

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 28 April 2026 09:30 – 11:10

Module 4D7

CONCRETE AND PRESTRESSED CONCRETE

*Answer **all** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Attachment: 4D7 Data Sheet (5 pages)

Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A 450 mm $\times$ 300 mm rectangular reinforced concrete section is shown in Fig. 1. The beam is reinforced with 10 mm diameter vertical stirrups at 150 mm spacings. The factored design yield stress of the steel stirrups is 435 MPa. The beam can be modelled as a hollow section with an equivalent wall thickness of 80 mm, and it can be assumed that all the steel is yielding. A design torque of $T_{Ed} = 60$ kNm is applied to the section.

- (a) What additional longitudinal reinforcement, with a design yield stress of 435 MPa, is required to sustain the applied torque? [30%]
- (b) Find the minimum design concrete strength required to ensure that the beam can sustain the applied torque. Assume $\nu_1 = 0.5$, and the additional longitudinal reinforcement found in part (a). [30%]
- (c) Calculate the shear capacity of the section using the variable angle truss model given the stirrup diameter and spacing, calculated longitudinal reinforcement from part (a), and compressive strength determined in part (b). Assume $\alpha_{cw} = 1.0$ and $\nu_1 = 0.5$. [30%]
- (d) What is the maximum design shear force the section can carry in combination with a design torque of 30 kNm? [10%]

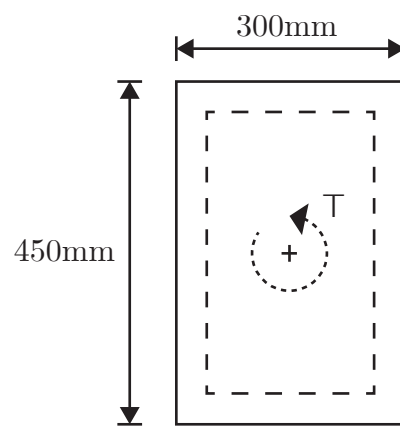


Fig. 1

2 The prestressed bridge beam shown in Fig. 2 is loaded by a sagging moment which varies from 600 kNm to 2850 kNm.

- (a) Making suitable simplifications, determine the position of the centroid and second moment of area for the section. [10%]
- (b) If the prestressing tendon, with area 3000 mm^2 , carries an effective prestress of 900 MPa, determine the top and bottom fibre stresses under the applied moments. [20%]
- (c) The permissible stresses in the concrete are 18 MPa in compression and 1 MPa in tension. Draw a Magnel diagram for the section. [50%]
- (d) Determine the minimum and maximum permissible prestress if the tendon can be placed anywhere in the section. [10%]
- (e) Comment on your results. [10%]

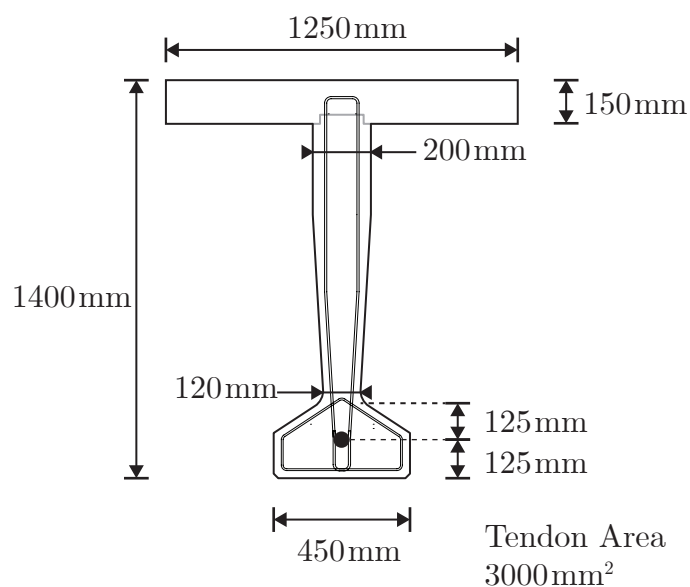


Fig. 2

3 A reinforced concrete office building built in 1960 in Tokyo is to be extended and refurbished. In order to justify the existing structure an assessment must be made of the likely future lifespan of the concrete due to the effects of carbonation. All elements in the structure have a cover to reinforcement of 25 mm. In 2020, cores were taken from a section of the frame, split in half, and a phenolphthalein solution was applied. On average, 17 mm of the core section was clear (with a standard deviation of 2 mm) and the remaining turned a deep pink colour.

