EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 6 May 2025 2.00 to 3.40

Module 4B23

OPTICAL FIBRE COMMUNICATION

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachment: 4B23 Optical Fibre Communication formula sheet (2 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Fused fibre couplers are a key component for modern optical fibre communication systems.

(a) The electric field amplitudes $A_1(z)$ and $A_2(z)$ in a 2×2 coupler can be modelled by the following differential equation

$$\frac{d}{dz}\mathbf{A}(z) = -j\kappa \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathbf{A}(z)$$

where κ is the coupling coefficient, $j = \sqrt{-1}$ and $\mathbf{A}(z) = [A_1(z), A_2(z)]^T$.

(i) Obtain the scattering matrix $S(z)$ such that $\mathbf{A}(z) = S(z)\mathbf{A}(0)$. [20%]

(ii) Hence or otherwise show that $S^H S = I$ where S^H denotes the Hermitian (complex transpose) of S and I is the identity matrix. [10%]

(iii) The scattering matrix $S_L = S(z = L)$ may be written in the form

$$S_L = \begin{bmatrix} \sqrt{1-R} & a \\ a & \sqrt{1-R} \end{bmatrix}$$

where R the proportion of power coupled into the other port. Determine an expression for a in terms of R. [15%]

(b) The scattering matrix approach may also be used to analyse a 3×3 coupler where the scattering matrix S_3 is given by

$$S_3 = \begin{bmatrix} \sqrt{1-R} & b & b \\ b & \sqrt{1-R} & b \\ b & b & \sqrt{1-R} \end{bmatrix}$$

where R represents the total proportion of power coupled into the other two ports. Given power conservation requires $S_3^H S_3 = I_3$ where I_3 is the 3x3 identity matrix, determine an expression for b in terms of R and hence deduce the maximum value R may take for a 3×3 coupler. [25%]

(c) An optical access network uses a coupler network to passively distribute the signal to 24 users. Design a coupler network with minimum loss using cascaded symmetric couplers where the loss of a realistic symmetric 2×2 coupler and a realistic symmetric 3×3 coupler is 3.5 dB and 5.8 dB respectively (compared to the ideal of 3 dB and 4.8 dB respectively). Your design should calculate the overall loss of the coupler network and include details (type and number) of the couplers used in the design. [30%]

2 A 1600ZR transceiver transmits 1.6 Tbit s^{-1} using PDM-16QAM at a symbol rate of 240 GBd. Transmitter based digital signal processing is used to create a rectangular Nyquist spectrum, with the chromatic dispersion compensated digitally. The transceiver generates a noise-to-signal ratio (NSR) of -15.2 dB, with a maximum NSR of -13.6 dB required for operation. At the operating wavelength of 1550 nm, the optical fibre has an attenuation of 0.2 dB km^{-1}, a dispersion coefficient of 17 ps nm^{-1} km^{-1} and a nonlinear coefficient of 1.3 W^{-1} km^{-1}.

(a) Calculate the minimum number of taps N_{CD} required to compensate all the chromatic dispersion at the receiver in the 240 GBd signal transmitted over an 120 km link if the digital coherent transceiver uses an oversampling rate of 8/7. [25%]

(b) Assuming appropriate frequency domain implementation, estimate the minimum power consumption required to realise digital chromatic dispersion compensation for the 240 GBd signal assuming the energy required to perform a complex multiply is 0.1 pJ. [25%]

(c) An erbium doped fibre amplifier with a gain of 24 dB and $n_{sp} = 2$ is used at the receiver to compensate the loss of the 120 km link. Estimate the minimum optical power at the transmitter assuming the fibre may be considered linear. [25%]

(d) Finally determine whether the assumption that the optical fibre may be considered linear is valid when transmitting the minimum required optical power. [25%]

3 A step index optical fibre has a core radius $a = 4\ \mu$m with core refractive index $n_{co} = 1.45$ and cladding refractive index $n_{cl} = 1.443$. The attenuation α_{dB} measured in dB km^{-1} as a function of optical wavelength λ may be modelled as

$$\alpha_{dB} = \alpha_0 \frac{A^4}{\lambda^4} + \alpha_0 \exp\left[B\left(\frac{1}{\lambda_{IR}} - \frac{1}{\lambda}\right)\right]$$

where α_0=1 dB km^{-1}, $A = 1\ \mu$m, $B = 50\ \mu$m and $\lambda_{IR} = 1.8\ \mu$m.

(a) State what is meant by the term cut-off wavelength with regards to a single mode optical fibre and hence calculate the cut-off wavelength of the step index optical fibre with the given parameters. [10%]

(b) The signal to noise ratio is 27 dB over the entire C-U band (1530 nm to 1675 nm). Estimate the total data capacity of the C-U band, stating any assumptions made. [20%]

(c) The attenuation follows the equation above. Estimate the minimum value of attenuation and the minimum attenuation wavelength. [20%]

(d) The maximum power in the optical fibre is 1 W. Assuming nonlinear effects may be neglected, estimate to one significant figure the capacity of the C-U band operating over 25 km of optical fibre. [30%]

(e) It is now proposed to also use the O-band extending from 1260 nm to 1360 nm. The total maximum power in the optical fibre remains as 1 W and nonlinear effects may be neglected. Estimate to one significant figure the increased capacity that can be obtained by using the O-band in conjunction with the C-U band operating over 25 km of optical fibre. [20%]

