EGT3
ENGINEERING TRIPOS PART IIB

Thursday 30 April 2026 09.30 to 11.10

Module 4B29

WIRELESS COMMUNICATION

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) An outdoor long-range Wi-Fi access point operates at a carrier frequency of 2.4 GHz. The propagation environment exhibits log-normal shadowing superimposed on free-space path loss. The access point transmits 10 W and has an antenna gain of 8 dB. The user device has an antenna gain of 2 dB. The receiver noise floor is −95 dBm. The Q-function values are given in Table 1.

(i) The received power in dBm at distance d is modelled as:

$$P_r(d) = P_t + G_t + G_r - PL(d) + \psi$$

where $\psi \sim \mathcal{N}(0, \sigma^2)$ is the shadowing component. The shadowing is modelled as a zero-mean Gaussian random variable with standard deviation $\sigma = 6$ dB. Derive the expression for the free-space path loss term $PL(d)$ in dB as a function of distance d. [10%]

(ii) Due to log-normal shadowing, the instantaneous SNR at distance d is given by

$$\text{SNR}(d) = M(d) + \psi,$$

where $M(d)$ denotes the average SNR (in dB) and $\psi \sim \mathcal{N}(0, \sigma^2)$ with $\sigma = 6$ dB. A reliable connection requires that the probability $\mathbb{P}$ satisfies

$$\mathbb{P}(\text{SNR}(d) \geq 15\,\text{dB}) \geq 0.98.$$

Determine the minimum average SNR $M(d)$ that satisfies this reliability requirement. [25%]

(iii) Treat the minimum average SNR found in part (ii) as the mean SNR at the cell edge, i.e., at the maximum coverage distance d_{max}. Determine the maximum distance d_{max} that satisfies [20%]

$$\mathbb{P}(\text{SNR}(d_{max}) \geq 15\,\text{dB}) \geq 0.98.$$

Friis free-space path loss: $P_r(d) = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2$

Wavelength–frequency relation: $\lambda = \frac{c}{f}$ where the speed of light is $c = 3 \times 10^8$ m s^{-1}.

(b) Consider a wideband wireless channel with an autocorrelation function

$$A_c(\tau, \Delta t) = \begin{cases} \text{sinc}(W\Delta t), & 0 \leq \tau \leq 5\ \mu\text{s} \\ 0, & \text{otherwise} \end{cases}$$

where $W = 200$ Hz and $\text{sinc}(x) = \sin(\pi x)/(\pi x)$. The variable τ denotes the excess delay and Δt denotes the time separation.

(i) Calculate the mean delay spread, the root-mean-square (RMS) delay spread, and the Doppler spread of the channel. [15%]

(ii) Determine the approximate range of signal bandwidths for which a transmitted signal experiences frequency-selective fading in this channel. Clearly state the criterion used. [15%]

(iii) Based on the channel description, state whether the small-scale fading is expected to be Rayleigh or Ricean and briefly justify your answer. [15%]

Table 1: Values of $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2}\, du$ for $0 \le x \le 9$.

