EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 30 April 2024 14.00 to 15.40

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 4D6 Data Sheet (8 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A two-storied structure is to be constructed on a hill slope as shown in Fig. 1. The floor masses are 20,000 kg each and each column has a flexural stiffness *EI* of 6 MN m^2. The base of each of the columns is rigidly attached to the foundations. A mode shape of [0.34 1] is assumed by the designer.

(a) Estimate the natural frequency of the structure. Comment on the accuracy of your estimate. [20%]

(b) The earthquake spectrum shown in Fig. 2 is to be used for the preliminary design of this structure. Consider two design cases where the damping in the structure is either 2% or 10% of the critical damping. Calculate the floor displacements and floor accelerations that can be anticipated for the two design cases. [40%]

(c) Estimate the shear force at the base of each of the columns using your answers in part b). Comment on how the shear force changes for the two design cases. [25%]

(d) Discuss how the actual response of this structure may differ in a different earthquake. [15%]

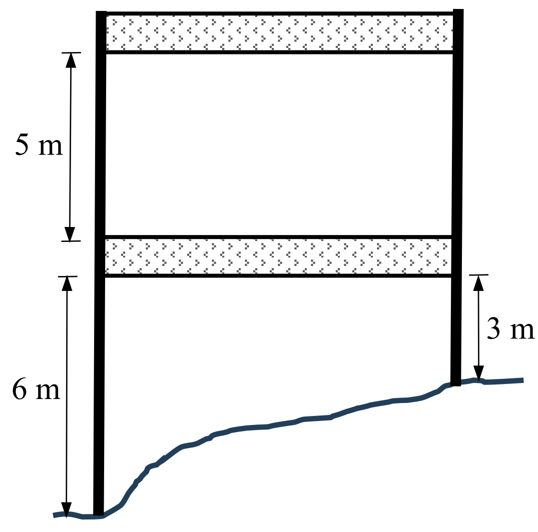

Fig. 1

Fraction of critical damping, ξ = 0, 2, 5, 10, 20%

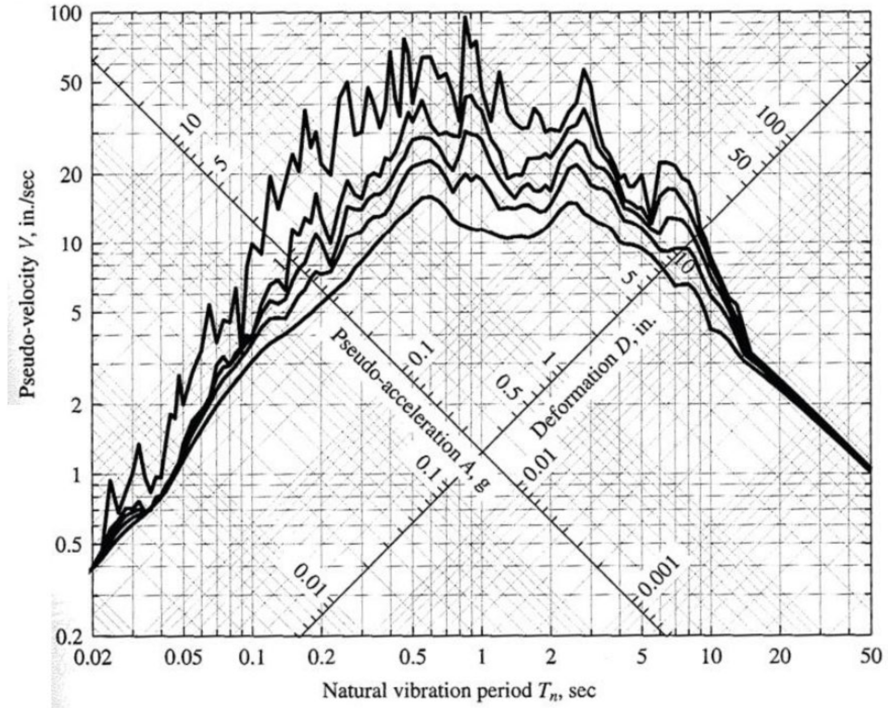

Fig. 2

 (TURN OVER

2 A footbridge is constructed by building outwards from each abutment. Just prior to completion, each half of the bridge behaves as a uniform cantilever beam of length L = 20 m, bending stiffness $EI = 1 \times 10^{10}$ N m^2 and mass per unit length m = 500 kg m^{-1}, as shown in Fig. 3(a).

