4D10 - 2026 - Cribs

Q1.

(a)

- Local buckling

  $\frac{c}{t} = \frac{97}{6} = 16.7 > 14\varepsilon$ Class 4 10

- Flexural buckling 5

- Flexural-torsional buckling 5

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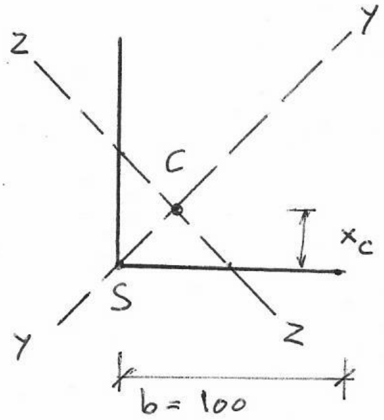
1 (b) $\frac{c}{t} = \frac{97}{6} = 16.7 > 14\varepsilon \rightarrow$ Class 4



$$x_c = \frac{b^2 t}{2} / 2bt = \frac{b}{4}$$

$$I_{zz} = 2\left(\frac{b}{\sqrt{2}}\right)^3 \sqrt{2}\, t / 12 = \frac{b^3 t}{12} = 0.5\,(10^6)\ \mathrm{mm}^4$$

$$I_{yy} = \sqrt{2}\, t \left(\frac{2b}{\sqrt{2}}\right)^3 / 12 = \frac{b^3 t}{3}$$

$$y_0 = -\frac{\sqrt{2}}{4} b \qquad z_0 = 0$$ 10

$$I_0 = I_{yy} + I_{zz} + A y_0^2 = \frac{b^3 t}{3} + \frac{b^3 t}{12} + \frac{b^3 t}{4} = \frac{2}{3} b^3 t = 4(10^6)\ \mathrm{mm}^4$$

Cross-sectional capacity (Local buckling)

$$\sigma_{cr} = 0.43 \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{6}{100}\right)^2 = 280\ \mathrm{MPa}$$

$$\lambda = \sqrt{\frac{355}{280}} = 1.126 \rightarrow \rho = \frac{1}{\lambda}\left(1 - \frac{0.188}{\lambda}\right) = 0.74$$

$$A_{eff} = 2\rho b t = 888\ \mathrm{mm}^2$$

$$N_c = A_{eff} \cdot f_y = 315\ \mathrm{kN}$$ 30

Flexural buckling about z-z

$$P_z = \frac{\pi^2 E I_{zz}}{L_e^2} = 987\ \mathrm{kN} \qquad (L_e = 1\ \mathrm{m})$$

$$\lambda = \sqrt{\frac{N_c}{P_z}} = 0.57 \qquad \text{curve b : } \alpha = 0.34$$

$$\phi = \frac{1}{2}\left(1 + 0.34(\lambda - 0.2) + \lambda^2\right) = 0.725$$

$$\chi = \left(\phi + \sqrt{\phi^2 - \lambda^2}\right)^{-1} = 0.85 \rightarrow N_{b,z} = \underline{267}\ \mathrm{kN}$$ 20

Flexural-torsional buckling

$$P_y = \frac{\pi^2 E I_{yy}}{L_e^2} = 3948\ \mathrm{kN}$$

$$P_\phi = \frac{A}{I_0} GJ = G \frac{2bt}{\cancel{\frac{2}{3}} b^3 t} \cancel{\frac{2}{3}} b t^3 = 2G\frac{t^3}{b} = 332\ \mathrm{kN}$$

$$P_{\phi,y} = \frac{1}{2\left(1 - \frac{A y_0^2}{I_0}\right)} \left(P_y + P_\phi - \sqrt{(P_y - P_\phi)^2 + 4 \frac{A y_0^2}{I_0} P_y P_\phi} \right)$$

$$\frac{A y_0^2}{I_0} = \cancel{2bt} \frac{\cancel{b^2}}{8} \frac{3}{\cancel{2 b^3 t}} = \frac{3}{8}$$

$$P_{\phi,y} = \frac{4}{5} \left(P_\phi + P_y - \sqrt{(P_y - P_\phi)^2 + \frac{3}{2} P_y P_\phi} \right) = 321.6 \text{ kN}$$

$$\lambda = \sqrt{\frac{N_c}{P_{\phi,y}}} = \sqrt{\frac{315}{322}} = 0.99 \qquad \text{curve b : } \alpha = 0.34$$

