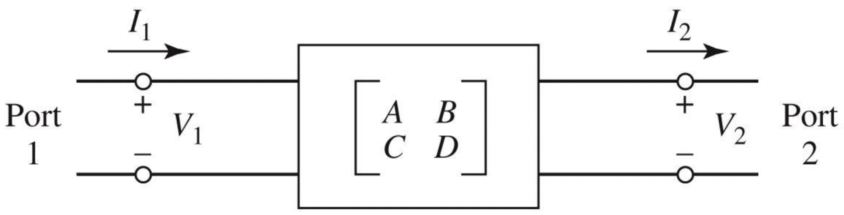
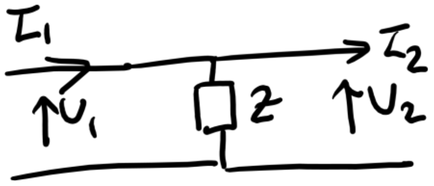
1. a) i)



(a)



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = V_2 \rightarrow A = 0, B = 0$$

$$I_1 = I_2 + \frac{V_2}{Z}$$

$$D = 1, C = \frac{1}{Z}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix}$$

[20%]

ii) Convert S params to ABCD then we can multiply matrixes and convert back.

ABCD parameters of the provided device $\begin{bmatrix} 1 & 100 \\ -0.04 & -3 \end{bmatrix}$

Combine ABCD with the shunt resistor by multiplying matrices.

$$\begin{bmatrix} 1 & 0 \\ 1/25 & 1 \end{bmatrix} \begin{bmatrix} 1 & 100 \\ -0.04 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 100 \\ 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

Therefore new S_{21}=0.5, original S_{21}=-1, so the gain is reduced by 6dB (remember that S parameters are voltage ratios)

[35%]

iii) Original S matrix has gain and is reciprocal as $S_{21} = S_{12}$

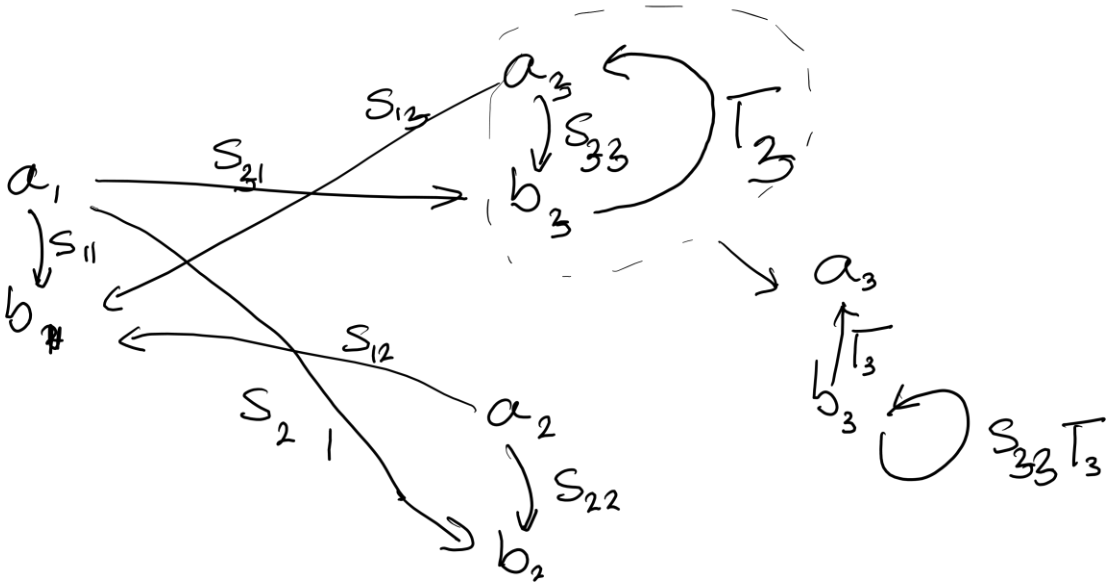
Resulting S matrix is reciprocal. But is lossy as $|S_{11}|^2 + |S_{21}|^2 = 0.5$ which isn't 1