(a) Using Rayleigh's principle and assuming a third-order polynomial form for the fundamental mode shape, estimate the fundamental natural frequency of one half of the bridge. [45%]

(b) Wind loading causes an unacceptable resonant response of the cantilevers. To mitigate this, it is suggested that a temporary mass M = 1000 kg is added to the tip of each cantilever, as shown in Fig. 3(b). Estimate the new fundamental frequency. [15%]

(c) An alternative suggestion is to anchor the tip of each cantilever to the ground using a tensioned cable of length l = 5 m, cross-sectional area $A = 1 \times 10^{-4}$ m^2 and Young's modulus E = 210 GPa, as shown in Fig. 3(c). Estimate the new fundamental frequency in this case. [20%]

(d) Comment on the viability of the two proposals and suggest an alternative option. [20%]

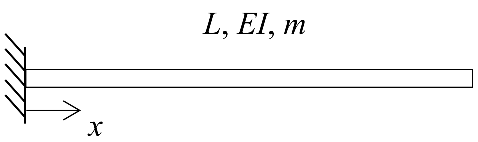

Fig. 3(a)

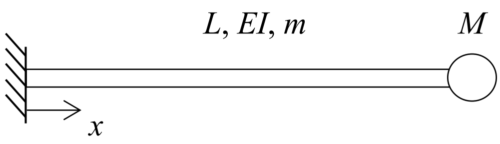

Fig. 3(b)

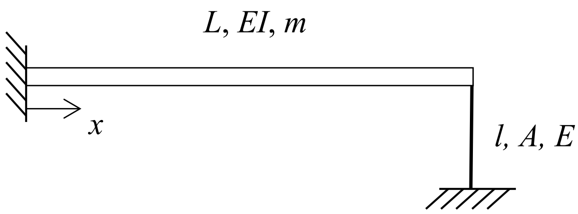

Fig. 3(c)

3 (a) Explain why loose saturated sands may liquefy while dense saturated sands may not, when subjected to earthquake loading. [10%]

(b) Explain briefly how finite element analyses can be carried out for two-phase materials like saturated soil. [15%]

(c) What is meant by the term semi-infinite extent of a soil layer? How is this modelled in the finite element analyses? [15%]

(d) A shallow foundation with a footprint of $2 \text{ m} \times 1 \text{ m}$ and embedded to a depth of 0.5 m as shown in Fig. 4 is to be constructed on a loose, saturated sand layer that is 10 m thick overlying bedrock. The sand has a saturated unit weight of 19.6 kN m^{-3}. The unit weight of water can be taken as 10 kN m^{-3}. The specific gravity of sand particles is 2.65 and the Poisson's ratio of sand is 0.3. Estimate the small-strain shear modulus and hence the shear wave velocity in the sand layer. Consider a reference plane 3 m below the sand surface and assume the water table is at the sand surface. [20%]

(e) The shallow foundation in part d) will be subjected to a peak dynamic vertical load of 1500 kN and a peak dynamic moment load of 300 kN m during a design earthquake. An excess pore pressure of 20 kPa is expected to be generated during this event at the reference plane. The design earthquake is expected to produce shear strains that will reduce the shear modulus by a factor of 10. Estimate the maximum vertical displacement and maximum rotation that this foundation may suffer. Comment on the accuracy of your estimates. [40%]

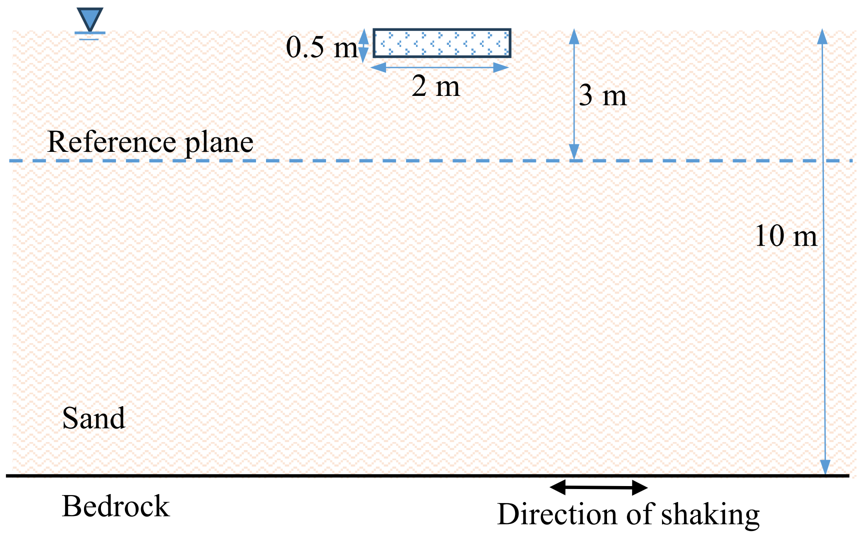

Fig. 4

4 Consider the bluff cross section shown in Fig. 5, representing a 1 m segment of a bridge deck. The deck width is $B = 31$ m, which represents the normalising dimension for all aerodynamic coefficients. The air density is 1.2 kg m^{-3}.

