Version KAS/4

EGT3
ENGINEERING TRIPOS PART IIB

Friday 1 May 2026 9.30 to 11.10

Paper 4D17

PLATE & SHELL STRUCTURES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: None
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 At a general point in a shallow surface, the out-of-plane variation is typically described by the equation

$$-z = \frac{1}{2}k_{11}x^2 + k_{12}xy + \frac{1}{2}k_{22}y^2$$

where the coordinate, z, is normal to the (x, y) plane.

(a) Comment on the significance of the minus sign before z and describe what is meant by k_{11}, k_{12} and k_{22}. [20%]

(b) At a particular point in a surface, $k_{11} = 3$, $k_{12} = 3$ and $k_{22} = 3$.

(i) Draw a Mohr's Circle of curvature *vs* twist. [20%]

(ii) Determine the principal curvatures and their orientation relative to the (x, y) coordinate system. Hence, describe the special property of the surface. [20%]

(c) At another point in the surface, $k_{11} = -2$, $k_{12} = 0$ and $k_{22} = 4$. Show that two straight lines pass through the point in the surface, and determine their relative orientation. [40%]

2 A shell of revolution is formed by rotating about the downwards z-axis, a meridian generated by the parabola, $z = Ar^2$. The shell is subjected to an internal pressure p.

(a) Describe, without calculation, the nature of the Membrane Hypothesis on thin shell behaviour. [10%]

(b) In view of the Membrane Hypothesis, determine the stress resultants, N_θ and N_ϕ, for the pressurised shell, where θ and ϕ are angular coordinates in the circumferential and meridional directions, respectively. [40%]

(c) Confirm that, for very small values of ϕ, N_θ and N_ϕ are approximately equal to each other, as defined in part (b): what is this stress-state more commonly known as? [10%]

(d) State Zick's Law and perform a check for very small values of ϕ as defined in part (b). [40%]

3 A thin square plate, of side-length a, undergoes a small transverse deflection, w, where

$$w = \frac{B}{2}\left[x^2 - \nu y^2\right] + Cxy$$

The coordinate system, Oxy, is located in the centre of the plate with the x- and y axes parallel to adjacent sides. The Young's modulus is E, the Poisson's ratio is ν, and the thickness of the plate is t.

(a) Determine the kinematical properties of the deformed shape. [20%]

(b) Determine the stress resultants everywhere within the plate. [20%]

(c) Determine the applied loading in terms of the constants, B and C, and other parameters of the plate. [40%]

(d) Compute the Gaussian curvature of the plate. In view of your result, what extra equilibrium considerations are needed when displacements become larger? [20%]

4 Figure 1 shows a long, thin strip in plan-view loaded by equal and opposite moments, M_0, which are inclined by angle, β, to each square end. The strip is made from linearly elastic, isotropic material, and has a very small thickness, t, compared to its width, w. The length of strip, L, is very much longer than w.

(a) Show that the applied loading corresponds to uniform torsion and uniform bending in ratio of $\tan\beta$. [10%]

(b) The deformation of the strip is indicated by the pattern of unifomly spaced, parallel generators, inclined by angle α to the long edge.

(i) When are generators a viable description of deformation? [10%]

(ii) Why are the generators parallel and uniformly spaced? [10%]

(iii) What is the significance of $L \gg w$ for the proposed generator pattern? [10%]

(iv) Why is it *not* appropriate to apply Mansfield's *Inextensibilty Theory*? [10%]

(v) Without calculation, state α when $\beta = 90°$. [10%]

(c) When β is not equal to zero, determine the bending moment, hence, curvature, across a given generator, as defined in part (b). [10%]

(d) Calculate the total strain energy stored in bending, U. By considering the variation of U with respect to α, find the relationship between β and α. [20%]

(e) For the case of pure torsion, verify that generators are inclined by 45° to the long edge. [10%]

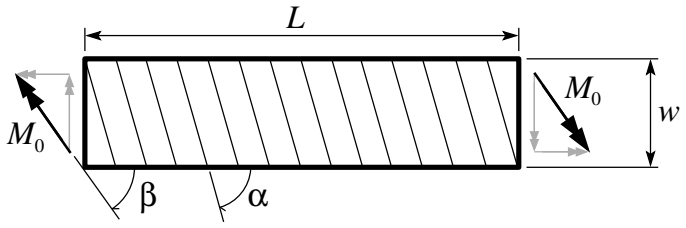

Fig. 1

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