, converted into a shared high-voltage DC for transport
    - Large and also expensive power electronic converter stations, but total cost over lifetime still lower

Question 3: Solar Cell Theory & Systems

(a) **Operational Parameters**

- **Short-circuit current (I_{sc}):** The current flowing through the solar cell when the voltage across it is zero (shorted). It is directly proportional to the incident light intensity.
- **Open-circuit voltage (V_{oc}):** The voltage across the cell when the current is zero (open circuit). It is determined by the bandgap and the ratio of light-generated current to saturation current.
- **Fill Factor (FF):** A measure of the "squareness" of the I-V curve. It is defined as the ratio of the maximum power point ($P_{max} = I_{mp}V_{mp}$) to the product of I_{sc} and V_{oc}.
$$FF = \frac{V_{mp}I_{mp}}{V_{oc}I_{sc}}$$
- **Efficiency (η):** The ratio of electrical power output to optical power input (P_{in}).
$$\eta = \frac{V_{oc}I_{sc}FF}{P_{in}}$$

(b) **Solar Concentrator**

- **Technology: Fresnel Lens**. A thin, lightweight lens with concentric grooves that acts like a large aperture prism, focusing parallel sunlight onto a small focal point where the PV cell is located.
- **Ray Diagram Description:** Parallel rays from the sun enter the flat surface of the lens. The angled grooves refract the light, converging the rays towards a single focal point on the receiver (solar cell).
- **Advantages:**
  - Reduces the area of expensive semiconductor material required (cost-effective for high-efficiency multi-junction cells).
  - Can achieve high concentration ratios (500$\times$ or more).
- **Disadvantages:**
  - Requires precise 2-axis mechanical tracking to keep the lens pointed at the sun.
  - Optical losses in the lens.
  - High temperatures at the focus require active cooling (passive or fluid).
  - Cannot utilise diffuse sunlight (cloudy days).

(c) **Effect of Concentration (1-sun to 10-sun)**

- **Short-circuit current (I_{sc}):** Increases **linearly** with concentration (X). $I_{sc}(X) \approx X \cdot I_{sc}(1)$. This is because the generation rate of electron-hole pairs is directly proportional to the photon flux.
- **Open-circuit voltage (V_{oc}):** Increases **logarithmically**.
$$V_{oc} = kT \ln\left(\frac{I_{sc}}{I_0} + 1\right) \approx kT \ln\left(\frac{X \cdot I_{sc,1-sun}}{I_0}\right) = V_{oc,1-sun} + kT \ln(X)$$
where X is the concentration factor of the sunlight. At room temperature ($kT \approx 26$ mV), V_{oc} increases by approx. 60 mV for a factor of $X = 10$ concentration.
However, if the temperature of the cell rises under concentration, then the V_{oc} will not rise as much as predicted. This is because n_i increases with temperature, increasing the reverse saturation current.

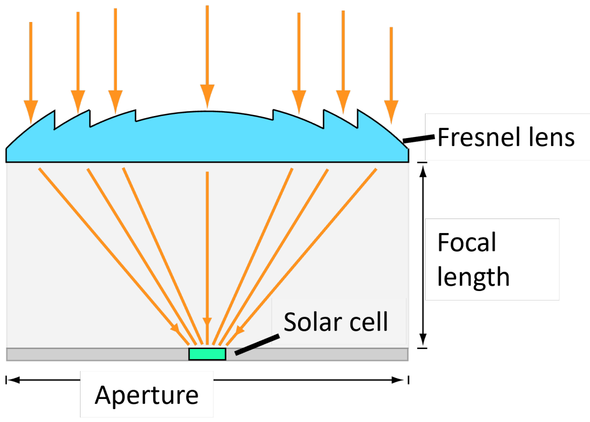


Figure 1: Fresnel lens

(d) **Fill Factor vs. Concentration (1 to 1000-sun)**

- **Initial Increase:** At low to moderate concentrations, FF increases because V_{oc} increases while the ideal diode shape is maintained.
- **Decrease at High Concentration:** As concentration approaches high levels (e.g., 1000-sun), resistive losses dominate. The high current causes significant voltage drop across the series resistance (I^2R_s loss), degrading the "squareness" of the curve and reducing the Fill Factor. In addition, Auger recombination can become more significant, further increasing losses.