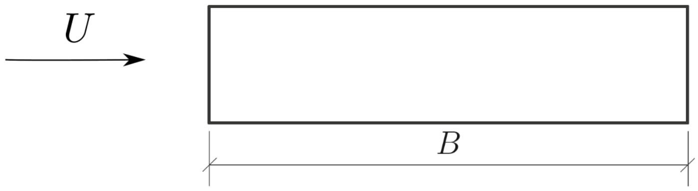

Fig. 5

(a) The dynamic properties of the first lateral mode of the deck are as follows: circular frequency $0.2\times2\pi$ rad s^{-1}; structural damping ratio $\xi_s = 0.002$; and deck mass is 22740 kg m^{-1}. The design mean wind speed is $U = 30$ m s^{-1}, with turbulence intensity in the along-wind direction of 0.1 and a normalised longitudinal spectrum S_{uu}/σ_u^2 given in Fig. 6. The drag coefficient is 0.4 and the aerodynamic admittance χ_D^2 is provided in Fig. 7.

Using a mean displacement of 0.185 metres, a peak factor of 3.5 and a background factor of $B_g^2 = 13.7$, estimate the peak lateral displacements due to along-wind turbulence using the Gust Factor Approach. [40%]

Note: The admittance is given with respect to circular frequency, rather than non-dimensional frequency.

(b) The circular frequency of the first vertical mode is $0.1\times2\pi$ rad s^{-1} and the circular frequency of the first torsional mode is $0.23\times2\pi$ rad s^{-1}. The aerodynamic derivatives are given in Fig. 8.

(i) What are the types of aeroelastic instabilities a bridge deck can experience? [10%]

(ii) Which type of instability is the deck experiencing? [10%]

(iii) Neglecting the structural damping, estimate the critical flutter wind speed. [10%]

(c) There have been 30 years of continuous measurements of the wind speed at the site. Out of these, the maximum hourly mean wind speed for each year is extracted and ranked in ascending order. Two out of the 30 values are representative of the extreme value distribution.

Rank [-]	Mean Wind speed [m s^{-1}]
6	18.1
30	28.6

Using these values, estimate the return period of the critical flutter wind speed from part b). [30%]

Note: Use a critical flutter wind speed of 40 m s^{-1} if you have not calculated the value in part b).

Hint: You may use either an analytical or a graphical method. In the case of the latter, set the abscissa (horizontal axis) to range from -2 to 10 and the ordinate (vertical axis) from 15 to 50.

