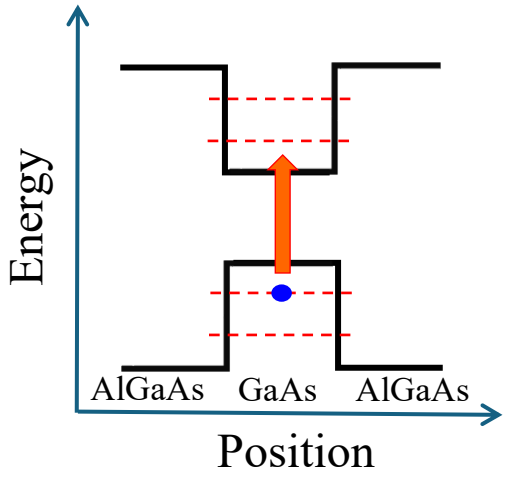
4B5 EXAM SOLUTIONS 2026

1. (a) The energy of the quantum well transitions will be the energy of the smaller bandgap material plus the confinement energy. For the quantum well to absorb energy corresponding to a wavelength of 800nm~1.55eV, the material of choice is therefore GaAs/AlGaAs.

[20%]

(b)



Interband transitions are needed to reach the wavelength of 800nm, since intraband transitions would occur at much shorter energies, with corresponding wavelengths in the mid-infrared or THz.
The S to S transition would provide the highest absorption thanks to the larger oscillator strength and larger overlap between the valence and conduction band confined carrier wavefunctions (as compared, for instance, to S to P or P to S transitions, if they are allowed). The quantum well could be designed for a P to P transition to be at 800nm but using excited states rather than ground states would make the transition strength, and therefore the collected signal, lower.

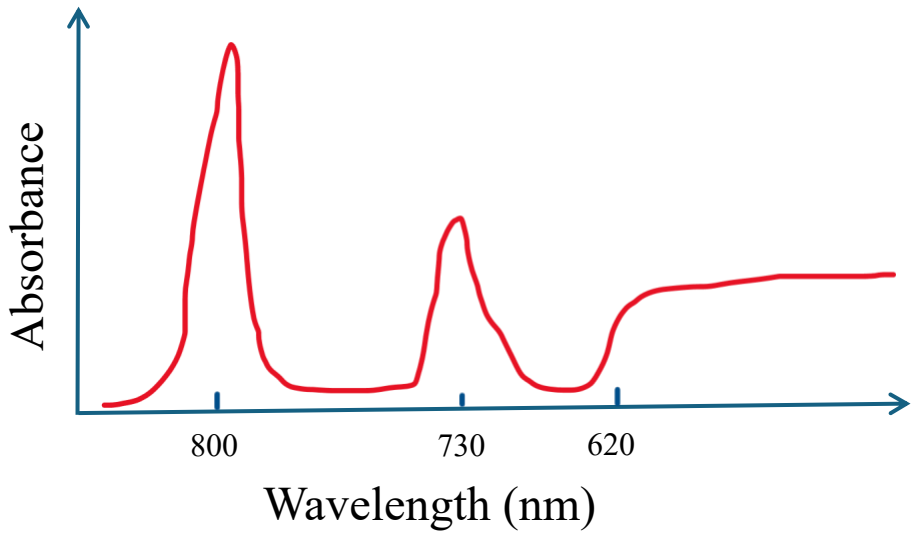
[20%]

(c) By using the infinite well approximation and by considering the free electron mass, one obtains:
GaAs bandgap = 1.4 eV
λ = 800 nm corresponds to 1.55 eV
[(1.55-1.4)/2] eV = 0.075 eV -> electron confinement energy
The electron confinement energy is much smaller than the total quantum well confinement energy (2-1.4)/2eV = 0.3eV, therefore one can use the infinite well approximation (valid when the bound states are deeply confined in the well, potential well height >> than confinement energy).

In the infinite well approximation:
$E = h^2 n^2/(8mL^2) = 0.075$ eV for n = 1.
Plugging in the numbers, $L = \sqrt{[h^2/(8m*0.075)]} = 2.2$ nm

[40%]

(d)



The S->S transition was designed to be at 800nm, the P->P transition can be calculated in the infinite well approximation to be at about 730nm, the onset of the continuum to continuum transitions is the band gap of AlGaAs, that is 2eV and therefore corresponds to a wavelength of about 620nm.