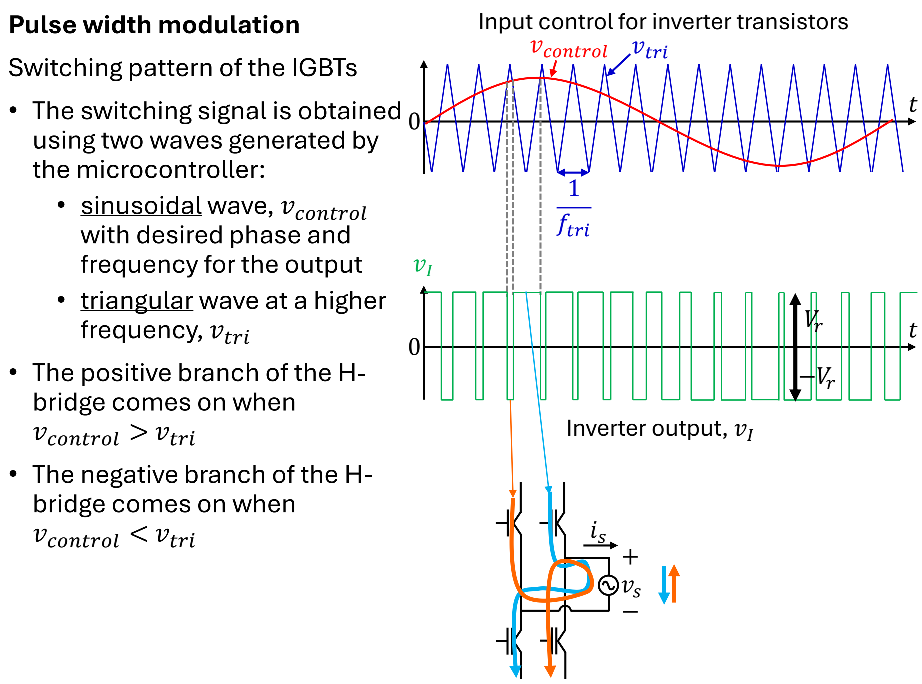
(e) **DC to AC Conversion (PWM)**

- **H-Bridge Inverter:** The DC output is connected to an H-bridge circuit (4 switches, typically IGBTs or MOSFETs).
- **Pulse-Width Modulation (PWM):** The switches are turned on and off at a high frequency (carrier frequency, e.g., 20 kHz).
- **Duty Cycle Control:** To generate a 50 Hz sine wave, the duty cycle of the pulses is varied sinusoidally.
  - Near the zero-crossing of the AC wave, the pulses are narrow (low average voltage).
  - Near the peak of the AC wave, the pulses are wide (high average voltage).
- **Filtering:** An output LC filter smooths the high-frequency pulsed waveform, resulting in a clean sinusoidal voltage and current that matches the grid frequency and phase.

Question 4: Device calculations and design

Constants & Parameters:

- $T = 300$ K $\Rightarrow kT/q \approx 0.02585$ V
- $\epsilon_r = 11.9$, $\epsilon_0 = 8.854 \times 10^{-14}$ F/cm $\Rightarrow \epsilon_0\epsilon_r \approx 1.05 \times 10^{-12}$ F/cm
- $q = 1.602 \times 10^{-19}$ C
- $n_i = 5 \times 10^{10}$ cm^{-3} (Given in paper)



- Area $A = 100 \text{ cm}^2$
- **Bottom (p-type):** $N_A = 10^{17} \text{ cm}^{-3}$, $L_n = 100\,\mu\text{m} = 0.01$ cm, $\tau_n = 1\,\mu\text{s} = 10^{-6}$ s.
- **Top (n-type):** $N_D = 10^{19} \text{ cm}^{-3}$, $L_p = 100$ nm $= 10^{-5}$ cm, $\tau_p = 1$ ns $= 10^{-9}$ s.