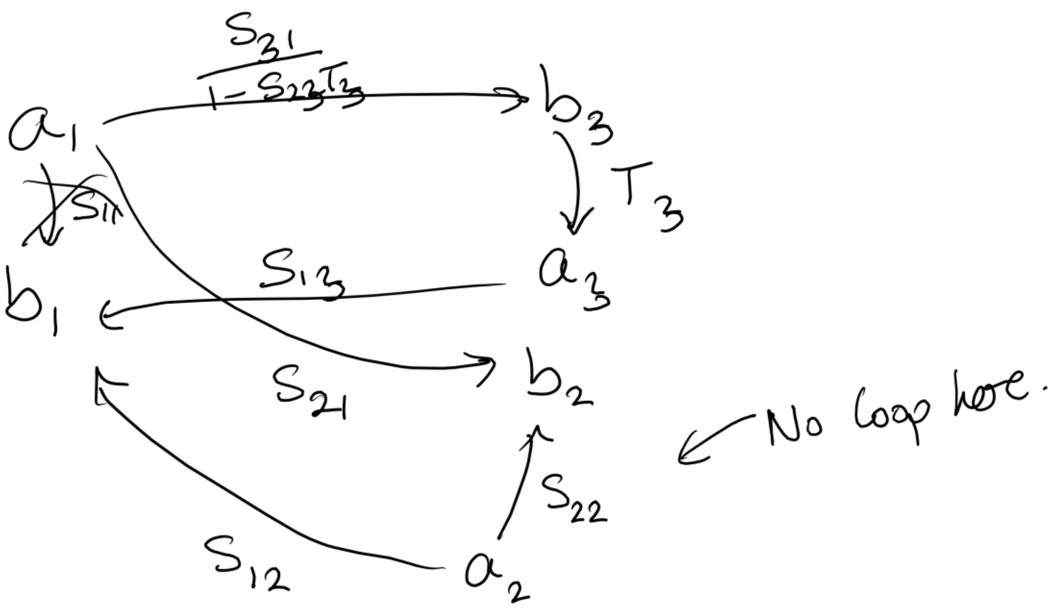
[15%]

b) i) S_{11} is the Γ_{in} of port 1

Can see that since S_{23} *and* S_{32} are both zero, the termination on port 3 has no impact on port 1-2 transmission so S_{21} remains $1/\sqrt{2}$

For an open $\Gamma_3 = 1$





$$\Gamma_{in} = \left(\frac{S_{31}}{1 - S_{23}\Gamma_3}\right)\Gamma_3 S_{31} = 1.707$$

[20%]

ii)

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-\Gamma_S S_{11}|^2(1-|\Gamma_{out}|^2)}$$

We can assume $\Gamma_s = 0$ for the normal case of a source at Z_0 so

$$G_A = \frac{|S_{21}|^2}{(1-|\Gamma_{out}|^2)}$$

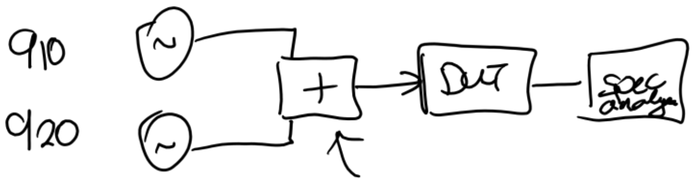
And $\Gamma_{out} = \frac{b_2}{a_2}$, so

$$G_A = \frac{|S_{21}|^2}{(1-\left|\frac{b_2}{a_2}\right|^2)} = \frac{1}{2(1-\left|\frac{b_2}{a_2}\right|^2)}$$

[10%]

A very popular question done by all candidates. Almost everyone could find the ABCD parameters by inspection and realised that in (ii) s-parameters could be converted to ABCD to cascade with matrix multiplications. (b) separated the candidate more with the signal flow diagrams. A common minor mistake was getting the sign of the reflection co-efficient of the open wrong. Many had over complicated diagrams due to not eliminating paths with s-parameters of zero. The final part really separated those who understood available gain.

2. a) (i)



Need a high isolation in the splitter/ combiner to ensure that no intermodulation of the sources occurs.

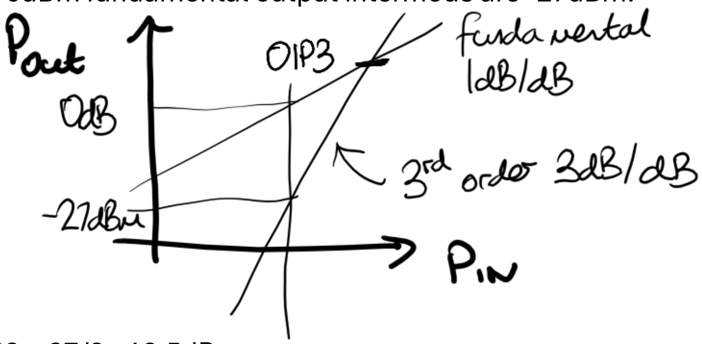
910 and 915MHz scale as 1dB per dB input so are the fundamental tones, arise due to the inputs and linear gain.