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$$\phi = \frac{1}{2} \left(1 + 0.34 (\lambda - 0.2) + \lambda^2 \right) = 1.136$$

$$\chi = \left(\phi + \sqrt{\phi^2 - \lambda^2} \right)^{-1} = 0.59$$

$$N_{b,\phi} = \chi N_c = \underline{186 \text{ kN}} \qquad \underline{\text{critical}}$$

Q2

(a)

$$EI\frac{d^2v}{dx^2} + N(v+v_o) = 0 \qquad v_o = e_o \sin\frac{\pi x}{L}$$

$$EI\frac{d^2v}{dx^2} + Nv = -Ne_o\sin\frac{\pi x}{L} \qquad (1)$$ 20

(b)

① Homogeneous eqn:

$$EI\frac{d^2v}{dx^2} + Nv = 0 \qquad \text{Set } \lambda = \sqrt{\frac{N}{EI}}$$

$$v'' + \lambda^2 v = 0$$

$$\rightarrow \quad v = A\sin\lambda x + B\cos\lambda x$$ 10

② Particular solution

$$v_p = C\sin\frac{\pi x}{L} \xrightarrow{(1)} -EI\frac{\pi^2}{L^2}C\cancel{\sin\frac{\pi x}{L}} + NC\cancel{\sin\frac{\pi x}{L}} = -Ne_o\cancel{\sin\frac{\pi x}{L}}$$

$$C\left(N - \frac{\pi^2 EI}{L^2}\right) = -Ne_o \quad \rightarrow \quad C = -\frac{Ne_o}{N-N_E} = e_o\frac{N/N_E}{1-N/N_E}$$

$$\rightarrow \quad v = A\sin\lambda x + B\cos\lambda x + e_o\frac{N/N_E}{1-N/N_E}\sin\frac{\pi x}{L}$$ 10

B.C. $\quad x=0 \quad v=0 \quad \rightarrow \quad B=0$

$\qquad x=L \quad v=0 \quad \rightarrow \quad A=0$

$$\boxed{v = e_o\frac{N/N_E}{1-N/N_E}\sin\frac{\pi x}{L}}$$ 10

$$M = -N(v+v_o) = -Ne_o\left(1+\frac{N/N_E}{1-N/N_E}\right)\sin\frac{\pi x}{L} = -\frac{Ne_o}{1-N/N_E}\sin\frac{\pi x}{L}$$

(c)

failure when:

$$\frac{N}{A} + \frac{M_{max}}{W} = f_y$$

$$N + \frac{A}{W}N\frac{e_o}{1-N/N_E} = N_p \qquad N_p = Af_y \qquad W = \frac{I}{d}$$ 20

d ← max. distance to edge

$$N + \frac{d}{r^2} e_0 \frac{N}{1 - N/N_E} = N_p \qquad r = \sqrt{\frac{I}{A}}$$

Set $\chi = \frac{N}{N_p} \qquad \lambda = \sqrt{\frac{N_p}{N_E}} \qquad \eta = \frac{d}{r^2} e_0$

$$\rightarrow \quad \chi + \eta \frac{\chi}{1 - \lambda^2 \chi} = 1$$

$$\rightarrow \quad \chi (1 - \lambda^2 \chi + \eta) = 1 - \lambda^2 \chi \quad \rightarrow \quad -\lambda^2 \chi^2 + \overbrace{(1 + \eta + \lambda^2)}^{= 2\phi \text{ (definition)}} \chi - 1 = 0$$

$$-\lambda^2 \chi^2 + 2\phi \chi - 1 = 0$$

$$\chi = \frac{2\phi \pm \sqrt{4\phi^2 - 4\lambda^2}}{2\lambda^2} = \frac{\phi - \sqrt{\phi^2 - \lambda^2}}{\lambda^2} = \frac{\cancel{\phi^2} - \cancel{\phi^2} + \lambda^2}{\lambda^2 (\phi + \sqrt{\phi^2 - \lambda^2})}$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}}$$