(a) The designers would like the refurbished structure to be capable of resisting the effects of corrosion for at least 60 years (from 2026). Estimate the age at which corrosion of steel is likely to be first initiated due to carbonation, and determine if their ambition is achievable. [20%]

(b) What other factors affect the durability related lifespan of reused concrete, and how are they assessed? [35%]

(c) You are to act as an advisor to the engineering team. What other key considerations should they consider in the extension of life of the existing concrete structure, and how would you advise that they assess the performance of the structure. You do not need to undertake any calculations. [45%]

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Module 4D7: Data Sheet

Ultimate limit states

It shall be verified that:

$$E_d \leq R_d \quad (1)$$

The design value of the effect of actions, E_d , for a specific combination of actions should be calculated by:

$$E_d = E \left\{ \sum_{j \geq 1} \gamma_{G,j} G_{k,j} + \gamma_P P + \gamma_{Q,1} Q_{k,1} + \sum_{i > 1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \right\} \quad (2)$$

Material partial factors are normally $\gamma_s = 1.15$ for steel and $\gamma_c = 1.5$ for concrete;

Partial factors on actions are normally $\gamma_{G,j} = 1.35$ and $\gamma_{Q,1} = 1.5$.

Serviceability limit states

It shall be verified that:

$$E_d \leq C_d \quad (3)$$

The characteristic combination is:

$$E_d = E \left\{ \sum_{j \geq 1} G_{k,j} + P + Q_{k,1} + \sum_{i > 1} \psi_{0,i} Q_{k,i} \right\} \quad (4)$$

The frequent combination is:

$$E_d = E \left\{ \sum_{j \geq 1} G_{k,j} + P + \psi_{1,1} Q_{k,1} + \sum_{i > 1} \psi_{2,i} Q_{k,i} \right\} \quad (5)$$

Probability of failure

Design values of actions:

$$F_d = F_k \gamma_f \quad (6)$$

$$F_k = \mu_s + 1.645 \sigma_s \quad (7)$$

$$\sigma_s = CoV \times \mu_s \quad (8)$$

Where F_d is the design value; γ_f is the partial safety factor; F_k is the characteristic value, μ_s is the mean value, σ_s is the standard deviation, and CoV is the coefficient of variation.

Design values of product properties:

$$X_d = \frac{X_k}{\gamma_m} \quad (9)$$

$$X_k = \mu_R - 1.645 \sigma_R \quad (10)$$

$$\sigma_R = CoV \times \mu_R \quad (11)$$

Where X_d is the design value; γ_m the partial safety factor; X_k the characteristic value, μ_R the mean value, σ_R the standard deviation, and CoV the coefficient of variation.

Reliability index, β

$$\beta = \frac{\mu_R - \mu_s}{\sqrt{\sigma_R^2 + \sigma_s^2}} \quad (12)$$

Probability of failure:

$$P_f = \Phi(-\beta) \quad (13)$$

Where Φ is the standard normal cumulative distribution function.

The difference between two normally distributed variables is itself normally distributed, with mean equal to the difference of the means, and variance the sum of the squares of the standard deviations.

Durability considerations

Uniaxial diffusion into a homogenous material:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \quad (14)$$

Solution:

$$C_x = C_0 [1 - \operatorname{erf}(z)] \quad (15)$$

$$z = \frac{x}{2(Dt)^{0.5}} \quad (16)$$

Table of $\operatorname{erf}(z)$

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	∞
$\operatorname{erf}(z)$	0	0.11	0.22	0.33	0.43	0.52	0.60	0.68	0.74	0.80	0.84	0.88	0.91	0.93	0.95	0.97	1.00

Deflections

Interpolated curvature:

$$\alpha = \zeta \alpha_{||} + (1 - \zeta) \alpha_{\perp} \quad (17)$$

Where α is a deflection, $\alpha_{\perp}$ and $\alpha_{||}$ are the values for the uncracked and fully cracked conditions, ζ is a distribution coefficient:

$$\zeta = 1 - \beta \left(\frac{\sigma_{sr}}{\sigma_s} \right)^2 \quad (18)$$

Where σ_{sr} is the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking; σ_s is the stress in the tension reinforcement calculated on the basis of a cracked section; $\beta = 1.0$ for single short term loading and $\beta = 0.5$ for sustained loads or many cycles of repeated loading.