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0.00	0.5	2.30	0.010724	4.55	2.6823×10^{-6}	6.80	5.231×10^{-12}
0.05	0.48006	2.35	0.0093867	4.60	2.1125×10^{-6}	6.85	3.6925×10^{-12}
0.10	0.46017	2.40	0.0081975	4.65	1.6597×10^{-6}	6.90	2.6001×10^{-12}
0.15	0.44038	2.45	0.0071428	4.70	1.3008×10^{-6}	6.95	1.8264×10^{-12}
0.20	0.42074	2.50	0.0062097	4.75	1.0171×10^{-6}	7.00	1.2798×10^{-12}
0.25	0.40129	2.55	0.0053861	4.80	7.9333×10^{-7}	7.05	8.9459×10^{-13}
0.30	0.38209	2.60	0.0046612	4.85	6.1731×10^{-7}	7.10	6.2378×10^{-13}
0.35	0.36317	2.65	0.0040246	4.90	4.7918×10^{-7}	7.15	4.3389×10^{-13}
0.40	0.34458	2.70	0.003467	4.95	3.7107×10^{-7}	7.20	3.0106×10^{-13}
0.45	0.32636	2.75	0.0029798	5.00	2.8665×10^{-7}	7.25	2.0839×10^{-13}
0.50	0.30854	2.80	0.0025551	5.05	2.2091×10^{-7}	7.30	1.4388×10^{-13}
0.55	0.29116	2.85	0.002186	5.10	1.6983×10^{-7}	7.35	9.9103×10^{-14}
0.60	0.27425	2.90	0.0018658	5.15	1.3024×10^{-7}	7.40	6.8092×10^{-14}
0.65	0.25785	2.95	0.0015889	5.20	9.9644×10^{-8}	7.45	4.667×10^{-14}
0.70	0.24196	3.00	0.0013499	5.25	7.605×10^{-8}	7.50	3.1909×10^{-14}
0.75	0.22663	3.05	0.0011442	5.30	5.7901×10^{-8}	7.55	2.1763×10^{-14}
0.80	0.21186	3.10	0.0009676	5.35	4.3977×10^{-8}	7.60	1.4807×10^{-14}
0.85	0.19766	3.15	0.00081635	5.40	3.332×10^{-8}	7.65	1.0049×10^{-14}
0.90	0.18406	3.20	0.00068714	5.45	2.5185×10^{-8}	7.70	6.8033×10^{-15}
0.95	0.17106	3.25	0.00057703	5.50	1.899×10^{-8}	7.75	4.5946×10^{-15}
1.00	0.15866	3.30	0.00048342	5.55	1.4283×10^{-8}	7.80	3.0954×10^{-15}
1.05	0.14686	3.35	0.00040406	5.60	1.0718×10^{-8}	7.85	2.0802×10^{-15}
1.10	0.13567	3.40	0.00033693	5.65	8.0224×10^{-9}	7.90	1.3945×10^{-15}
1.15	0.12507	3.45	0.00028029	5.70	5.9904×10^{-9}	7.95	9.3256×10^{-16}
1.20	0.11507	3.50	0.00023263	5.75	4.4622×10^{-9}	8.00	6.221×10^{-16}
1.25	0.10565	3.55	0.00019262	5.80	3.3157×10^{-9}	8.05	4.1397×10^{-16}
1.30	0.0968	3.60	0.00015911	5.85	2.4579×10^{-9}	8.10	2.748×10^{-16}
1.35	0.088508	3.65	0.00013112	5.90	1.8175×10^{-9}	8.15	1.8196×10^{-16}
1.40	0.080757	3.70	0.0001078	5.95	1.3407×10^{-9}	8.20	1.2019×10^{-16}
1.45	0.073529	3.75	8.8417×10^{-5}	6.00	9.8659×10^{-10}	8.25	7.9197×10^{-17}
1.50	0.066807	3.80	7.2348×10^{-5}	6.05	7.2423×10^{-10}	8.30	5.2056×10^{-17}
1.55	0.060571	3.85	5.9059×10^{-5}	6.10	5.3034×10^{-10}	8.35	3.4131×10^{-17}
1.60	0.054799	3.90	4.8096×10^{-5}	6.15	3.8741×10^{-10}	8.40	2.2324×10^{-17}
1.65	0.049471	3.95	3.9076×10^{-5}	6.20	2.8232×10^{-10}	8.45	1.4565×10^{-17}
1.70	0.044565	4.00	3.1671×10^{-5}	6.25	2.0523×10^{-10}	8.50	9.4795×10^{-18}
1.75	0.040059	4.05	2.5609×10^{-5}	6.30	1.4882×10^{-10}	8.55	6.1544×10^{-18}
1.80	0.03593	4.10	2.0658×10^{-5}	6.35	1.0766×10^{-10}	8.60	3.9858×10^{-18}
1.85	0.032157	4.15	1.6624×10^{-5}	6.40	7.7688×10^{-11}	8.65	2.575×10^{-18}
1.90	0.028717	4.20	1.3346×10^{-5}	6.45	5.5925×10^{-11}	8.70	1.6594×10^{-18}
1.95	0.025588	4.25	1.0689×10^{-5}	6.50	4.016×10^{-11}	8.75	1.0668×10^{-18}
2.00	0.02275	4.30	8.5399×10^{-6}	6.55	2.8769×10^{-11}	8.80	6.8408×10^{-19}
2.05	0.020182	4.35	6.8069×10^{-6}	6.60	2.0558×10^{-11}	8.85	4.376×10^{-19}
2.10	0.017864	4.40	5.4125×10^{-6}	6.65	1.4655×10^{-11}	8.90	2.7923×10^{-19}
2.15	0.015778	4.45	4.2935×10^{-6}	6.70	1.0421×10^{-11}	8.95	1.7774×10^{-19}
2.20	0.013903	4.50	3.3977×10^{-6}	6.75	7.3923×10^{-12}	9.00	1.1286×10^{-19}
2.25	0.012224						