905 and 920MHz are the 3rd order products 2F1-F2 and 2F2-F1. They scale as 3dB per dB input. Arise due to the 3rd (or higher odd) order non linearities. Since scaling is 3dB per dB exactly only 3rd order is present here.

[15%]

ii) Gain = -20dB

For 0dBm fundamental output intermods are -27dBm.



OIP3 = 27/2 =13.5dBm

(Looking at the triangles, a 13.5dB increase in the fundamental will result in a 13.5*3 = 40.5dB increase in 3rd harmonics -27+40.5= 13.5dBm, so the fundamental is equal to the intermods at 13.5dBm output)

IIP3 = OIP3-gain = 33.5 dBm

[15%]

b) (i) T a will scale by the proportion of the temperatures seen, but then needs adjusting for additional thermal noise of the antenna due to the efficiency.

$$T_a = \mu * T_B + (1-\mu)T_{phys}$$

$$T_B = (0.88 \times 30) + (0.12 \times 300) = 62.4K$$

$$T_a = 86.16K$$

[15%]

(ii) Feeder: $T_e = (L-1)T = 453.6K$

Preamp $T = 175K$ (given)

Main amp $T = 275K$

Also need gains. $G_{feed} = 0.4, G_{pre} = 5.01, G_{main} = 100$

$$T_{sys} = T_{ant} + T_{feed} + \frac{1}{G_{feed}} T_{pre} + \frac{1}{G_{feed} G_{pre}} T_{main}$$

$$T_{sys} = 1114.5K$$

$$F = 1 + \frac{T_{sys}}{T_0}$$

$$F = 4.8 = 6.8\ dB$$

$$N_o = kGB(T_{sys})$$

$$N_o = 3.0687 \times 10^{-12}\, W = -85.13 dBm\ (in\ 1MHz)$$

[30%]

iii) $OIP3_{pre} = IIP3_{pre} + 7 = -23dBm$

$$OIP3_{main} = IIP3_{main} + 20 = -5dBm$$

$$OIP_3 = \left[\frac{1}{G_2\ (OIP_3')} + \frac{1}{(OIP_3'')}\right]^{-1}$$

$$OIP3_{cas} = \left(\frac{1}{100 \times 10^{-2.3}} + \frac{1}{10^{-0.5}}\right) = -7.12\ dBm$$

(this is worst case as the intermods are adding in phase)

$$SFDR = \frac{2}{3}(OIP3 - N_o) = \frac{2}{3}\left(-7.12 - (-85.13 - 60)\right)$$

$$SFDR = 92 dBm/Hz^{\frac{2}{3}}$$

Max input with no intermods means that the intermod products are at the noise floor (-85dBm).

This is 77.88 dB below OIP3, fundamentals at this point are 77.88/2 = 38.9 dB below OIP3 or -7.12 – 38.9 = -46.02dBm at the output of the amplifier chain.

At the antenna output this corresponds to :

-46.02 – 23dB = -69.02dBm

[25%]

3.a) Transducer gain

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2}$$

Assuming unilteral max gain when $S_{11} = \Gamma_{in}, \Gamma_s = S_{11}^*, \Gamma_L = S_{22}^*$

This gives $G_{TU,max} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2}$=6.19

Clearly this device isn't unilateral S_{12} isn't that much smaller than S_{21}, so in practice would want a simultaneous conjugate match.

Using unilateral figure of merit.

$$u \equiv \frac{|S_{11}S_{12}S_{21}S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = 0.28$$

$$\frac{1}{(1+u)^2} < \frac{G_{TU}}{G_{TU,max}} < \frac{1}{(1-u)^2}$$

Shows the relative error in transducer gain is between 0.61 and 1.92, so the assumption quite poor given that the gain is also fairly low (gain is between 3.72 and 11.88)

[20%]

b) We have:

$$\Gamma_s = \frac{Z_S - Z_0}{Z_S + Z_0} = -0.25$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_l + Z_0} = 0.1667$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}\Gamma_L S_{21}}{1-S_{22}\Gamma_L} = 0.72\angle 45.9°$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}\Gamma_s S_{21}}{1-S_{11}\Gamma_s} = 0.51\angle 39.8°$$

Results in Power gain 6.6dB power delivered into the 70 ohm / power taken from source and not returned.

Available gain 2.84dB power available at output in none reflected / available power from source.

Trans gain 2.01dB power delivered/ power available

Unilateral trans gain 1.97dB (found by setting S12 =0) so the actual error is only 0.04dB which is very small.