ULS Flexure

A doubly reinforced concrete section when flexural strength is reached is shown in Figure 1. It is usual to assume that failure occurs when the extreme fibre compressive strain in the concrete reaches a limiting value of 0.0035. Forces are found by equilibrium of the section.

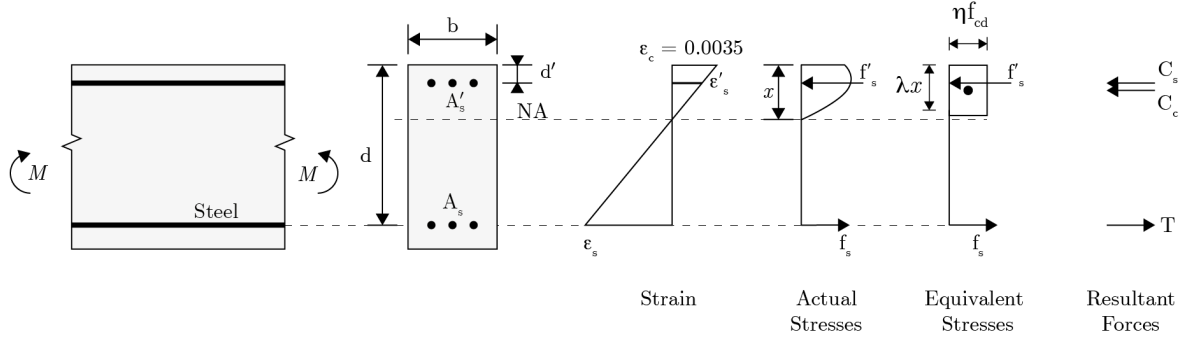


Figure 1

	$f_{ck} \leq 50 \text{ MPa}$	$50 \text{ MPa} < f_{ck} \leq 90 \text{ MPa}$
λ	0.8	$0.8 - (f_{ck} - 50)/400$
η	1.0	$1.0 - (f_{ck} - 50)/200$

ULS Shear and Torsion

For unreinforced webs at ULS:

$$V_{Rd,c} = \left[\frac{0.18}{\gamma_c} k (100\rho_1 f_{ck})^{\frac{1}{3}} + 0.15\sigma_{cp} \right] b_w d \quad (19)$$

$$\geq (v_{min} + 0.15\sigma_{cp}) b_w d$$

$$k = 1 + (200/d)^{0.5} \leq 2.0$$

$$\gamma_c = 1.5$$

$$\rho_1 = A_s / b_w d \quad (\rho_1 \leq 0.02)$$

$$v_{min} = 0.035k^{3/2} f_{ck}^{1/2}$$

For reinforced webs at ULS:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{yw} \cot \theta \quad (20)$$

$$V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd} / (\cot \theta + \tan \theta) \quad (21)$$

$$\alpha_{cw} = 1 \text{ for non-prestressed structures}$$

$$\nu_1 = 0.6 \text{ for } f_{ck} \leq 60 \text{ MPa and } \nu_1 = 0.9 - f_{ck}/200 > 0.5 \text{ for } f_{ck} \geq 60 \text{ MPa}$$

The shear stress in a wall of a section subject to pure torsion:

$$\tau_{t,i} t_{ef,i} = \frac{T_{Ed}}{2A_k} \quad (22)$$

$\tau_{t,i}$ = torsional stress in wall i; $t_{ef,i}$ = effective wall thickness (= total area of cross section / outer circumference), A_k = area enclosed by centrelines of the walls including inner hollow areas.

The shear force $V_{Ed,i}$ in wall i due to torsion is given by:

$$V_{Ed,i} = \tau_{t,i} t_{ef,i} z_i \quad (23)$$

z_i = side length of wall i

The required area of longitudinal reinforcement for torsion is given by:

$$\frac{\sum A_{sl} f_{yd}}{u_k} = \frac{T_{Ed}}{2A_k} \cot\theta \quad (24)$$

A_{sl} = reinforcement area; u_k = perimeter of area A_k ; f_{yd} = design yield stress of longitudinal reinforcement A_{sl} ; $\cot\theta$ = angle of the compression struts.