[20%]

2. (a) The electron mean free path λ is the average distance an electron travels between scattering events (collisions) in a material.
When the dimensionality of a wire is reduced to the nanoscale, its dimensions can become comparable to or smaller than the electron mean free path: current conduction is significantly affected, with a dramatic increase in electrical resistivity.
The physical boundaries of the wire can limit the average distance an electron can travel before colliding with the wire's surface. This leads to a reduction in the effective mean free path within the nanostructure compared to the bulk material.
The electron mean free path dictates the nature of electron transport, shifting it from a diffusive, bulk-like regime, to a surface-dominated as the wire dimensions are reduced to the nanoscale.

[10%]

(b) The mean grain size D of a conductor is the average length of the individual crystalline regions (grains), separated by grain boundaries, within the wire.
When an electron moving through a conductor encounters a grain boundary, the boundary can act as a scattering centre for the free electrons that carry electrical current, and the electron's path can be altered, even changing its direction of travel significantly.
These scattering events impede the flow of electrons, effectively increasing the material's electrical resistivity, thus reducing the overall electrical conductivity of the material.

[10%]

(c) The reflection probability R in a conductor quantifies the fraction of incident charge carriers that is reflected at a boundary, rather than being transmitted through it.
The reflection probability can increase the electrical resistance: a higher reflection probability means a larger fraction of charge carriers is scattered backward at interfaces or imperfections, rather than continuing to move in the intended direction. This opposition to the flow of charge constitutes an increase in the conductor's resistance: if electrons are frequently reflected, their net forward velocity decreases, which reduces the overall current.

[10%]

(d) The parameter p is defined as the probability that a charge carrier will be specularly reflected from the surface, a perfect specular scattering ($p = 1$) implies that the momentum parallel to the surface is not modified, while when the electron impinges on a rough surface, the momentum is randomised ($p = 0$).
Surface scattering provides an additional scattering mechanism that impedes the flow of current and increases the material's electrical resistivity compared to its bulk value.

[10%]

(e) A larger value of λ, implies fewer collisions and therefore lower resistivity.
A larger value of D implies fewer reflections at grain boundaries, therefore lower resistivity.
A lower value of R implies that the electrons are reflected less, thus providing a lower resistivity.

A lower value of p (more diffuse scattering) leads to higher resistivity, while a higher value of p (more specular scattering) results in a lower resistivity.

[25%]

(f) You might try to minimise grain boundaries and decrease roughness of nanowire edges. You might want to engineer the size of grain boundaries and roughness compared to the wavelength of the travelling electrons. You could reduce the operation temperature to minimise phonon scattering.

[10%]

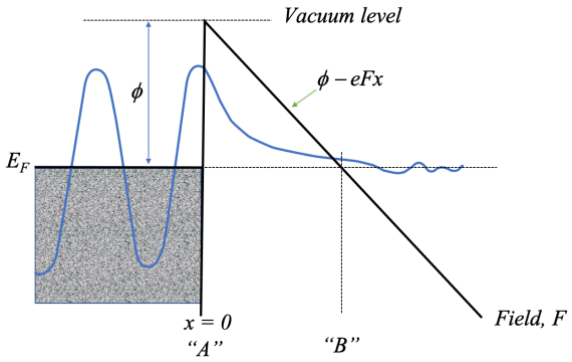
(g) Charge carriers in graphene can move with very high mobility, which translates to faster device operation speeds compared to traditional semiconductors like silicon, since graphene does not have a bandgap and possesses a linear energy-momentum relationship (known as Dirac cones). This allows electrons to behave as if they are massless relativistic particles, enabling them to accelerate quickly and move with minimal resistance.
Furthermore, graphene exhibits a very high saturation velocity for charge carriers, meaning that even at high electric fields, the carriers can maintain high speeds without significant slowdowns due to scattering.