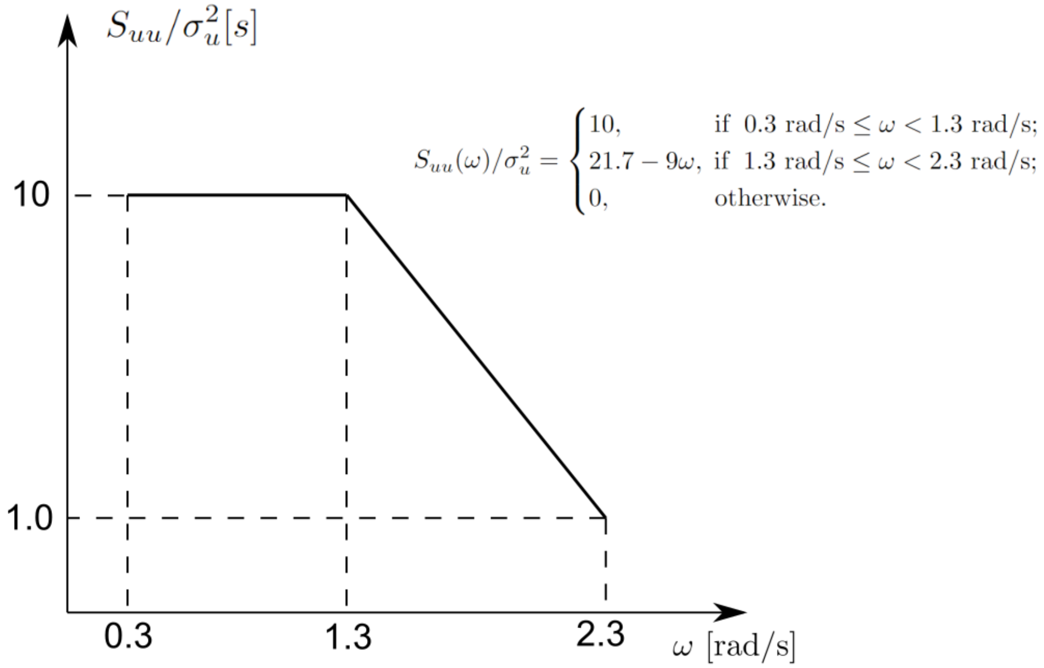

Fig. 6

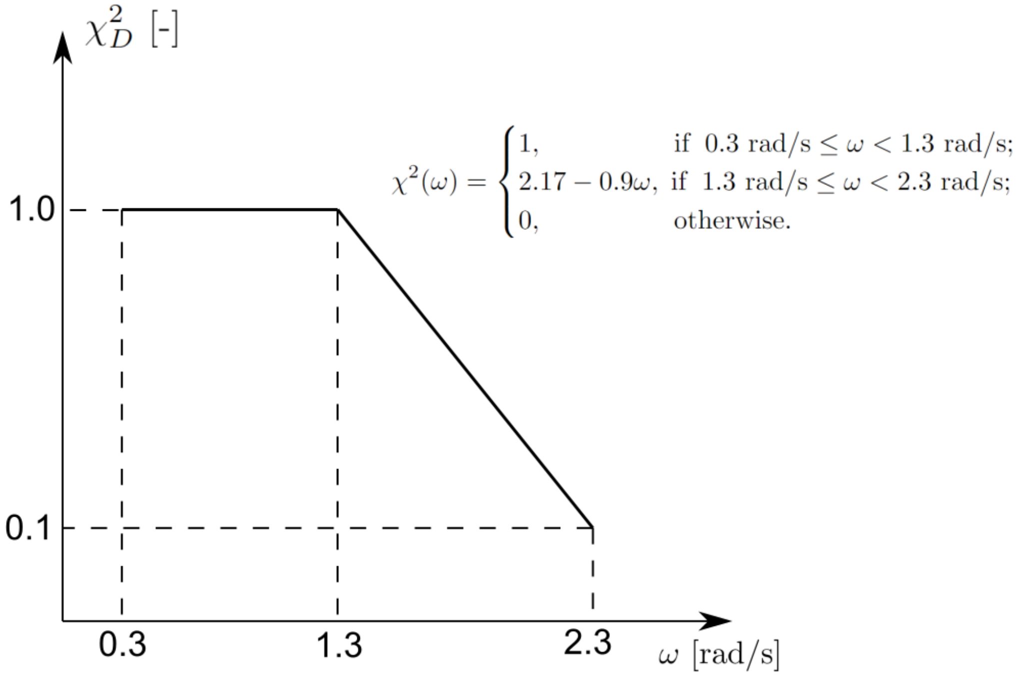
χ_D^2 [-]
1.0
0.1
0.3
1.3
2.3
ω [rad/s]
$\chi^2(\omega) = \begin{cases} 1, & \text{if } 0.3 \text{ rad/s} \leq \omega < 1.3 \text{ rad/s}; \\ 2.17 - 0.9\omega, & \text{if } 1.3 \text{ rad/s} \leq \omega < 2.3 \text{ rad/s}; \\ 0, & \text{otherwise.} \end{cases}$

Fig. 7

(cont.