3D3 - 2025 - Steel Question 3

$$A = (1700)(18) + 2(600)(40) = 78\,600 \ \text{mm}^2$$

$$I_{zz} = 2(40)(600)^3/12 = 1.44\,(10^9) \ \text{mm}^4$$

$$I_{yy} = (18)(1700)^3/12 + 2(40)(600)\left(\frac{1700}{2} + \frac{40}{2}\right)^2 = 4.37\,(10^{10}) \ \text{mm}^4$$ 10

$$I_T = 2.89\,(10^7) \ \text{mm}^4 \quad \text{(given)}$$

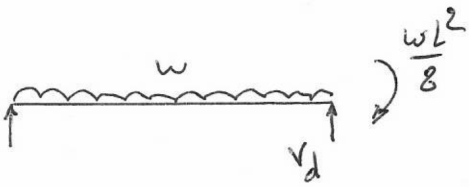
$$I_w = 1.09\,(10^{15}) \ \text{mm}^6 \quad \text{(given)}$$

$$\text{s.w.} = A\,(78.5 \ \text{kN/m}^3) = 6.17 \ \text{kN/m}$$

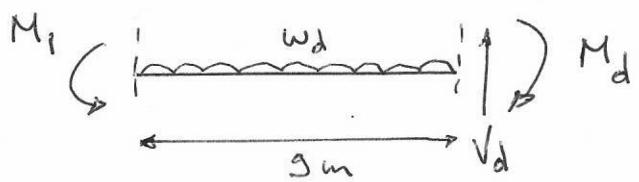
$$w_d = 1.35\,(15 + 6.17) + 1.5\,(12.5) = 47.3 \ \text{kN/m}$$

$$M_d = w_d \frac{(45)^2}{8} = 11\,980 \ \text{kNm}$$

$$V_d = \left(\frac{wL^2}{2} + \frac{wL^2}{8}\right)/L = \frac{5wL}{8} = 1331 \ \text{kN}$$



(i) $$M_{cr,0} = \frac{\pi^2 E I_{zz}}{L_{cr}^2}\sqrt{\frac{I_w}{I_{zz}} + \frac{L_{cr}^2 G I_T}{\pi^2 E I_{zz}}} = 31\,782 \ \text{kNm} \quad (L_{cr} = 9 \ \text{m})$$ 10



$$M_1 = M_d - V_d \cdot 9 + w_d \frac{9^2}{2} = 2154 \ \text{kNm}$$

Data sheets: $\psi = \frac{M_1}{M_d} = 0.18 \rightarrow C_1 \approx 1.65$ 10

$$\rightarrow M_{cr} = C_1 M_{cr,0} = 52\,500 \ \text{kNm}$$

Class?

web: $c/t = \frac{1700}{18} < 100 \rightarrow$ Class 3

flange: $c/t = \frac{(600-18)/2}{40} = 7.27 < 10\varepsilon \rightarrow$ Class 2

$\rightarrow$ 3 10

$$W_{el} = \frac{I_{yy}}{1700/2 + 40} = 4.91\,(10^7) \ \text{mm}^3$$

$$M_c = (355)\,W_{el} = 17\,431 \ \text{kNm}$$

$$\lambda_{LT} = \sqrt{\frac{M_c}{M_{cr}}} = 0.576 \qquad \text{curve } d : \ \alpha = 0.76$$

$$\phi = \tfrac{1}{2}\left(1 + \alpha(\lambda_{LT} - 0.2) + \lambda_{LT}^2\right) = 0.809$$

$$\chi = \left(\phi + \sqrt{\phi^2 - \lambda_{LT}^2}\right)^{-1} = 0.726$$ 10

$$M_{b,Rd} = \chi M_c = 12660 \text{ kNm} \qquad \underline{\text{Ok}}$$

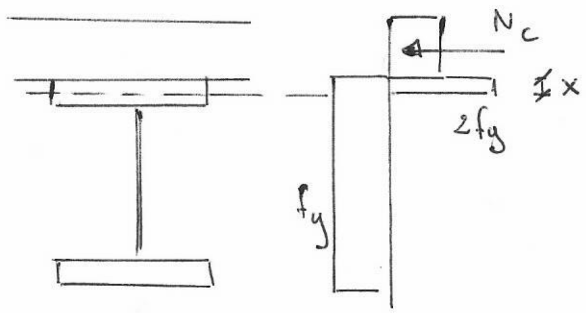
(ii) $b_{eff} = \min\left(\frac{0.85(45)}{4} + b_o,\ 2.5\right) = 2.5 \text{ m}$

$$f_{cd} = \frac{30}{1.5} = 20 \text{ MPa}$$ 10

$$0.85 f_{cd}\, b_{eff}\, h_c = 10\,625 \text{ kN}$$

$$A \cdot f_y = 25\,063 \text{ kN}$$

Assume N.A. in flange

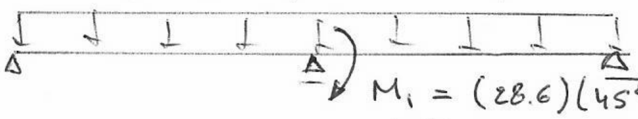


$$x = \frac{25063 - 10625}{2(600)(355)} = 33.9 \text{ mm} < 40 \quad \underline{\text{Ok}}$$

