EGT3
ENGINEERING TRIPOS PART IIB

Thursday 30 April 2026   09.30 to 11.10

Module 4B29

WIRELESS COMMUNICATION

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) An outdoor long-range Wi-Fi access point operates at a carrier frequency of 2.4 GHz. The propagation environment exhibits log-normal shadowing superimposed on free-space path loss. The access point transmits 10 W and has an antenna gain of 8 dB. The user device has an antenna gain of 2 dB. The receiver noise floor is −95 dBm. The Q-function values are given in Table 1.

(i) The received power in dBm at distance d is modelled as:

$$P_r(d) = P_t + G_t + G_r - PL(d) + \psi$$

where $\psi \sim \mathcal{N}(0, \sigma^2)$ is the shadowing component. The shadowing is modelled as a zero-mean Gaussian random variable with standard deviation $\sigma = 6$ dB. Derive the expression for the free-space path loss term $PL(d)$ in dB as a function of distance d. [10%]

Solution:
The free-space path loss $PL(d)$ is given by:

$$PL(d) = 20 \log_{10}\left(\frac{4\pi d}{\lambda}\right)$$

(ii) Due to log-normal shadowing, the instantaneous SNR at distance d is given by

$$\mathrm{SNR}(d) = M(d) + \psi,$$

where $M(d)$ denotes the average SNR (in dB) and $\psi \sim \mathcal{N}(0, \sigma^2)$ with $\sigma = 6$ dB. A reliable connection requires that the probability $\mathbb{P}$ satisfies

$$\mathbb{P}(\mathrm{SNR}(d) \geq 15\,\mathrm{dB}) \geq 0.98.$$

Determine the minimum average SNR $M(d)$ that satisfies this reliability requirement. [25%]

Solution:
Let $M(d)$ be the average SNR at distance d. The reliability condition is:

$$\mathbb{P}(M(d) + \psi \geq 15) \geq 0.98$$

$$\mathbb{P}(\psi \geq 15 - M(d)) \geq 0.98$$

Normalizing the random variable ψ (dividing by $\sigma = 6$):

$$Q\left(\frac{15 - M(d)}{6}\right) \geq 0.98$$

Since $0.98 > 0.5$, the argument of the Q-function must be negative. Let $x = \frac{M(d)-15}{6}$, so that:

$$Q(-x) = 1 - Q(x) = 0.98 \implies Q(x) = 0.02$$

Using the table, $Q(2.05) \approx 0.02$. Therefore $x \approx 2.05$.

$$\frac{M(d) - 15}{6} = 2.05 \implies M(d) = 15 + 12.3 = 27.3 \text{ dB}$$

(iii) Treat the minimum average SNR found in part (ii) as the mean SNR at the cell edge, i.e., at the maximum coverage distance d_{max}. Determine the maximum distance d_{max} that satisfies [20%]

$$\mathbb{P}(\text{SNR}(d_{max}) \geq 15\,\text{dB}) \geq 0.98.$$

Solution:
First, convert the transmit power to dBm:

$$P_t(\text{dBm}) = 10\log_{10}(10000) = 40 \text{ dBm}$$

The wavelength is:

$$\lambda = \frac{3 \times 10^8}{2.4 \times 10^9} = 0.125 \text{ m}$$

Equating the average SNR to the link budget expression:

$$M(d) = P_t + G_t + G_r - PL(d) - N$$

$$27.3 = 40 + 8 + 2 - PL(d) - (-95)$$

$$27.3 = 145 - PL(d)$$

$$PL(d) = 145 - 27.3 = 117.7 \text{ dB}$$

Expanding the path loss term:

$$20\log_{10}\left(\frac{4\pi d}{0.125}\right) = 117.7$$

$$\log_{10}\left(\frac{4\pi d}{0.125}\right) = \frac{117.7}{20} = 5.885$$

$$\frac{4\pi d}{0.125} = 10^{5.885} \approx 767361.49$$

$$d = \frac{767361.49 \times 0.125}{4\pi} \approx 7633.083 \text{ m}$$

The maximum coverage radius is $d = 7633.083$ m ≈ 7.63 km.