END OF PAPER

Formula	**Notes**
$\langle S \rangle = \frac{1}{2}\frac{n}{\eta_0}\lvert E_0 \rvert^2$	$\langle S \rangle$ - time averaged Poynting Vector, E_0 - complex electric field, $\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$, n is refractive index.
$P(z) = P(0)\exp(-\alpha z)$	α in nepers/km, related to loss in terms of α_{dB}, the loss in dB/km via $\alpha \approx 0.23\alpha_{dB}$
$L_{eff} = [1 - \exp(-\alpha L)]/\alpha$	Effective length L_{eff} associated with L and loss α
$v_p = \frac{\omega_0}{\beta} = \frac{c}{n}$	Phase velocity v_p, for electric field $E = E_0 e^{j(\omega_0 t - \beta z)}$, where $\beta = k_0 n$ and n is refractive index and $k_0 = 2\pi/\lambda$ (note λ always refers to the wavelength in vacuo). $c = 3 \times 10^8$ m/s
$v_g = \frac{d\omega}{d\beta}$	Group velocity v_g of a modulated wave $e^{j(\omega t - \beta z)}$
$n_g = c/v_g$	Group refractive index $n_g = n + \omega\frac{dn}{d\omega} = n - \lambda\frac{dn}{d\lambda}$
$D = -\frac{2\pi c}{\lambda^2}\beta_2 = -\frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$	Dispersion coefficient D: $\beta_2 = \frac{d^2\beta}{d\omega^2}$. For λ =1550 nm $D\ (ps/nm/km) = -0.78 \times \beta_2 (ps^2/km)$
$\Delta t = D\Delta\lambda L$	Dispersion Δt, with dispersion coefficient D, spectral width $\Delta\lambda$, over a distance L. For 1550 nm 100 GHz spectrum corresponds to 0.8 nm
$V = k_0 a\sqrt{n_{co}^2 - n_{cl}^2}$	Normalised wavenumber for step index fibre. Core radius a, core refractive index n_{co}, cladding refractive index n_{cl}
$J_{m-1}(V) = 0$	Cutoff criterion for the modes. LP_{mn} is n^{th} solution of $J_{m-1}(V) = 0$ where J_m is the m^{th} order Bessel function of the first kind
$F(r) = \exp\left(-\frac{r^2}{r_0^2}\right)$	Gaussian approximation for fundamental mode with mode field radius: $r_0^2 = \frac{a^2}{\ln V}$
$\eta = \frac{2\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2}\exp\left(-\frac{\Delta^2}{\sigma_1^2 + \sigma_2^2}\right)$	Overlap integral between two normalised Gaussian fields separated by Δ with the mode field radii σ_1 and σ_2

First n zeros for $J_k(x)$

k \ n	1	2	3	4	5
0	2.405	5.520	8.654	11.792	14.931
1	3.832	7.016	10.173	13.324	16.471
2	5.136	8.417	11.620	14.796	17.960
3	6.380	9.761	13.015	16.223	19.409
4	7.588	11.065	14.373	17.616	20.827
5	8.771	12.339	15.700	18.980	22.218

Formula	**Notes**
$S_{3dB} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & j\\ j & 1\end{bmatrix}$	Scattering matrix of a 3 dB coupler
$i = RP = R\|A\|^2$	Current in a photodiode, responsivity R, electric field amplitude A, optical power $P = \|A\|^2$
$\phi = \gamma P$	Kerr nonlinear phase shift: $\gamma = n_2 k_0/A_{eff}$ where for optical fibres it can be assumed $n_2 = 2.6 \times 10^{-20}$ m²/W
$\frac{\partial A}{\partial z} = j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}$	Effect of dispersion in retarded frame of reference. With loss and nonlinearity becomes the NLSE $\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - j\gamma\|A\|^2 A$
$P(\Delta\tau > x) \approx \frac{4}{\pi}\frac{x}{\langle\Delta\tau\rangle}\exp\left(-\frac{4x^2}{\pi\langle\Delta\tau\rangle^2}\right)$	Probability that with mean DGD $\langle\Delta\tau\rangle$ the instantaneous DGD $\Delta\tau$ exceeds x
$N_{NLI} = C_{NLI}G_{TX}^3$	Nonlinear noise power density for input PSD of G_{TX} with $C_{NLI} = \frac{8\gamma^2 L_{eff}^2\alpha}{27\pi\|\beta_2\|}\ln\left(\frac{\|\beta_2\|}{\alpha}\pi^2 B^2\right)$
$N_{ASE} = 10^{NF/10}h\nu(G-1)$	Power spectral density for ASE amplifier with gain G and noise figure NF. $h = 6.634 \times 10^{-34}$ Js and for $\lambda \approx$1550 nm, $h\nu \approx 1.3 \times 10^{-19}$ J = 0.8 eV.
$G_{opt} = \sqrt[3]{\frac{N_{ASE}}{2C_{NLI}}}$	Optimum power spectral density
$N_q = h\nu P$	PSD for quantum noise
$N_{cm} = \frac{N\log_2(N) + N}{N - N_f + 1}$	Number of complex multiplications per sample for overlap and save implementation of a filter of length N_f using N point FFT (that in turn requires $0.5N\log_2 N$ complex multiplications)
$\boldsymbol{h} \coloneqq \boldsymbol{h} - \mu\frac{\partial\|\epsilon\|^2}{\partial\boldsymbol{h}^*}$	Stochastic gradient update for taps $\boldsymbol{h}$ with error ϵ and convergence parameter μ
$C = 1 - H_2(p_b)$	Capacity of binary symmetric channel where $H_2(p_b) = -(1-p_b)\log_2(1-p_b) - p_b\log_2 p_b$
$C = B\log_2(1 + SNR)$	Shannon capacity (for one polarisation), with bandwidth B and signal to noise ratio SNR