(a) **Built-in Potential** (V_{bi})

$$V_{bi} = kT/e \ln\left(\frac{N_A N_D}{n_i^2}\right)$$
$$V_{bi} = 0.02585 \ln\left(\frac{10^{17} \cdot 10^{19}}{(5 \times 10^{10})^2}\right) = 0.02585 \ln\left(\frac{10^{36}}{25 \times 10^{20}}\right)$$
$$V_{bi} = 0.02585 \ln(4 \times 10^{14}) \approx 0.02585 \times 33.62$$
$$V_{bi} \approx 0.87 \text{ V}$$

(b) **Depletion Width & Electric Field** Since $N_D \gg N_A$, the junction is a "one-sided" step junction. The depletion region extends almost entirely into the lightly doped p-side ($W \approx x_p$).

Poisson's equation

$$\frac{\partial^2 V}{\partial x^2} = \frac{\rho}{\epsilon_r \epsilon_0}$$
$$\therefore \frac{\partial E}{\partial x} = \frac{\rho}{\epsilon_r \epsilon_0}$$

On the p-side of the depletion region:

$$\frac{\partial E}{\partial x} = \frac{-eN_A}{\epsilon_r \epsilon_0}$$

On the n-side of the depletion region:

$$\frac{\partial E}{\partial x} = \frac{eN_D}{\epsilon_r \epsilon_0}$$

Integrating with boundary condition that the electric field is zero outside the depletion region:

$$E(x) = \begin{cases} \frac{-eN_A}{\epsilon_r\epsilon_0}(x + x_p) & \text{on p-side } x_p \geq x \geq 0 \\ \frac{eN_D}{\epsilon_r\epsilon_0}(x - x_n) & \text{on n-side } 0 \geq x \geq x_n \end{cases}$$

$$V_{bi} = \int_{-x_p}^{x_n} E(x)\,dx = \frac{e}{2\epsilon_r\epsilon_0}(N_D x_n^2 + N_A x_p^2)$$

Charge neutrality requires $x_n N_D = x_p N_A$. Knowing this, we can eliminate x_n from equation for V_{bi}:

$$V_{bi} = \frac{e}{2\epsilon_r\epsilon_0}(\frac{N_D^2}{N_A}x_p^2 + N_A x_p^2)$$

$$\therefore x_p = \sqrt{\frac{2\epsilon_0\epsilon_r V_{bi}}{e}\frac{N_D}{N_A(N_A + N_D)}}$$

Depletion width on p-side (x_p):

$$x_p \approx \sqrt{\frac{2\epsilon_0\epsilon_r V_{bi}}{eN_A}}$$

$$x_p = \sqrt{\frac{2 \cdot (1.05 \times 10^{-12}) \cdot 0.87}{1.602 \times 10^{-19} \cdot 10^{17}}}$$

$$x_p \approx 1.07 \times 10^{-5} \text{ cm} = 0.107\,\mu\text{m}$$

Peak Electric Field (E_{max}):

$$E_{max} = \frac{qN_A x_p}{\epsilon_s}$$

$$E_{max} = \frac{1.602 \times 10^{-19} \cdot 10^{17} \cdot 1.07 \times 10^{-5}}{1.05 \times 10^{-12}}$$

$$E_{max} \approx 1.63 \times 10^5 \text{ V/cm}$$

(c) **Reverse Saturation Current (I_0)** First, calculate diffusion coefficients $D = L^2/\tau$:

- Electrons (p-side): $D_n = (0.01)^2/10^{-6} = 100 \text{ cm}^2/\text{s}$
- Holes (n-side): $D_p = (10^{-5})^2/10^{-9} = 0.1 \text{ cm}^2/\text{s}$