Friis free-space path loss: $P_r(d) = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2$

Wavelength–frequency relation: $\lambda = \frac{c}{f}$ where the speed of light is $c = 3 \times 10^8 \text{ m s}^{-1}$.

(b) Consider a wideband wireless channel with an autocorrelation function

$$A_c(\tau, \Delta t) = \begin{cases} \text{sinc}(W\Delta t), & 0 \le \tau \le 5\ \mu\text{s} \\ 0, & \text{otherwise} \end{cases}$$

where $W = 200$ Hz and $\text{sinc}(x) = \sin(\pi x)/(\pi x)$. The variable τ denotes the excess delay and Δt denotes the time separation.

(i) Calculate the mean delay spread, the root-mean-square (RMS) delay spread, and the Doppler spread of the channel. [15%]

Solution:

The power delay profile is uniform over the interval $0 \le \tau \le 5\ \mu$s and zero elsewhere. Hence, the mean delay spread is

$$\bar{\tau} = \frac{\int_0^{5\mu s} \tau \, d\tau}{\int_0^{5\mu s} d\tau} = \frac{(5\mu s)^2/2}{5\mu s} = 2.5\ \mu\text{s}.$$

The second moment of the delay is

$$E[\tau^2] = \frac{\int_0^{5\mu s} \tau^2 \, d\tau}{\int_0^{5\mu s} d\tau} = \frac{(5\mu s)^3/3}{5\mu s} = \frac{(5\mu s)^2}{3}.$$

The RMS delay spread is therefore

$$\tau_{\text{rms}} = \sqrt{E[\tau^2] - \bar{\tau}^2} = \sqrt{\frac{(5\mu s)^2}{3} - (2.5\mu s)^2} \approx 1.44\ \mu\text{s}.$$

The Doppler spread is determined from the time autocorrelation function $\text{sinc}(W\Delta t)$. Since the Doppler spectrum corresponding to a sinc correlation has support over $[-W, W]$, the Doppler spread is approximately

$$B_D \approx W = 200 \text{ Hz}.$$

(ii) Determine the approximate range of signal bandwidths for which a transmitted signal experiences frequency-selective fading in this channel. Clearly state the criterion used. [15%]

Solution:
A channel is frequency-selective when the signal bandwidth exceeds the coherence bandwidth of the channel. The coherence bandwidth can be approximated as

$$B_c \approx \frac{1}{5\tau_{\text{rms}}}.$$

Using $\tau_{\text{rms}} \approx 1.44\ \mu$s,

$$B_c \approx \frac{1}{5 \times 1.44 \times 10^{-6}} \approx 1.39 \times 10^5 \text{ Hz}.$$

Therefore, signals with bandwidths (or equivalently data rates) significantly larger than approximately 140 kHz will experience frequency-selective fading. Signals with bandwidths much smaller than this value will experience flat fading.

(iii) Based on the channel description, state whether the small-scale fading is expected to be Rayleigh or Ricean and briefly justify your answer. [15%]

Solution:
The channel model specifies a uniform multipath delay profile over a finite delay interval and does not indicate the presence of a deterministic line-of-sight (LOS) component. In the absence of a dominant LOS path, the received signal is the sum of many independently scattered components.
Therefore, the small-scale fading is expected to follow a Rayleigh distribution rather than a Ricean distribution.

Table 1: Values of $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2}\, du$ for $0 \le x \le 9$.