[25%]

3. (a) You will need to have an array of small, pointed electrodes that will emit electrons when a high electric field is applied, through the field effect, an extreme form of electron tunnelling that ejects electrons from a metal.
The emitted electrons are then accelerated toward a target. The target is coated with a phosphor material, typically arranged in red, green, and blue pixels. When the electrons hit the phosphor, the material luminesces, producing the visible light that forms the image on the screen.

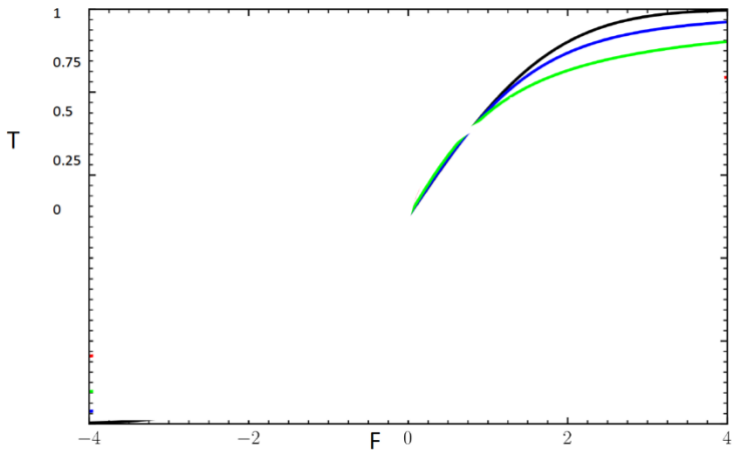
[20%]

(b) The work function Φ of a metal is the minimum energy required for an electron to escape from the surface of that metal into the vacuum, overcoming the binding forces holding it within the material.
$V(x) = \Phi - eFx$
F = electric field, x = distance from metal surface, e = electron charge



[30%]

(c)



The work function will define the intensity of the electric field F that will need to be applied to the tips (and therefore to the device), in order to obtain a certain number of emitted electrons. The number of emitted electrons will define the brightness of the pixels whose light emission the flux of electrons will trigger.

[30%]

(d) The main component that will need to be modified is the fluorescent screen that will now need to emit light at infra-red wavelengths. In order for the screen to function, you will need phosphors that emit a sufficient intensity of light when hit by a flux of electrons that can be reached with an applied voltage that does not damage the tips. If all these can be achieved, a field-effect infra-red screen can be realised.

[20%]

4. (a) A bit-based computer utilises bits that can only have two values, 0 and 1. A qubit instead can be a one, a zero, or, crucially, any quantum superposition of these.

[10%]

(b) Superposition: a qubit can exist in a combination of both 0 and 1 states simultaneously. As more qubits are added, the processing power increases exponentially. Quantum computer with n qubits can be in an arbitrary superposition of up to 2^n different states simultaneously (this compares to a normal computer that can only be in *one* of these 2^n states at any one time).

Entanglement: this is a strong correlation between two or more qubits, such that the state of one instantly influences the state of the others, regardless of the distance separating them. Entanglement allows qubits to work together on a problem in a highly coordinated way, enabling complex calculations to be performed in parallel in a single operation.

[20%]

(c) A quantum computer with 50 qubits can evaluate over a quadrillion states simultaneously, a task impossible for classical computers that would deal with 50 single-valued bits at a time. This gives quantum systems a great advantage for solving complex problems such as simulating molecular interactions.

[10%]

(d) Trapped ions, photons, superconducting circuits. They all provide access to controllable 2-level systems: electron spin, light polarisation, magnetic flux.

[15%]

(e) Initialisation (preparing qubits into a known, desired starting state, usually the $|0\rangle$ state, before algorithms run), readout (the process of measuring the final state of qubits $|0\rangle$ or $|1\rangle$, after a quantum computation), and manipulation (to induce superposition and entanglement, altering the states without collapsing them, via a gate operation).

[30%]

(f) Decoherence in quantum computers is a process whereby quantum states (in superposition and entanglement) lose or modify their quantum properties in an uncontrolled way. This causes errors and computational failure, essentially collapsing the quantum state into classical bits. It's the main hurdle for building large-scale quantum computers, requiring sophisticated quantum error correction and/or better isolation of the qubits from the environment or from other sources of decoherence, in order to extend "coherence times" for time spans long enough to be able to carry out reliable operations.

[15%]