2024-2025 4I10 Exam Crib

Question 1

(a) 40%

Pumping power is the work required to raise the coolant pressure per unit time or force through distance per unit time, i.e. a product of force and flow velocity:

$$W_p = \Delta p\, A\, v = \Delta p\, \frac{\dot{m}}{\rho}$$

Most of the pressure losses are frictional and for a channel with hydraulic diameter D and length L, and flow rate $\dot{m}$ are given by:

$$\Delta p = \frac{\rho v^2}{2} f \frac{L}{D} \quad \text{where } f \text{ is the Moody friction factor.}$$

From NE Databook (p.14) $$f = 0.184 \left(\frac{\rho v D}{\mu}\right)^{-0.2} = 0.184 \left(\frac{\dot{m} D}{A\mu}\right)^{-0.2}$$

Then, $$\Delta p = \frac{1}{2\rho}\left(\frac{\dot{m}}{A}\right)^2 0.184 \left(\frac{\dot{m} D}{A\mu}\right)^{-0.2} \frac{L}{D}$$

Mass flow rate can be expressed from the heat balance and substituted to the pumping power along with the expression for Δp:

$$Q = \dot{m} c_p\, \Delta T \quad \text{or} \quad \dot{m} = \frac{Q}{c_p\, \Delta T}$$

$$W_P = \frac{1}{\rho^2 A^{1.8}} \left(\frac{Q}{C_p \Delta T}\right)^{2.8} \frac{L}{D^{1.2}} \frac{0.184}{2} \mu^{0.2}$$

Regrouping to combine the geometry, heat and properties terms:

$$W_P = 0.184 \left(\frac{L}{2\, A^{1.8} D^{1.2}}\right) \left(\frac{Q}{\Delta T}\right)^{2.8} \left(\frac{\mu^{0.2}}{\rho^2\, c_p^{2.8}}\right)$$

For water: $$\frac{1}{F} = \left(\frac{\mu^{0.2}}{\rho^2\, c_p^{2.8}}\right) = \frac{(8\times10^{-5})^{0.2}}{700^2 \times 5800^{2.8}} = 9.2 \times 10^{-18}$$

For sodium: $$\frac{1}{F} = \left(\frac{\mu^{0.2}}{\rho^2\, c_p^{2.8}}\right) = \frac{(23\times10^{-5})^{0.2}}{800^2 \times 1300^{2.8}} = 5.6 \times 10^{-16}$$

Given the pumping power, core heat generation and core geometry are the same:

$$\left(\frac{1}{\Delta T}\right)^{2.8}_{water} \left(\frac{1}{F}\right)_{water} = \left(\frac{1}{\Delta T}\right)^{2.8}_{sodium} \left(\frac{1}{F}\right)_{sodium}$$

$$\Delta T_{sodium} = \Delta T_{water} \left(\frac{(1/F)_{sodium}}{(1/F)_{water}}\right)^{\frac{1}{2.8}} = 50 \left(\frac{5.6 \times 10^{-16}}{9.2 \times 10^{-18}}\right)^{\frac{1}{2.8}} = 217\ K$$

(b) 30%

Helium is a gas with much lower density even if pressurised. It has very high specific heat, but this is insufficient to compensate for the very low density. The main implication of that is much higher temperature rise required to remove heat. High temperature will be challenging for structural materials which will require to maintain strength at those temperatures and under irradiation. Failure of the components due to pressurisation stress, while operating at high temperature is a safety concern. Depressurisation is typically very rapid and would require reliance on other means of heat removal (like conduction through graphite core matrix) which will restrict power density and challenge the reactor economics. High ΔT can be compensated to an extent by higher flow rate but it will be costly due to high pumping power requirements since frictional pressure loss is roughly proportional to square of flow velocity. Finally, helium is very leaky and will require maintenance and make-up of the inventory, which can also be costly.