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0.00	0.5	2.30	0.010724	4.55	2.6823×10^{-6}	6.80	5.231×10^{-12}
0.05	0.48006	2.35	0.0093867	4.60	2.1125×10^{-6}	6.85	3.6925×10^{-12}
0.10	0.46017	2.40	0.0081975	4.65	1.6597×10^{-6}	6.90	2.6001×10^{-12}
0.15	0.44038	2.45	0.0071428	4.70	1.3008×10^{-6}	6.95	1.8264×10^{-12}
0.20	0.42074	2.50	0.0062097	4.75	1.0171×10^{-6}	7.00	1.2798×10^{-12}
0.25	0.40129	2.55	0.0053861	4.80	7.9333×10^{-7}	7.05	8.9459×10^{-13}
0.30	0.38209	2.60	0.0046612	4.85	6.1731×10^{-7}	7.10	6.2378×10^{-13}
0.35	0.36317	2.65	0.0040246	4.90	4.7918×10^{-7}	7.15	4.3389×10^{-13}
0.40	0.34458	2.70	0.003467	4.95	3.7107×10^{-7}	7.20	3.0106×10^{-13}
0.45	0.32636	2.75	0.0029798	5.00	2.8665×10^{-7}	7.25	2.0839×10^{-13}
0.50	0.30854	2.80	0.0025551	5.05	2.2091×10^{-7}	7.30	1.4388×10^{-13}
0.55	0.29116	2.85	0.002186	5.10	1.6983×10^{-7}	7.35	9.9103×10^{-14}
0.60	0.27425	2.90	0.0018658	5.15	1.3024×10^{-7}	7.40	6.8092×10^{-14}
0.65	0.25785	2.95	0.0015889	5.20	9.9644×10^{-8}	7.45	4.667×10^{-14}
0.70	0.24196	3.00	0.0013499	5.25	7.605×10^{-8}	7.50	3.1909×10^{-14}
0.75	0.22663	3.05	0.0011442	5.30	5.7901×10^{-8}	7.55	2.1763×10^{-14}
0.80	0.21186	3.10	0.0009676	5.35	4.3977×10^{-8}	7.60	1.4807×10^{-14}
0.85	0.19766	3.15	0.00081635	5.40	3.332×10^{-8}	7.65	1.0049×10^{-14}
0.90	0.18406	3.20	0.00068714	5.45	2.5185×10^{-8}	7.70	6.8033×10^{-15}
0.95	0.17106	3.25	0.00057703	5.50	1.899×10^{-8}	7.75	4.5946×10^{-15}
1.00	0.15866	3.30	0.00048342	5.55	1.4283×10^{-8}	7.80	3.0954×10^{-15}
1.05	0.14686	3.35	0.00040406	5.60	1.0718×10^{-8}	7.85	2.0802×10^{-15}
1.10	0.13567	3.40	0.00033693	5.65	8.0224×10^{-9}	7.90	1.3945×10^{-15}
1.15	0.12507	3.45	0.00028029	5.70	5.9904×10^{-9}	7.95	9.3256×10^{-16}
1.20	0.11507	3.50	0.00023263	5.75	4.4622×10^{-9}	8.00	6.221×10^{-16}
1.25	0.10565	3.55	0.00019262	5.80	3.3157×10^{-9}	8.05	4.1397×10^{-16}
1.30	0.0968	3.60	0.00015911	5.85	2.4579×10^{-9}	8.10	2.748×10^{-16}
1.35	0.088508	3.65	0.00013112	5.90	1.8175×10^{-9}	8.15	1.8196×10^{-16}
1.40	0.080757	3.70	0.0001078	5.95	1.3407×10^{-9}	8.20	1.2019×10^{-16}
1.45	0.073529	3.75	8.8417×10^{-5}	6.00	9.8659×10^{-10}	8.25	7.9197×10^{-17}
1.50	0.066807	3.80	7.2348×10^{-5}	6.05	7.2423×10^{-10}	8.30	5.2056×10^{-17}
1.55	0.060571	3.85	5.9059×10^{-5}	6.10	5.3034×10^{-10}	8.35	3.4131×10^{-17}
1.60	0.054799	3.90	4.8096×10^{-5}	6.15	3.8741×10^{-10}	8.40	2.2324×10^{-17}
1.65	0.049471	3.95	3.9076×10^{-5}	6.20	2.8232×10^{-10}	8.45	1.4565×10^{-17}
1.70	0.044565	4.00	3.1671×10^{-5}	6.25	2.0523×10^{-10}	8.50	9.4795×10^{-18}
1.75	0.040059	4.05	2.5609×10^{-5}	6.30	1.4882×10^{-10}	8.55	6.1544×10^{-18}
1.80	0.03593	4.10	2.0658×10^{-5}	6.35	1.0766×10^{-10}	8.60	3.9858×10^{-18}
1.85	0.032157	4.15	1.6624×10^{-5}	6.40	7.7688×10^{-11}	8.65	2.575×10^{-18}
1.90	0.028717	4.20	1.3346×10^{-5}	6.45	5.5925×10^{-11}	8.70	1.6594×10^{-18}
1.95	0.025588	4.25	1.0689×10^{-5}	6.50	4.016×10^{-11}	8.75	1.0668×10^{-18}
2.00	0.02275	4.30	8.5399×10^{-6}	6.55	2.8769×10^{-11}	8.80	6.8408×10^{-19}
2.05	0.020182	4.35	6.8069×10^{-6}	6.60	2.0558×10^{-11}	8.85	4.376×10^{-19}
2.10	0.017864	4.40	5.4125×10^{-6}	6.65	1.4655×10^{-11}	8.90	2.7923×10^{-19}
2.15	0.015778	4.45	4.2935×10^{-6}	6.70	1.0421×10^{-11}	8.95	1.7774×10^{-19}
2.20	0.013903	4.50	3.3977×10^{-6}	6.75	7.3923×10^{-12}	9.00	1.1286×10^{-19}
2.25	0.012224						

