Version SJS/3

EGT3
ENGINEERING TRIPOS PART IIB

Monday 4 May 2026   14.00 to 15.40

Module 4B23

OPTICAL FIBRE COMMUNICATION

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachment: 4B23 Optical Fibre Communication formula sheet (2 pages)

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 EX2500 optical fibre is an ultra-low loss optical fibre that is used for submarine systems. In the C-band EX2500 optical fibre has an attenuation of 0.15 dB km^{-1}, an effective area of 125 μm^2, a chromatic dispersion of 22 ps nm^{-1}km^{-1} and a polarisation mode dispersion (PMD) coefficient of 0.2 ps km$^{-1/2}$. 11,000 km of EX2500 optical fibre is used in an optically amplified transpacific link. The link employs wavelength division multiplexing (WDM) with 20 WDM channels operating in the C-band. Each WDM channel transmits 1.0 Tbit s^{-1} with a symbol rate of 200 GBd. Every 100 km the optical signal is amplified using an erbium doped fibre amplifier (EDFA) with $n_{sp} = 1.5$ and gain of 15 dB. The transceiver employs digital signal processing (DSP) to create a rectangular Nyquist spectrum, with the chromatic dispersion compensated digitally at the receiver. A rate of 225 GSa s^{-1} is used in both the receiver analogue to digital converters and the transmitter digital to analogue converters.

(a) Explain why the fibre parameters of the EX2500 optical fibre make it ideal for submarine systems. [10%]

(b) Calculate the minimum number of taps N_{CD} that should be included in the transceiver to compensate the chromatic dispersion for any WDM channel when the 200 GBd signal is transmitted over the 11,000 km link. [20%]

(c) Assuming appropriate frequency domain implementation, calculate the optimum FFT size and hence estimate the minimum power consumption required to realise digital chromatic dispersion compensation for the 200 GBd signal given the energy required to perform a complex multiply is 0.3 pJ. [30%]

(d) In addition to compensating chromatic dispersion, PMD is compensated digitally. Estimate the number of complex filter taps required in the adaptive filter if the outage probability is to be less than 10^{-12}. [20%]

(e) Finally, carefully explaining the rationale for any assumptions made, estimate the minimum required optical output power of the EDFAs used in the link. [20%]

2 (a) The electric field amplitudes $A_1(z)$ and $A_2(z)$ in a 2×2 coupler can be modelled by the following differential equation

$$\frac{d}{dz}\mathbf{A}(z) = \begin{bmatrix} 0 & -j\kappa \\ -j\kappa & 0 \end{bmatrix} \mathbf{A}(z)$$

where κ is the coupling coefficient, $j = \sqrt{-1}$ and $\mathbf{A}(z) = [A_1(z), A_2(z)]^T$.

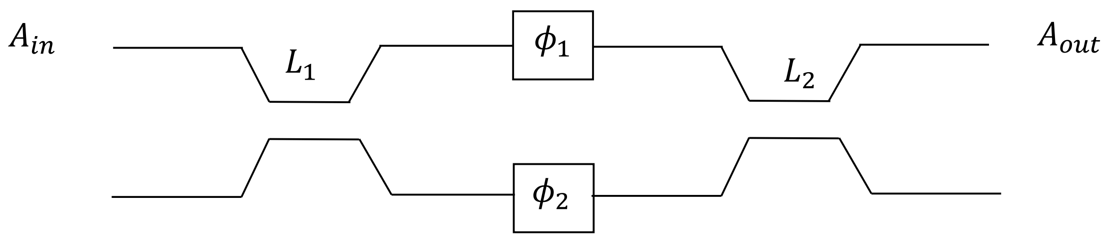
(i) Show that the solution is given by

$$\mathbf{A}(z) = \begin{bmatrix} \cos(\kappa z) & -j\sin(\kappa z) \\ -j\sin(\kappa z) & \cos(\kappa z) \end{bmatrix} \mathbf{A}(0)$$

[20%]

(ii) Calculate the minimum coupling length if the optical power from one input is split equally between the two outputs. [15%]

(b) A Mach-Zehnder modulator illustrated below is formed from two couplers, which have coupling lengths of L_1 and L_2 respectively, separated by two phase elements ϕ_1 and ϕ_2 modelled as $\exp(j\phi_1)$ and $\exp(j\phi_2)$ respectively.



