

EGT3
ENGINEERING TRIPOS PART IIB

Monday 27 April 2026 9.30 to 11.10

Module 4B5

QUANTUM AND NANO-TECHNOLOGIES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Attachment: 4B5 Quantum and Nano-technologies data sheet (2 pages)

Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) You are tasked with developing a quantum well photo-detector operating at the wavelength of interest of 800 nm. You need to choose what materials to use between GaAs/AlGaAs and GaN/AlGaN.

The bandgaps of the materials are:

GaAs, 1.4 eV

AlGaAs, 2.0 eV

GaN, 3.4 eV

AlGaN, 4.0 eV

Explain how you choose which material system to use.

[20%]

(b) Make a sketch of the valence and conduction bands and of the energy levels of the quantum well.

What transition would you target and why? [Interband (between valence and conduction bands: S to S, S to P, P to S, where S is the ground state and P the first excited state); or intraband (between confined states in the conduction or the valence band)].

[20%]

(c) Calculate the size of the quantum well that is needed for the detector to be optimised for the absorption of the target wavelength of 800 nm. Explain what approximations you make and check their validity.

[40%]

(d) Sketch the expected absorption spectrum and explain its features.

[20%]

- 2 (a) Define the mean free path λ of an electron travelling in a metallic wire and explain its impact on current conduction when the dimensionality of the wire is reduced down to the nanoscale. [10%]
- (b) Define the mean grain size D of a conductor and explain what happens when an electron encounters a grain boundary. [10%]
- (c) Define the reflection probability R in a conductor and explain its impact on current transport. [10%]
- (d) Define the surface scattering parameter p and explain its impact on current transport. [10%]
- (e) Describe how these parameters affect the resistivity of a nanowire. [25%]
- (f) You want to fabricate a device where resistivity is minimised. Explain how you would engineer its property and what will limit current transport in metal nanowires. [10%]
- (g) Explain why graphene has been seen as a potential candidate for the development of fast devices based on current transport. [25%]

3 (a) You want to build a screen based on field emission, operating at visible wavelengths. Explain how you would proceed, what the main components will be needed to build such a device, and how it will function. [20%]

(b) Define the work function ϕ of the metal and considering that you will apply an electric field F to the device, give a definition of the potential felt by the electrons in the metal and plot a schematic of it. [30%]

(c) Recall that the transmission probability is given by:

$$T = e^{-\frac{4\sqrt{2m}\phi^{3/2}}{3eF\hbar}} \quad (1)$$

where m is the mass of an electron and e (in the exponent) is the charge of an electron. Sketch T as a function of F for various decreasing values of ϕ ($\phi_1 > \phi_2 > \phi_3$) and explain the role of the work function in the operation of the device. [30%]

(d) You decide to realise a new screen, operating at infra-red wavelengths. Explain if this is possible and, if so, what changes and consideration are required, and if not, why. [20%]

- 4 (a) You want to design a new drug and you realise that you will not be able to solve the calculations that you need with a standard computer. You consider using a quantum computer instead. Explain why you expect a qubit-based processor to be more powerful than your current bit-based computer. [10%]
- (b) Define superposition and entanglement and explain their roles in quantum computing. [20%]
- (c) You can access a 50-qubit quantum computer, explain how its performance would compare to a 50-bit classical computer and provide a quantitative evaluation of the potential increase in performance. [10%]
- (d) Provide three examples of qubits that are currently used in quantum computers and explain why such platforms have been chosen. [15%]
- (e) Define the three steps that are essential for the operation of a quantum computer. [30%]
- (f) Explain what decoherence is and why it is an issue faced by quantum computers. [15%]

END OF PAPER

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4B5 Quantum Technologies Formula sheet – 2 pages

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

1-D time-independent Schrödinger equation,
 $V(x)$ is the potential, E is the total energy

$$\hat{E} = i\hbar \frac{\partial}{\partial t}$$

Quantum Energy Operator

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

Quantum Momentum Operator

$$\widehat{K.E.} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$$

Quantum Kinetic Energy Operator

$$h$$

Planck's constant = 6.626×10^{-34} Js

$$\hbar$$

Reduced Planck's constant = 1.05×10^{-34} Js

$$e$$

Electron charge = 1.6×10^{-19} C

$$\text{eV}$$

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-19} \text{ kg m}^2 \text{ s}^{-2}$

$$m$$

Free electron rest mass = 9.11×10^{-31} kg

$$c$$

Speed of light in vacuum = $2.998 \times 10^8 \text{ ms}^{-1}$

$$k_B$$

Boltzmann's constant = $1.38 \times 10^{-23} \text{ J/K}$

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega_c$$

Spectrum of energy levels in quantum
harmonic oscillator, where ω_c is the natural
resonant frequency of the system in rad s^{-1}

$$E_n = \frac{n^2 h^2}{8mL^2}$$

Spectrum of energy levels in
an infinite quantum well of width L

Prefix Tera = 10^{12}

Prefix Atto = 10^{-18}

$$\Delta x(t) = \sqrt{\Delta x_0 + \left(\frac{\hbar t}{m\Delta x_0}\right)^2}$$

Width (standard deviation) of matter wave-
packet of initial width Δx_0
as a function of time, t .

$$\frac{d|\psi(x)|^2}{dt} = \frac{i\hbar}{2m} \nabla \cdot \mathbf{j}$$

Quantum continuity equation,
where $\mathbf{j}$ = probability flux.

$$\{\psi^*(\nabla\psi) - (\nabla\psi^*)\psi\}$$

1-D Probability flux.

$$T \cong e^{-\frac{2}{\hbar} \int_A^B \sqrt{2m(V(x)-E)} dx}$$

WKB approximation to transmission
probability, T .
The integration limits are the classical turning
points, A & B .