2 (a) Consider a frequency-hopping (FH) communication system operating over a two-ray channel consisting of a line-of-sight (LOS) path and a single reflected path. The reflected path has an excess delay of $\tau = 10\ \mu$s relative to the LOS path. The receiver is assumed to be synchronized to the hopping pattern of the LOS path. Determine the range of hopping rates for which the FH system experiences no fading. [15%]

Solution:

The channel will exhibit no fading if the delayed reflected path does not overlap in time with the LOS path within a hop. This requires the hop duration T_c to be smaller than the excess delay τ, so that each hop contains energy from only one propagation path. The condition for no fading is

$$T_c < \tau.$$

Given $\tau = 10\ \mu$s, the hop duration must satisfy

$$T_c < 10\ \mu\text{s}.$$

Equivalently, the hopping rate must satisfy

$$R_c = \frac{1}{T_c} > 100\ \text{kHz}.$$

(b) Consider a single-user, point-to-point wireless communication system operating over a flat-fading channel with bandwidth B. The receiver noise power is $N_0 B$, and the average transmit signal-to-noise ratio is $\bar{\gamma} = \frac{P_t}{N_0 B}$. The total transmit power is fixed at P_t and, when multiple transmit antennas are used, the power is equally divided among them. All channel coefficients are independent and identically distributed (i.i.d.) Rayleigh fading and remain constant over one symbol interval. Perfect channel state information is available at the receiver. Let h_{ij} denote the complex channel gain between transmit antenna i and receive antenna j.

(i) Consider a system with one transmit antenna ($i = 1$) and two receive antennas ($j = 1, 2$). Derive expressions for the instantaneous received SNR when the receiver employs Maximal Ratio Combining (MRC) and Equal Gain Combining (EGC). Briefly explain the difference between the two schemes. [30%]

Solution:
The instantaneous SNR on receive branch j is

$$\gamma_j = \bar{\gamma}|h_{1j}|^2.$$

With MRC, the received signals are weighted by the conjugate of their channel gains. The resulting instantaneous SNR is

$$\gamma_{\text{MRC}} = \sum_{j=1}^{2} \gamma_j = \bar{\gamma}\left(|h_{11}|^2 + |h_{12}|^2\right).$$

With EGC, the received signals are co-phased but combined with equal magnitudes. The instantaneous SNR is

$$\gamma_{\text{EGC}} = \frac{\bar{\gamma}}{2}\left(|h_{11}| + |h_{12}|\right)^2.$$

MRC maximizes the output SNR by weighting each branch according to its channel magnitude and phase, whereas EGC uses only phase information and therefore provides suboptimal performance.