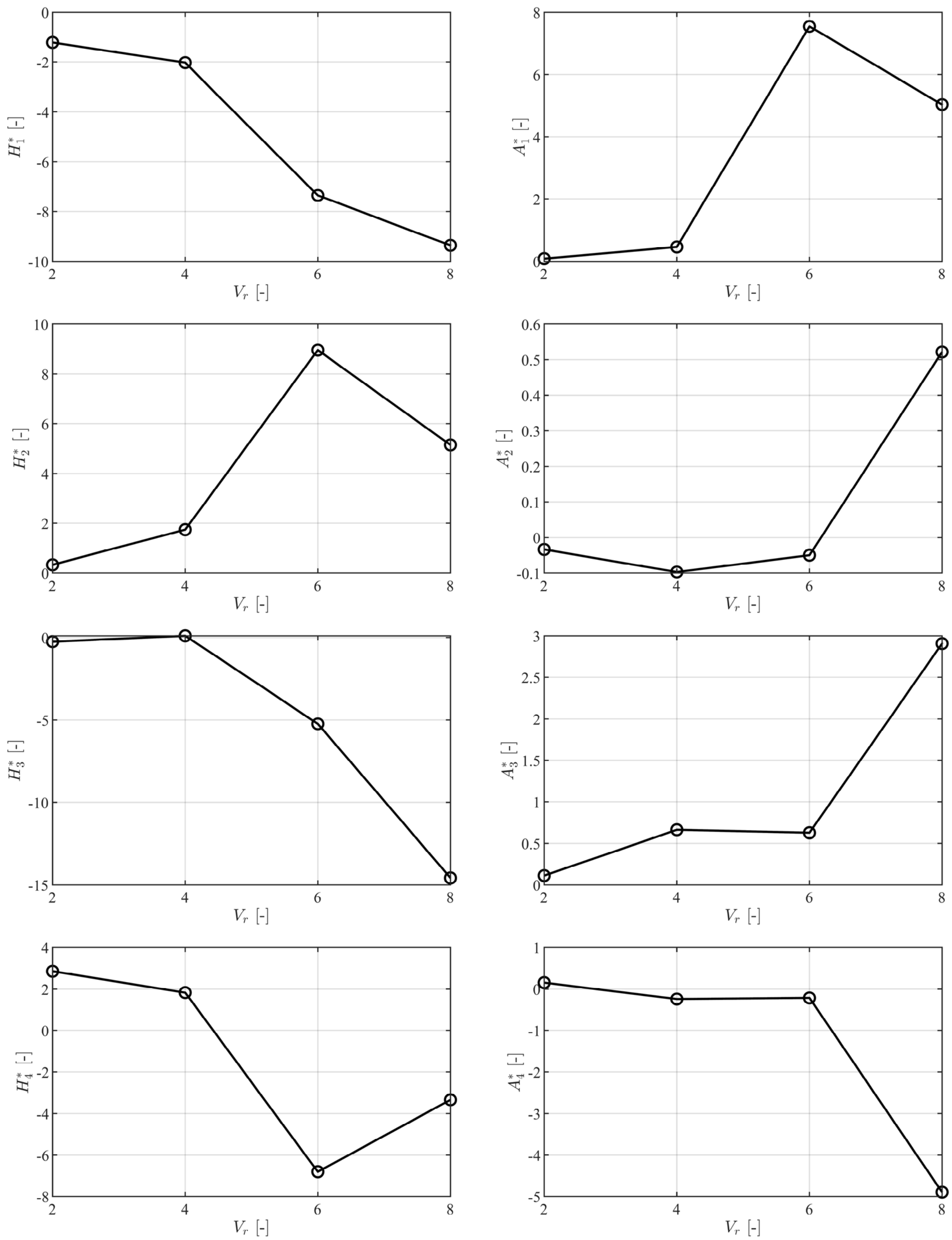

Fig. 8

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Module 4D6: Dynamics in Civil Engineering

Data Sheets

Equivalent SDOF Systems

For an n-DOF system, with mass matrix $\underline{\underline{M}}$ and stiffness matrix $\underline{\underline{K}}$, responding in mode shape $\underline{\overline{u}}$ to an applied force $\underline{F}$, the parameters of the equivalent SDOF system are:

$$M_{eq} = \underline{\overline{u}}^T \underline{\underline{M}}\, \underline{\overline{u}} \qquad K_{eq} = \underline{\overline{u}}^T \underline{\underline{K}}\, \underline{\overline{u}} \qquad F_{eq} = \underline{F}^T \underline{\overline{u}}$$

For a continuous beam, of length L, mass per unit length m and bending stiffness EI, responding in mode shape $\overline{u}(x)$ to an applied force $f(x)$, the parameters of the equivalent SDOF system are:

$$M_{eq} = \int_0^L m\overline{u}^2 dx \qquad K_{eq} = \int_0^L EI \left(\frac{d^2\overline{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\overline{u}\, dx$$

The corresponding natural frequency is $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}}$

Modal Analysis of a Simply-Supported Beam

$$\overline{u}_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\;eq} = \frac{mL}{2} \qquad K_{i\;eq} = \frac{(i\pi)^4 EI}{2L^3}$$