2 (a) Consider a frequency-hopping (FH) communication system operating over a two-ray channel consisting of a line-of-sight (LOS) path and a single reflected path. The reflected path has an excess delay of $\tau = 10\ \mu$s relative to the LOS path. The receiver is assumed to be synchronized to the hopping pattern of the LOS path. Determine the range of hopping rates for which the FH system experiences no fading. [15%]

(b) Consider a single-user, point-to-point wireless communication system operating over a flat-fading channel with bandwidth B. The receiver noise power is N_0B, and the average transmit signal-to-noise ratio is $\bar{\gamma} = \frac{P_t}{N_0B}$. The total transmit power is fixed at P_t and, when multiple transmit antennas are used, the power is equally divided among them. All channel coefficients are independent and identically distributed (i.i.d.) Rayleigh fading and remain constant over one symbol interval. Perfect channel state information is available at the receiver. Let h_{ij} denote the complex channel gain between transmit antenna i and receive antenna j.

(i) Consider a system with one transmit antenna ($i = 1$) and two receive antennas ($j = 1, 2$). Derive expressions for the instantaneous received SNR when the receiver employs Maximal Ratio Combining (MRC) and Equal Gain Combining (EGC). Briefly explain the difference between the two schemes. [30%]

(ii) Now consider a system with two transmit antennas ($i = 1, 2$) and one receive antenna ($j = 1$) employing the Alamouti space–time block code. Derive the equivalent input–output relationship after decoding and determine the resulting instantaneous SNR. [30%]

(iii) Compare the systems in parts (i) and (ii) in terms of diversity order, required channel knowledge, and robustness to fading. Using the derived instantaneous SNR expressions and the diversity orders of the systems, state which scheme is more prone to error and justify your answer. [25%]

 (TURN OVER

3 Consider a 3×3 narrowband Multiple Input Multiple Output (MIMO) system described by

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n},$$

where $\mathbf{H}$ is the channel matrix and $\mathbf{n}$ is additive white Gaussian noise with variance σ^2 per receive antenna. The channel matrix is approximately

$$\mathbf{H} = \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.1 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.7 \end{bmatrix}.$$

The Singular Value Decomposition (SVD) of $\mathbf{H}$ is given by

$$\mathbf{H} = \mathbf{U\Sigma V}^H,$$

where

$$\mathbf{U} = \begin{bmatrix} -0.508 & 0.775 & 0.377 \\ -0.467 & 0.120 & -0.876 \\ -0.724 & -0.621 & 0.301 \end{bmatrix}, \mathbf{\Sigma} = \begin{bmatrix} 1.041 & 0 & 0 \\ 0 & 0.358 & 0 \\ 0 & 0 & 0.166 \end{bmatrix}$$

and

$$\mathbf{V}^H = \begin{bmatrix} -0.379 & -0.583 & -0.719 \\ 0.553 & 0.480 & -0.681 \\ 0.742 & -0.655 & 0.140 \end{bmatrix}.$$

(a) Determine an equivalent representation of the MIMO channel as a set of independent parallel scalar channels. Clearly state the transmit precoding matrix, the receive combining matrix, and the gains of the resulting parallel channels. [20%]

(b) Assume the transmitter has perfect channel state information and a total power-to-noise ratio of

$$\frac{P_{\text{tot}}}{\sigma^2} = 10 \quad \text{(linear scale)}.$$

The system bandwidth is $B = 100$ kHz. Determine the optimal power allocation across the three parallel channels using water-filling and the resulting total channel capacity. [50%]

(c) Now assume the channel is unknown at the transmitter and the total power is equally divided among the three transmit antennas. Compute the resulting channel capacity and compare it with the result obtained in part (b). [30%]

END OF PAPER