(ii) Now consider a system with two transmit antennas ($i = 1, 2$) and one receive antenna ($j = 1$) employing the Alamouti space–time block code. Derive the equivalent input–output relationship after decoding and determine the resulting instantaneous SNR. [30%]

Solution:
Let h_{11} and h_{21} denote the channel gains from the two transmit antennas to the receive antenna.
Since the total transmit power is constrained to P_t, it is divided equally between the two antennas. Therefore, the transmit power per antenna is $P_t/2$.
After Alamouti encoding and linear decoding at the receiver, the equivalent input–output relationship is:

$$\tilde{y} = \sqrt{\frac{P_t}{2}}\left(|h_{11}|^2 + |h_{21}|^2\right) s + \tilde{n},$$

where s is the transmitted symbol (normalized to unit energy) and $\tilde{n}$ is the effective noise term with variance $N_0 B\left(|h_{11}|^2 + |h_{21}|^2\right)$.

The resulting instantaneous SNR is the ratio of signal power to noise power:

$$\gamma_{\text{Alamouti}} = \frac{\left|\sqrt{\frac{P_t}{2}}(|h_{11}|^2 + |h_{21}|^2)\right|^2}{N_0 B(|h_{11}|^2 + |h_{21}|^2)} = \frac{\frac{P_t}{2}(|h_{11}|^2 + |h_{21}|^2)^2}{N_0 B(|h_{11}|^2 + |h_{21}|^2)}.$$

Simplifying and substituting $\bar{\gamma} = P_t/(N_0 B)$:

$$\gamma_{\text{Alamouti}} = \frac{P_t}{2N_0 B}\left(|h_{11}|^2 + |h_{21}|^2\right) = \frac{\bar{\gamma}}{2}\left(|h_{11}|^2 + |h_{21}|^2\right).$$

(iii) Compare the systems in parts (i) and (ii) in terms of diversity order, required channel knowledge, and robustness to fading. Using the derived instantaneous SNR expressions and the diversity orders of the systems, state which scheme is more prone to error and justify your answer. [25%]

Solution:

Diversity Order: All three schemes (MRC, EGC, and Alamouti) achieve a full diversity order of two.

Channel Knowledge:

- MRC requires full CSI (magnitude and phase) at the receiver.
- EGC requires only phase information at the receiver.
- Alamouti requires CSI at the receiver for decoding but does *not* require CSI at the transmitter (open-loop diversity).

Performance Comparison: MRC provides the best error-rate performance. Although MRC and Alamouti share the same diversity order, MRC achieves an instantaneous SNR that is exactly double (3 dB higher) than that of the Alamouti scheme:

$$\gamma_{\text{MRC}} = 2 \times \gamma_{\text{Alamouti}}.$$

This is because MRC benefits from **receive array gain** (collecting total energy from two antennas), whereas the Alamouti scheme splits the fixed total transmit power between two antennas to achieve diversity. EGC performs strictly worse than MRC due to non-optimal weighting.

3 Consider a 3×3 narrowband Multiple Input Multiple Output (MIMO) system described by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

where $\mathbf{H}$ is the channel matrix and $\mathbf{n}$ is additive white Gaussian noise with variance σ^2 per receive antenna. The channel matrix is approximately

$$\mathbf{H} = \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.1 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.7 \end{bmatrix}.$$

The Singular Value Decomposition (SVD) of $\mathbf{H}$ is given by

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H,$$

where

$$\mathbf{U} = \begin{bmatrix} -0.508 & 0.775 & 0.377 \\ -0.467 & 0.120 & -0.876 \\ -0.724 & -0.621 & 0.301 \end{bmatrix}, \mathbf{\Sigma} = \begin{bmatrix} 1.041 & 0 & 0 \\ 0 & 0.358 & 0 \\ 0 & 0 & 0.166 \end{bmatrix}$$

and

$$\mathbf{V}^H = \begin{bmatrix} -0.379 & -0.583 & -0.719 \\ 0.553 & 0.480 & -0.681 \\ 0.742 & -0.655 & 0.140 \end{bmatrix}.$$