Ground Motion Participation Factor

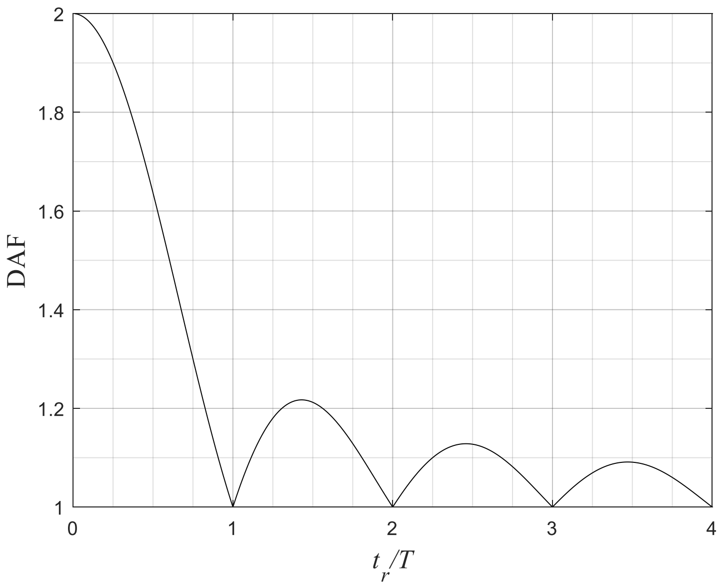
For an n-DOF system, with mass matrix $\underline{\underline{M}}$ and stiffness matrix $\underline{\underline{K}}$, responding to ground acceleration $\ddot{u}_g$, the parameters of the equivalent SDOF system are:

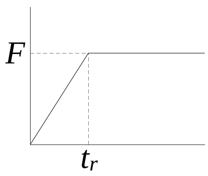
$$M_{eq} = \underline{\overline{u}}^T \underline{\underline{M}}\, \underline{\overline{u}} \qquad K_{eq} = \underline{\overline{u}}^T \underline{\underline{K}}\, \underline{\overline{u}} \qquad F_{eq} = -\Gamma M_{eq}\ddot{u}_g$$

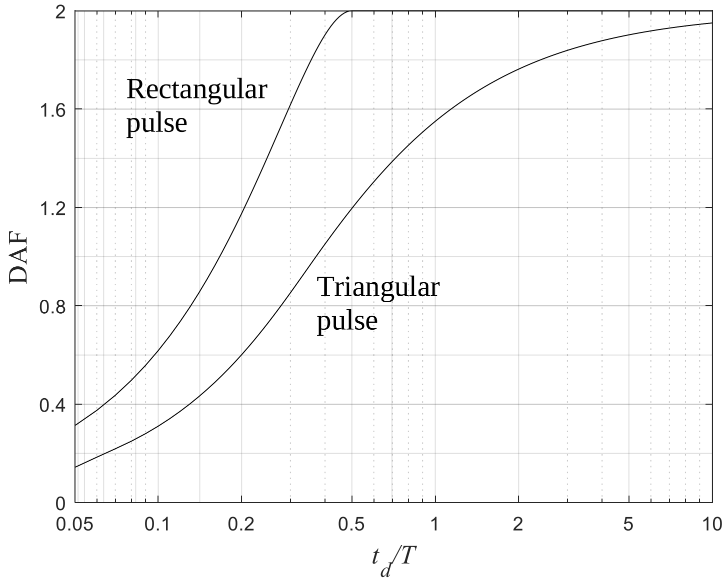
where $\underline{\overline{u}}$ is defined relative to ground and Γ is the modal participation factor

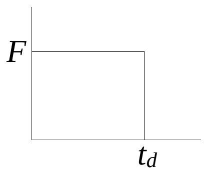
$$\Gamma = \frac{M_1\overline{u}_1 + M_2\overline{u}_2 + \cdots + M_n\overline{u}_n}{M_{eq}}$$