Prestressed concrete

Elastic analysis: compression is positive. Eq.(25) applies for both top and bottom fibres since Z_i has sign:

$$\sigma = \frac{P}{A} + \frac{Pe}{Z_i} - \frac{M}{Z_i} \quad (25)$$

To design prestress, stress inequalities take the form:

$$f_c \geq \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} \geq f_t \quad (26)$$

For fibre 1 (top):

$$-\frac{Z_1}{A} + \frac{f_c Z_1}{P} + \frac{M}{P} \leq e \leq -\frac{Z_1}{A} + \frac{f_t Z_1}{P} + \frac{M}{P} \quad (27)$$

For fibre 2 (bottom):

$$-\frac{Z_2}{A} + \frac{f_c Z_2}{P} + \frac{M}{P} \geq e \geq -\frac{Z_2}{A} + \frac{f_t Z_2}{P} + \frac{M}{P} \quad (28)$$

Cumulative normal distribution function

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^u e^{-\frac{x^2}{2}} dx \quad \text{FOR } 0.00 \leq u \leq 4.99.$$

<i>u</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.990097	.990358	.990613	.990863	.991106	.991344	.991576
2.4	.991802	.992024	.992240	.992451	.992656	.992857	.993053	.993244	.993431	.993613
2.5	.993790	.993963	.994132	.994297	.994457	.994614	.994766	.994915	.995060	.995201
2.6	.995339	.995473	.995604	.995731	.995855	.995975	.996093	.996207	.996319	.996427
2.7	.996533	.996636	.996736	.996833	.996928	.997020	.997110	.997197	.997282	.997365
2.8	.997445	.997523	.997599	.997673	.997744	.997814	.997882	.997948	.998012	.998074
2.9	.998134	.998193	.998250	.998305	.998359	.998411	.998462	.998511	.998559	.998605
3.0	.998650	.998694	.998736	.998777	.998817	.998856	.998893	.998930	.998965	.998999
3.1	.9990324	.9990646	.9990957	.9991260	.9991553	.9991836	.9992112	.9992378	.9992636	.9992886
3.2	.9993129	.9993363	.9993590	.9993810	.9994024	.9994230	.9994429	.9994623	.9994810	.9994991
3.3	.9995166	.9995335	.9995499	.9995658	.9995811	.9995959	.9996103	.9996242	.9996376	.9996505
3.4	.9996631	.9996752	.9996869	.9996982	.9997091	.9997197	.9997299	.9997398	.9997493	.9997585
3.5	.9997674	.9997759	.9997842	.9997922	.9997999	.9998074	.9998146	.9998215	.9998282	.9998347
3.6	.9998409	.9998469	.9998527	.9998583	.9998637	.9998689	.9998739	.9998787	.9998834	.9998879
3.7	.9998922	.9998964	.99990039	.99990426	.99990799	.99991158	.99991504	.99991838	.99992159	.99992468
3.8	.99992765	.99993052	.99993327	.99993593	.99993848	.99994094	.99994331	.99994558	.99994777	.99994988
3.9	.99995190	.99995385	.99995573	.99995753	.99995926	.99996092	.99996253	.99996406	.99996554	.99996696
4.0	.99996833	.99996964	.99997090	.99997211	.99997327	.99997439	.99997546	.99997649	.99997748	.99997843
4.1	.99997934	.99998022	.99998106	.99998186	.99998263	.99998338	.99998409	.99998477	.99998542	.99998605
4.2	.99998665	.99998723	.99998778	.99998832	.99998882	.99998931	.99998978	.99999022	.999990655	.999991066
4.3	.999991460	.999991837	.999992199	.999992545	.999992876	.999993193	.999993497	.999993788	.999994066	.999994332
4.4	.999994587	.999994831	.999995065	.999995288	.999995502	.999995706	.999995902	.999996089	.999996268	.999996439
4.5	.999996602	.999996759	.999996908	.999997051	.999997187	.999997318	.999997442	.999997561	.999997675	.999997784
4.6	.999997888	.999997987	.999998081	.999998172	.999998258	.999998340	.999998419	.999998494	.999998566	.999998634
4.7	.999998699	.999998761	.999998821	.999998877	.999998931	.999998983	.9999990320	.9999990789	.9999991235	.9999991661
4.8	.9999992067	.9999992453	.9999992822	.9999993173	.9999993508	.9999993827	.9999994131	.9999994420	.9999994696	.9999994958
4.9	.9999995208	.9999995446	.9999995673	.9999995889	.9999996094	.9999996289	.9999996475	.9999996652	.9999996821	.9999996981

Example: $\Phi(3.57) = .998215 = 0.998215$.