Current density formula:

$$J_0 = qn_i^2\left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D}\right)$$

p-side term: $\frac{100}{0.01 \cdot 10^{17}} = 10^{-13}$. n-side term: $\frac{0.1}{10^{-5} \cdot 10^{19}} = 10^{-15}$ (Negligible).

$$qn_i^2 = 1.6 \times 10^{-19} \cdot 25 \times 10^{20} = 400 \text{ C/cm}^6$$

$$J_0 \approx 400 \cdot (10^{-13}) = 4 \times 10^{-11} \text{ A/cm}^2$$

$$I_0 = J_0 \cdot A = 4 \times 10^{-11} \cdot 100 = 4 \text{ nA}$$

(d) **Current under Forward Bias (0.55 V)**

(i) **Dark Conditions:**

$$I_{dark} = I_0 \left(e^{eV/kT} - 1\right)$$
$$I_{dark} = 4 \times 10^{-9} \left(e^{0.55/0.02585} - 1\right)$$
$$I_{dark} \approx 4 \times 10^{-9} \cdot (1.73 \times 10^9) \approx 6.9 \text{ A}$$

(ii) **Under Illumination ($G = 10^{17}$ cm^{-3}s^{-1}):** Calculate Photocurrent (I_{ph}). Assuming collection from Depletion Region + Diffusion Lengths. Dominant collection is from the base ($L_n = 100\,\mu$m) because the junction is deep ($2\,\mu$m) and L_p is short ($0.1\,\mu$m).

$$J_{ph} \approx q \cdot G \cdot L_n$$
$$J_{ph} = 1.6 \times 10^{-19} \cdot 10^{17} \cdot 0.01 = 1.6 \times 10^{-4} \text{ A/cm}^2$$
$$I_{ph} = J_{ph} \cdot A = 16 \text{ mA} = 0.016 \text{ A}$$

Net Current (Standard Diode Convention, flowing *into* anode):

$$I_{total} = I_{dark} - I_{ph} = 6.9 - 0.016 \approx 6.88 \text{ A}$$

(iii) **Junction Depth Decreased to 100 nm:**

- **Improved Blue Response:** The current design has a $2\,\mu$m deep "dead layer" where carriers generated by short-wavelength (blue) light recombine before reaching the junction (since $L_p = 100$ nm). Moving the junction to 100 nm eliminates this dead layer, significantly increasing quantum efficiency for high-energy photons.
- **Series Resistance:** A very thin emitter (100 nm) may increase the sheet resistance of the top layer, potentially requiring a finer contact grid to prevent Fill Factor loss.

(iv) **Increasing Shunt Resistance:**

- **Edge Isolation:** A manufacturing step such as **plasma etching** or **laser grooving** is performed around the perimeter of the cell. This removes the conductive n-type layer at the edges, preventing a short circuit between the front emitter and the rear contact, thereby increasing the shunt resistance (R_{sh}).

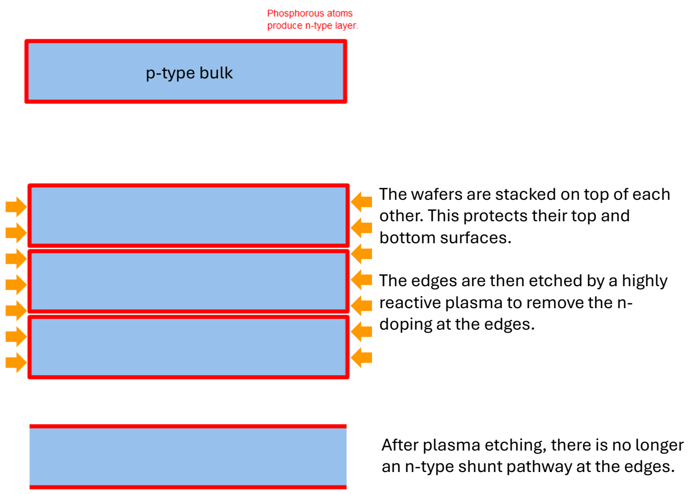


Figure 2: Removal of shunt pathways at edges