(i) Show that the output of the modulator is given by
$A_{out} = A_{in}\,[\cos(\kappa L_1)\cos(\kappa L_2)\exp(j\phi_1) - \sin(\kappa L_1)\sin(\kappa L_2)\exp(j\phi_2)]$ [20%]

(ii) The phase shifts ϕ_1 and ϕ_2 can be written as $\phi_1 = \phi_0 + \pi V/V_\pi$ and $\phi_2 = \phi_0 - \pi V/V_\pi$ where ϕ_0 is a bias phase, V is an applied voltage and V_π is the voltage required to achieve a phase shift of π radians. Determine appropriate values of ϕ_0, L_1 and L_2 such that $A_{out} = A_{in}\sin(\pi V/V_\pi)$ [20%]

(iii) Finally, it is proposed to combine this modulator with a phase modulator to enable an arbitrary complex signal to be generated. Sketch how this might be realised and discuss the relative advantages and disadvantages relative to a conventional Cartesian modulator formed from two nested Mach-Zehnder modulators. [25%]

3 Nonlinear effects in optical fibres present a key limitation for long-haul optical fibre communication systems. These nonlinear effects can be modelled as a nonlinear interference noise term which when combined with the amplified spontaneous emission (ASE) from the optical amplifiers.

(a) In an optical fibre communication system the noise-to-signal ratio (NSR) as a function of power P can be written as

$$NSR = NSR_0 + \frac{a}{P} + bP^2$$

where NSR_0 represents the internal (modem) noise, a represents the additive linear noise and b represents the nonlinear interactions that occur in the fibre.

(i) Determine the optimum launch power P_{opt} and the corresponding minimum noise-to-signal ratio NSR_{min} in terms of NSR_0, a and b, giving your answer in the simplest form. [20%]

(ii) Sketch $10\log_{10}(NSR)$ as a function of $10\log_{10}(P)$, highlighting salient features including the asymptotic behaviour (when $P \ll P_{opt}$ and $P \gg P_{opt}$ respectively). [15%]

(b) A system transmits 25 wavelength division multiplexed (WDM) channels in the C-band, providing a total capacity of 30 Tbit s^{-1} over 1000 km of single mode optical fibre. Each WDM channel transmits 1.2 Tbit s^{-1} at 200 GBd, with the transmitter employing digital signal processing to create a rectangular Nyquist spectrum. The transceiver generates an NSR of -20.0 dB, with a maximum NSR of -12.8 dB required for operation. The signal is amplified every 100 km using an erbium doped fibre amplifier (EDFA) with gain 20 dB, n_{sp} = 2 and flat gain window of 5 THz.
In the C-band, the optical fibre has an attenuation of 0.2 dB km^{-1}, dispersion coefficient of 17 ps nm^{-1}km^{-1} and nonlinear coefficient of 1.3 W^{-1}km^{-1}.

(i) Assuming that a 200 GHz WDM grid is used, calculate in dBm the optimum launch power per WDM channel. [30%]

(ii) The overall performance is dictated by launch power per WDM channel. Assuming all WDM channels have the same power, estimate the range of WDM channel launch power that will allow for acceptable operation. [35%]

END OF PAPER

Formula	**Notes**
$\langle S\rangle = \frac{1}{2}\frac{n}{\eta_0}\lvert E_0\rvert^2$	$\langle S\rangle$ - time averaged Poynting Vector, E_0 - complex electric field, $\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$, n is refractive index.
$P(z) = P(0)\exp(-\alpha z)$	α in nepers/km, related to loss in terms of α_{dB}, the loss in dB/km via $\alpha \approx 0.23\alpha_{dB}$
$L_{eff} = [1-\exp(-\alpha L)]/\alpha$	Effective length L_{eff} associated with L and loss α
$v_p = \frac{\omega_0}{\beta} = \frac{c}{n}$	Phase velocity v_p, for electric field $E = E_0 e^{j(\omega_0 t-\beta z)}$, where $\beta = k_0 n$ and n is refractive index and $k_0 = 2\pi/\lambda$ (note λ always refers to the wavelength in vacuo). $c = 3\times10^8$ ms^{-1}
$v_g = \frac{d\omega}{d\beta}$	Group velocity v_g of a modulated wave $e^{j(\omega t-\beta z)}$
$n_g = c/v_g$	Group refractive index $n_g = n + \omega\frac{dn}{d\omega} = n - \lambda\frac{dn}{d\lambda}$
$D = -\frac{2\pi c}{\lambda^2}\beta_2 = -\frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$	Dispersion coefficient D: $\beta_2 = \frac{d^2\beta}{d\omega^2}$. For λ =1550 nm $D\ (\mathrm{ps\ nm^{-1}km^{-1}}) = -0.78\times\beta_2(\mathrm{ps^2\ km^{-1}})$
$\Delta t = D\Delta\lambda L$	Dispersion Δt, with dispersion coefficient D, spectral width $\Delta\lambda$, over a distance L. For 1550 nm 100 GHz spectrum corresponds to 0.8 nm
$V = k_0 a\sqrt{n_{co}^2 - n_{cl}^2}$	Normalised wavenumber for step index fibre. Core radius a, core refractive index n_{co}, cladding refractive index n_{cl}
$J_{m-1}(V) = 0$	Cutoff criterion for the modes. LP_{mn} is n^{th} solution of $J_{m-1}(V) = 0$ where J_m is the m^{th} order Bessel function of the first kind
$F(r) = \exp\left(-\frac{r^2}{r_0^2}\right)$	Gaussian approximation for fundamental mode with mode field radius: $r_0^2 = \frac{a^2}{\ln V}$
$\eta = \frac{2\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}\exp\left(-\frac{\Delta^2}{\sigma_1^2+\sigma_2^2}\right)$	Overlap integral between two normalised Gaussian fields separated by Δ with the mode field radii σ_1 and σ_2