[35%]

c) Max unambiguous range is PRI*c/2 = 150m

$$S_t = \frac{P_t G}{4\pi R^2}, \sigma = \frac{P_s}{s_t}$$

$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

So $\sigma_{min} = \frac{1\times10^{-12}\times(4\pi)^3\times\left(1.5\times10^3\right)^4}{1\times100^2\times\frac{3\times10^8}{20\times10^9}} = 4.5 \times 10^{-3}\ m^2$

[25%]

d) Power received by antenna into a matched load

$$P_l = S_t A_e = \frac{S_t \lambda^2 G}{4\pi}$$

Some of this power is reflected back to the antenna, some is dissipated in the resistor.

$$\Gamma = \frac{25 - 50}{25 + 50} = \frac{1}{3}$$

$$P_s = |\Gamma|^2 P_l G$$

$$\sigma = \frac{P_s}{S_t} = \frac{\lambda^2 G^2}{4\pi} |\Gamma|^2 = 3.15 \times 10^{-5}\ m^2$$

[20%]

4. i)

Max gain with conjugate match $\Gamma_S = S_{11}^* = 0.86\angle + 78°, \Gamma_L = S_{22} = 0.74\angle + 35°$. Can assume unilateral since S_12 is so small.

K = 0.67, $|\Delta| = 0.65$

This means that it is not unconditionally stable so circles have to be plotted.

Using datasheet formulae.

$$C_l = 1.61\angle 59°, R_l = 0.76$$

$$C_s = 1.21\angle 88.2°, R_s = 0.29$$

Since |S_11| and |S_22| both less than 1 stable area is outside circles.

Can see that we are well away from the proposed operating point (S_{11}^* and S_{22}^*) though so we are fine.

Unilateral transducer gain is 16.78dB.

[20%]

ii) $N = \frac{|\Gamma_S - \Gamma_{opt}|^2}{1-|\Gamma_S|^2} = \frac{F-F_{min}}{4R_N/Z_0}\left|1+\Gamma_{opt}\right|^2$

N=0.7105

$$F = 3.2dB$$

[15%]

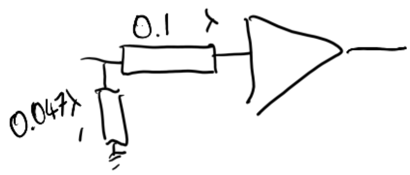
iii) See Smith chart

Want to produce a network which presents S_11* to the amplifier. This means that we need a stub to produce a reflection and then a length of line to modify the impedance seen by the amplifier.

Start at S_11*

Flip to admittance. Now add transmission line to meet unit R circle. This is towards the generator (as we are moving away from the stub). Required length is .108-.207 = 0.1λ

At this point we have a susceptance of -3.3j. This is the susceptance we need to generate with the stub. Now work from RHS (short in admittance) and wavelengths towards source length is 0.297 – 0.25 = 0.047λ