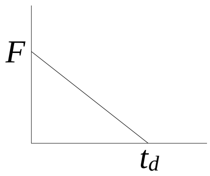
Dynamic Amplification Factors

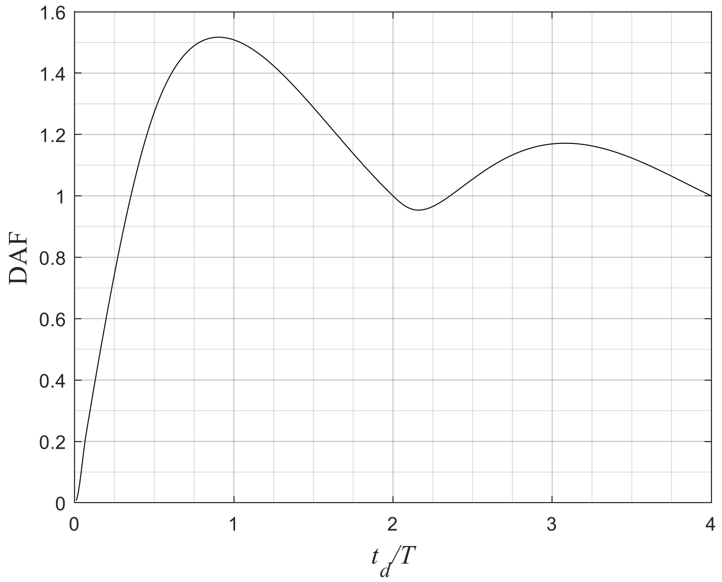


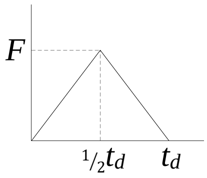




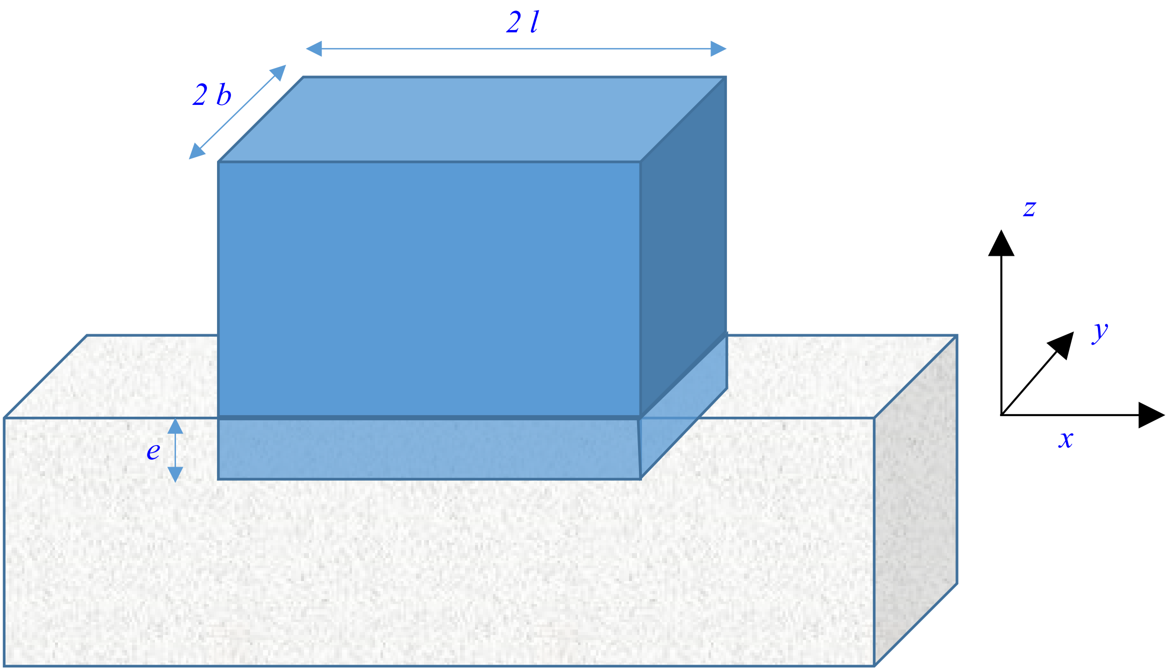








Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions $2l$ and $2b$, embedded to a depth e are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\frac{l}{b}}\left(\frac{e}{b}\right)^2\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\left(\frac{l}{b}\right)^4}\left(\frac{e}{b}\right)^2\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\,\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(\mathrm{G_s} + \mathrm{eS_r})\gamma_{\mathrm{w}}}{1+\mathrm{e}}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_{\mathrm{d}} = \frac{\mathrm{G_s}\gamma_{\mathrm{w}}}{1+\mathrm{e}}$$

Effective mean confining stress

$$\mathrm{p}' = \sigma'_{\mathrm{v}}\frac{\left(1+2\mathrm{K_o}\right)}{3}$$

where σ'_{v} is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$\mathrm{K_o} = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{\max} = 100\,\frac{(3 - e)^2}{(1 + e)}\,(p')^{0.5}$$

where p′ is the effective mean confining pressure in **MPa,** e is the void ratio and G_{max} is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{\max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a.\, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1 + K_o}{2}\sigma_v' \sin\phi'\right)^2 - \left(\frac{1 - K_o}{2}\sigma_v'\right)^2\right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity $\boldsymbol{v_s}$ as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Wind Engineering Data Sheet

1. *Extreme value statistics*

Empirical distribution:

$$P(X \leq U_i) = \frac{n_i}{N+1}$$

where: n_i – ordered sample i; N – number of samples; U_i –wind speed of sample i (samples here are maximum annual mean wind speeds).