First n zeros for $J_k(x)$

k \ n	1	2	3	4	5
0	2.405	5.520	8.654	11.792	14.931
1	3.832	7.016	10.173	13.324	16.471
2	5.136	8.417	11.620	14.796	17.960
3	6.380	9.761	13.015	16.223	19.409
4	7.588	11.065	14.373	17.616	20.827
5	8.771	12.339	15.700	18.980	22.218

Formula	**Notes**
$S_{3dB} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$	Scattering matrix of a 3 dB coupler
$i = RP = R\|A\|^2$	Current in a photodiode, responsivity R, electric field amplitude A, optical power $P = \|A\|^2$
$\phi = \gamma P$	Kerr nonlinear phase shift: $\gamma = n_2 k_0 / A_{eff}$ where for optical fibres it can be assumed $n_2 = 2.6 \times 10^{-20}$ m^{2}W^{-1}
$\frac{\partial A}{\partial z} = j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}$	Effect of dispersion in retarded frame of reference. With loss and nonlinearity becomes the NLSE $\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - j\gamma\|A\|^2 A$
$P(\Delta\tau > x) \approx \frac{4}{\pi}\frac{x}{\langle\Delta\tau\rangle}\exp\left(-\frac{4x^2}{\pi\langle\Delta\tau\rangle^2}\right)$	Probability that with mean DGD $\langle\Delta\tau\rangle$ the instantaneous DGD $\Delta\tau$ exceeds x
$N_{NLI} = C_{NLI} G_{TX}^3$	Nonlinear noise power density for input PSD of G_{TX} with $C_{NLI} = \frac{8\gamma^2 L_{eff}^2 \alpha}{27\pi\|\beta_2\|}\ln\left(\frac{\|\beta_2\|}{\alpha}\pi^2 B^2\right)$
$N_{ASE} = 2n_{sp}h\nu(G-1)$	Power spectral density of ASE amplifier with gain G and population-inversion factor n_{sp}. Noise figure $NF = 10\log_{10}(2n_{sp})$. $h = 6.634 \times 10^{-34}$ Js giving for $\lambda \approx$1550 nm, $h\nu \approx 1.3 \times 10^{-19}$ J = 0.8 eV.
$G_{opt} = \sqrt[3]{\frac{N_{ASE}}{2C_{NLI}}}$	Optimum power spectral density
$N_q = h\nu P$	Power spectral density for quantum noise
$N_{cm} = \frac{N\log_2(N) + N}{N - N_f + 1}$	Number of complex multiplications per sample for overlap and save implementation of a filter of length N_f using N point FFT (that in turn requires $0.5N\log_2 N$ complex multiplications)
$C = 1 - H_2(p_b)$	Capacity of binary symmetric channel where $H_2(p_b) = -(1-p_b)\log_2(1-p_b) - p_b\log_2 p_b$
$C = B\log_2(1 + SNR)$	Shannon capacity (for one polarisation), with bandwidth B and signal-to-noise ratio SNR
$NSR_{total} = NSR_0 + \sum_{i=1}^{N} NSR_i$	Additive nature of noise-to-signal ratio (in linear units – not decibels) over N spans with internal modem noise of NSR_0