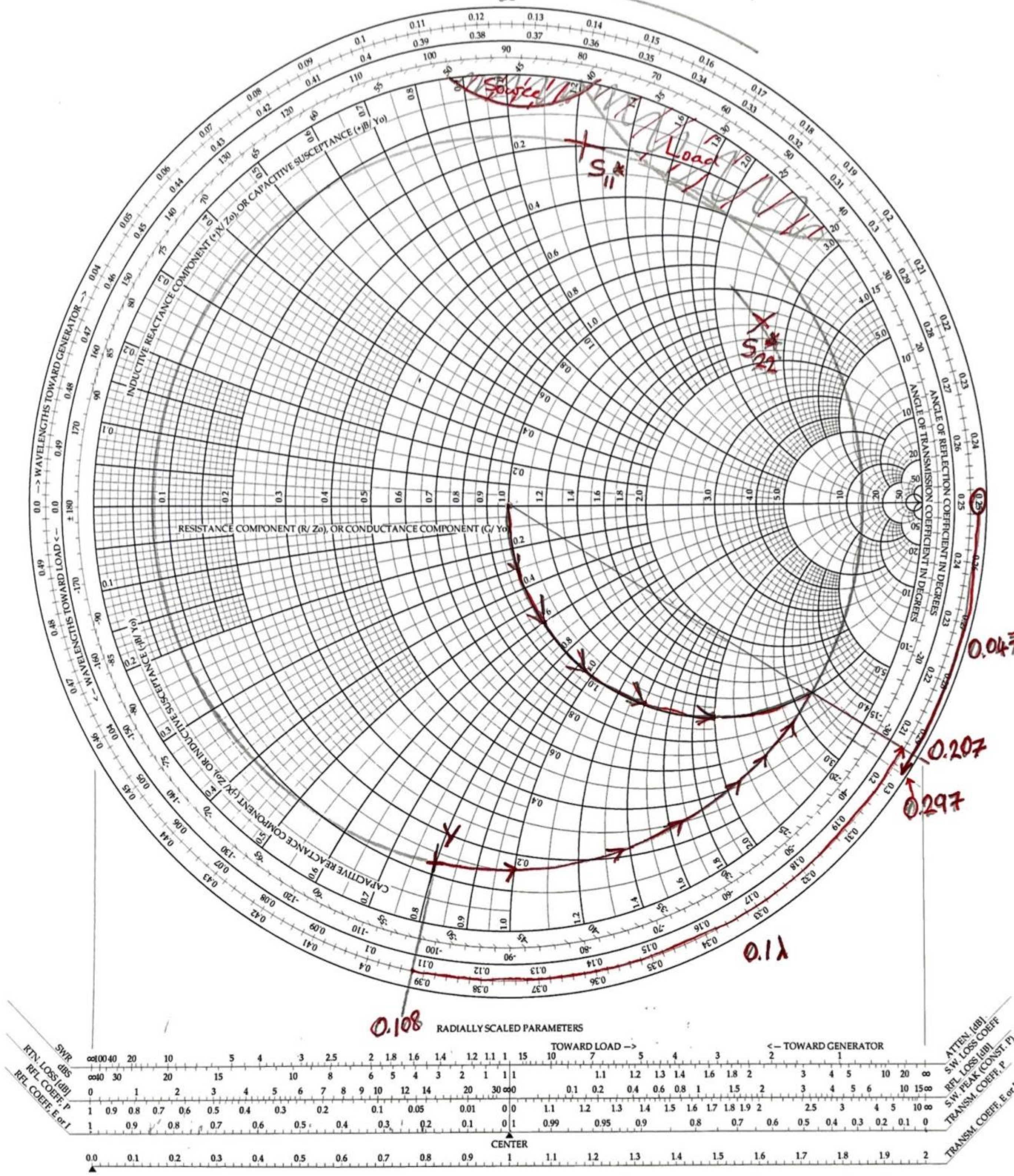


[25%]

Version MJC/3
EGT3
ENGINEERING TRIPOS PART IIB
Friday 2 May 2025, Module 4B24, Question 4

Candidate No.

Smith Chart for Question 4 to be detached and handed in with script



b) (i) we are told this is passive and clearly since S_11 and S_22 are non zero we have mismatch.

If we assume that we are at T_0 then the noise figure is equal to the available gain.

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)}$$

With no matching networks we have $\Gamma_S = 0$ $\Gamma_{out} = S_{2,2}$

$$G_A = \frac{|S_{21}|^2}{(1-|\Gamma_{out}|^2)}$$

$G_a = -9.21dB$ so NF = 9.21dB

[25%]

ii) minimum noise figure when the gain is maximised. Can't do this easily as it isn't unilateral, but we only need to improve. This means we can take the unilateral assumption.

Setting $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$.

Results in: $G_{max} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2}$

Gain with matching is now -9.16dB so noise figure has reduced to 9.16dB

[15%]

Examiners comments:

Q1:

A very popular question done by all candidates. Almost everyone could find the ABCD parameters by inspection and realised that in (ii) s-parameters could be converted to ABCD to cascade with matrix multiplications. (b) separated the candidate more with the signal flow diagrams. A common minor mistake was getting the sign of the reflection co-efficient of the open wrong. Many had over complicated diagrams due to not eliminating paths with s-parameters of zero. The final part really separated those who understood available gain.

Q2:

Not very popular (20/28 candidates). Most could explain which products were intermodulation and which were fundamentals, but measurement setups for the 2-tone test were variable. In (b) a common error was to miss noise at the antenna arising from the inefficiency of the antenna and to then attempt to solve with noise figures (which can't be done since the input noise isn't kT_0B. In the final part on SFDR many could find the SFDR but struggled with the worst case OIP3 and scaling to the receiver bandwidth.

Q3:

Attempted by 22 of 28 candidates. Most could find the transducer gain under conjugate matching with the unilateral assumption. The validity of the assumption required the unilateral figure of merit which some calculated in (b) (iv) instead. In the final part of (b), many did not realise that with the provided source and load impedances there is enough information to find the transducer gain without any unilateral assumption and the S12 can then be set to zero to find the actual error.

Q4:

This was the least popular and had the lowest average although there were a couple of excellent solutions. The stability parts were generally well done. A common error was to not plot the conjugate impedances to check they were outside the stability circles and to get the transmission line and stub the wrong way around in (iii). Many struggled to get started in (b) as they didn't realise that for the passive device the noise figure is equal to the transducer gain.