Gumbel distribution:

$$P(X \leq U) = \exp\left(-\exp(-y)\right)$$

$$y = a(m - U)$$

where: a, m – distribution parameters; U – mean wind speed.

Return period:

$$P(X \leq U) = 1 - \frac{1}{\mathrm{T}}$$

where: $T -$ return period.

2. *Longitudinal wind buffeting response*

Turbulence intensity:

$$I_u = \frac{\sigma_u}{U}$$

where: σ_u – standard deviation of longitudinal turbulence; U – mean wind speed.

Mean force:

$$F_D = \frac{1}{2}\rho U^2 C_D A$$

where: ρ – fluid density ($\approx 1.2\ \mathrm{kg/m^3}$); C_d – drag coefficient; A – representative dimension (for bridges typically deck width B) .

Peak response:

$$x_p = x_m + q_p \sigma_x$$

where: x_p – peak response; x_m – mean response; q_p – peak factor; σ_x – fluctuating response.

Fluctuating response:

$$\sigma_x^2 = 4x_m^2 \frac{\sigma_u^2}{U^2}(B_g{}^2 + R^2)$$

where: B_g − background response; R − resonant response.

Background response:

$$B_g = \frac{1}{2\pi}\int_0^\infty \chi_D^2(\omega)\frac{S_{uu}(\omega)}{\sigma_u^2}\mathrm{d}\omega$$

where: χ_D^2 − drag admittance due to longitudinal fluctuations; S_{uu} – longitudinal wind spectrum.

Resonant response:

$$R^2 = \chi_D^2(\omega_n)\frac{S_{uu}(\omega_n)}{\sigma_u^2}\frac{\omega_n}{8\xi}$$

where: ω_n − natural frequency; ξ – damping ratio.

Aerodynamic damping:

$$\xi_a = \frac{\rho U C_D B}{2m\omega_n}$$

where: m − mass per metre.

3. *Bridge flutter*

Coupled equation of motion for a 2 dimensional system (h - vertical; α – torsional):

$$m_h\ddot{h} + c_h\dot{h} + k_h h = \frac{1}{2}\rho U^2 B\left(KH_1^*\frac{\dot{h}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + KH_4^*\frac{h}{B}\right),$$

$$m_\alpha\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha\alpha = \frac{1}{2}\rho U^2 B^2\left(KA_1^*\frac{\dot{h}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + KA_4^*\frac{h}{B}\right)$$

where: m, c, k – mass, damping coefficient, and stiffness of the corresponding degree of freedom; ρ − fluid density; B − deck width; K − reduced frequency; H_i^*, A_i^* − aerodynamic derivatives.

Reduced velocity:

$$V_r = \frac{2\pi U}{\omega B} = \frac{2\pi}{K}$$

where: ω – oscilating frequency.

4. Vortex-induced vibrations

Strouhal number:

$$\mathrm{St} = \frac{f_v H_D}{U}$$

where: f_v – vortex shedding frequency (Hz); H_D – referent dimension (depth for bridge decks; diameter for cylinder). U – flow velocity.

For circular cylinders $\mathrm{St} \approx 0.2$.

Scrouton number number:

$$\mathrm{Sc} = \frac{2\delta_s m}{\rho H_D^2}$$

where: δ_s – logarithmic decrement (=$2\pi \times$ fraction of critical damping); m – actual mass per metre; H_D – referent dimension (depth for bridge decks; diameter for cylinder). U – flow velocity.

Peak response for vortex-induced vibrations:

$$\frac{h_{viv}}{H_D} = \frac{1}{4\pi}\frac{1}{\mathrm{St}^2}\frac{1}{\mathrm{Sc}}\sqrt{2}\ c_{lat}$$

where: c_{lat} – lateral force coefficient.

For rectangular cross sections $c_{lat} \approx 1.1$.