(a) Determine an equivalent representation of the MIMO channel as a set of independent parallel scalar channels. Clearly state the transmit precoding matrix, the receive combining matrix, and the gains of the resulting parallel channels. [20%]

Solution:

Applying SVD-based precoding and combining,

$$\mathbf{x} = \mathbf{V}\tilde{\mathbf{x}}, \qquad \tilde{\mathbf{y}} = \mathbf{U}^H\mathbf{y}.$$

Substituting into the input–output relation,

$$\tilde{\mathbf{y}} = \mathbf{U}^H\mathbf{H}\mathbf{V}\tilde{\mathbf{x}} + \tilde{\mathbf{n}} = \mathbf{\Sigma}\tilde{\mathbf{x}} + \tilde{\mathbf{n}}.$$

Therefore:

- The transmit precoding matrix is $\mathbf{V}$.
- The receive combining matrix is $\mathbf{U}^H$.

The MIMO channel is converted into three independent parallel scalar channels with gains equal to the singular values:

$$\lambda_1 = 1.041, \quad \lambda_2 = 0.358, \quad \lambda_3 = 0.166.$$

(b) Assume the transmitter has perfect channel state information and a total power-to-noise ratio of

$$\frac{P_{\text{tot}}}{\sigma^2} = 10 \quad \text{(linear scale)}.$$

The system bandwidth is $B = 100$ kHz. Determine the optimal power allocation across the three parallel channels using water-filling and the resulting total channel capacity. [50%]

Solution:

The squared singular values are:

$$\lambda_1^2 \approx 1.084, \quad \lambda_2^2 \approx 0.128, \quad \lambda_3^2 \approx 0.028.$$

Let $P_{\text{tot}} = 10\sigma^2$. Water-filling allocates power as $\frac{P_i}{\sigma^2} = \left(L - \frac{1}{\lambda_i^2}\right)^+$. The inverse squared singular values are:

$$\frac{1}{\lambda_1^2} \approx 0.923, \quad \frac{1}{\lambda_2^2} \approx 7.813, \quad \frac{1}{\lambda_3^2} \approx 35.714.$$

Assuming two active channels (since L for three channels would be < 35.714):

$$(L - 0.923) + (L - 7.813) = 10 \implies 2L = 18.736 \implies L = 9.368.$$

The power allocation is:

$$\frac{P_1}{\sigma^2} = 8.445, \quad \frac{P_2}{\sigma^2} = 1.555, \quad \frac{P_3}{\sigma^2} = 0.$$

The total capacity with $B = 100\,\text{kHz}$ is:

$$C = B \sum_{i=1}^{2} \log_2 \left(1 + \frac{P_i}{\sigma^2}\lambda_i^2\right) \approx 100\,\text{kHz}[3.344 + 0.261] \approx 360.5 \text{ kbps}.$$

(c) Now assume the channel is unknown at the transmitter and the total power is equally divided among the three transmit antennas. Compute the resulting channel capacity and compare it with the result obtained in part (b). [30%]

Solution:

With equal power allocation, $\frac{P_i}{\sigma^2} = \frac{10}{3} \approx 3.333$. The capacity is:

$$C_{\text{EP}} = B \sum_{i=1}^{3} \log_2 \left(1 + \frac{10}{3} \sigma_i^2 \right).$$

Substituting the channel gains:

$$C_{\text{EP}} \approx 100\,\text{kHz}[\log_2(4.613) + \log_2(1.427) + \log_2(1.093)]$$

$$C_{\text{EP}} \approx 100\,\text{kHz}[2.206 + 0.513 + 0.128] \approx 284.7 \text{ kbps}.$$

Compared to equal power allocation, water-filling achieves a significantly higher capacity (360.5 kbps vs 284.7 kbps) by identifying that the third channel is too weak to justify any power expenditure.

END OF PAPER