$$M = (25063)\left(\frac{1700}{2} + 40 + \frac{250}{2}\right) - (600)(33.9)(2)(355)\left(\frac{33.9}{2} + \frac{250}{2}\right)$$

$$= 23\,390 \text{ kNm}$$ 20

$$(6.17)(1.35) + (15)(1.35) = 28.6 \text{ kN/m}$$

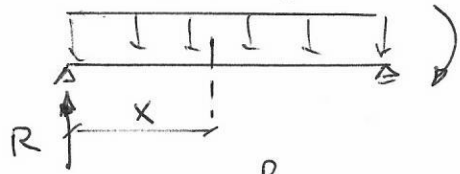


$$M_1 = (28.6)(45^2)/8 = 7234 \text{ kNm}$$

$$(12.5)(1.5) = 18.75 \text{ kN/m}$$

$$M_2 = (18.75)(45)^2/16 = 2373 \text{ kNm}$$

47.3 kN/m



$$2373 + 7234 = 9607 \text{ kNm}$$

$$R(45) - (47.3)\frac{(45)^2}{2} + 9607 = 0 \rightarrow R = 851 \text{ kN}$$

$$x = \frac{R}{47.3} = 18 \text{ m} \qquad M_d = 18R - \frac{(47.3)(18)^2}{2} = 7660 \text{ kNm}$$

$\underline{\text{Ok}}$ 20

Question 4

UB 356 × 127 × 33 : $t_w = 6$ mm , $t_f = 8.5$ mm , $b = 125.4$ mm

$h_w = 349 - 2(8.5) = 332$ mm

$r = 10.2$ mm

$$A_v = A - 2b\,t_f + (t_w + 2r)\,t_f = 4210 - 2(125.4)(8.5) + (6 + 20.4)(8.5)$$

$$= 2303 \text{ mm}^2 \quad \text{but} \geq h_w \cdot t_w = (332)(6) = 1992 \text{ mm}^2$$

$$V_d = A_v \cdot \frac{f_y}{\sqrt{3}} = 472 \text{ kN}$$

$$\frac{h_w}{t_w} = \frac{332}{6} = 55 < 72\varepsilon = (72)(0.81) = 58$$ No shear buckling

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(a) Try 4-M22 $\quad (0.6) A_s f_{ub} / \gamma_{M2} = (0.6)(303)(800)/1.1 = 132$ kN

$\times 4 = 529$ kN

Ok

(b) Bearing $\quad d_o = 24$

end bolt : $\alpha_d = \dfrac{50}{72} = 0.69$

inner bolt : $\alpha_d = \dfrac{67}{72} - \dfrac{1}{4} = 0.67$

$k_1 = \min\left(2.8\,\dfrac{50}{24} - 1.7,\ \cancel{14}\ 2.5\right) = 2.5$ 25

$F_{b,Rd} = (2.5)(0.67)(490)(22)\,t/1.1 = 16415\,t > 472000$ N

$\rightarrow t > 7.2$ mm

Plastic shear

$A_v = [300 - 4(24)]\,t = 204\,t$

$204\,\dfrac{f_y}{\sqrt{3}}\,t = 41812\,t > 472000$ N $\rightarrow t > 11.3$ mm

Take 12 mm

20

Block tear-out

$A_v = [300 - 3(24) - (50 + 12)]\,t = 166\,t$

$A_t = (50 - 12)\,t = 38\,t$

$0.5\,f_u A_t / 1.1 + \dfrac{f_y}{\sqrt{3}} A_v > 472$ kN

$\left[(0.5)(490)(38)/1.1 + \left(\dfrac{355}{\sqrt{3}}\right)(166)\right] t > 472000$ N

25

$\rightarrow t > 11.1$ mm Ok

Combined bending and shear

$V_d = 472 \text{ kN}$

$M_d = V_d \, (50 \text{ mm}) = 23.6 \text{ kNm}$

$V_{pl} = (12)(300)\dfrac{(355)}{\sqrt{3}} = 738 \text{ kN} \rightarrow V_d > \dfrac{V_{pl}}{2}$

$\rho = \left(\dfrac{2V_d}{V_{pl}} - 1\right)^2 = 0.078$

$f_{yr} = (1-\rho) f_y = 327 \text{ MPa}$

$M_r = \dfrac{(12)(300)^2}{4}(327) = 88 \text{ kNm}$ $\underline{\text{Ok}}$