(c) 30%

- High FoM which minimises pumping power for a given heat to be transported with a given temperature rise (high density, high specific heat and low viscosity).
- Low neutron absorption for minimal effect on neutron economy.
- High slowing down power $\xi\Sigma_s$ if coolant is used as a moderator to minimise the coolant volume fraction in the core and increase the core power density.
- Low thermal expansion to minimise the reactivity change from cold to hot full power conditions. A combination of absorbing and moderating properties should be such that the coolant thermal expansion reactivity coefficient is nevertheless negative.
- Low freezing point to avoid flow blockage in accident scenarios.
- High boiling point to minimise system pressure and stress on components.
- Chemical compatibility with surrounding materials at normal operation (minimal corrosion) and in accidents (e.g. exothermic sodium-water reaction in case of a leak).
- Stability under irradiation to avoid formation of corrosive species.
- Low activation to minimise maintenance costs and source term during accidents.
- Availability at low cost.

Question 2

(a) 40%

Steady state Heat Conduction Equation in a 1D slab (see NE Databook, p.13):

$$k\frac{d^2T}{dx^2} + q''' = 0$$

Integrating twice with respect to *x*:

$$T(x) = -\frac{q'''x^2}{2k} + C_1x + C_2$$

BC: $T(a) = T_R$; $T(-a) = T_L$

Substituting BC:

$$C_1 = \frac{T_R - T_L}{2a} \quad ; \quad C_2 = \frac{T_R + T_L}{2} + \frac{q'''a}{2k}$$

$$T(x) = \frac{q'''(a^2 - x^2)}{2k} + \frac{T_R - T_L}{2a}x + \frac{T_R + T_L}{2}$$

Heat flux is given by:

$$q'' = -k\frac{dT(x)}{dx} = q'''x - k\frac{T_R - T_L}{2a}$$

Note that we are interested only in the magnitude of outgoing heat fluxes which otherwise have opposite signs (directions) at the left and right boundaries. Since the factor of 2 refers to the magnitude of the heat flux, we have

$$|q''_L| = 2|q''_R|$$

At the right boundary, the heat flux is positive:

$$q''_R = -k\frac{dT}{dx}\Big|_{x=a} = q'''a - k\frac{T_R - T_L}{2a}$$

At the left boundary, the heat flux is negative, so needs reversing the sign to obtain positive value of the magnitude:

$$|q''_L| = k\frac{dT}{dx}\Big|_{x=-a} = -q'''x + k\frac{T_R - T_L}{2a} = q'''a + k\frac{T_R - T_L}{2a}$$

$$q'''a + k\frac{T_R - T_L}{2a} = 2q'''a - 2k\frac{T_R - T_L}{2a}$$

$$T_R - T_L = \frac{2}{3}\frac{q'''a^2}{k}$$

(b) 25%

To transfer all the heat through the left boundary, heat flux through the right boundary should be zero:

$$q''_R = -k\frac{dT}{dx}\Big|_{x=a} = q'''a - k\frac{T_R - T_L}{2a} = 0$$

Therefore: $$T_R - T_L = 2\,\frac{q'''a^2}{k}$$

If the temperature difference is larger than that, the slab will be removing heat from the coolant on the right side of the slab and conducting it to the coolant on the left side, in addition to all the heat generated within the slab.

(c) 35%

There are two factors that limit the core power density: DNBR and fuel temperature.
From NE Databook, we can write for the maximum ΔT across the fuel element:

$$\Delta T_{slab} = q'''\frac{a^2}{2k} \qquad ; \qquad \Delta T_{pin} = q'''\frac{R^2}{4k}$$

i.e. pin-type fuel can support twice the power density for the same ΔT as the plate fuel.
The heat flux expressed in terms of fuel power density for the two configurations:

$$q''_{slab} = q'''a \qquad ; \qquad q''_{pin} = q'''\frac{R}{2}$$

Here, again, the heat flux through the pin surface is half that of the plate if $a = R$.
Since coolant to fuel volume ratio is fixed, these conclusions translate directly from the fuel power density into the core power density through scaling it by the same factor.
Smaller surface area (or surface to volume ratio) of the plate fuel may have some neutronic advantage because of the stronger self-shielding of U238 and therefore lower resonance absorption and higher reactivity.
Other considerations such as cladding thickness and its parasitic absorption or effect on V_m/V_f may also affect the neutronics (e.g. plate fuel will have less cladding because of the smaller surface to volume ratio).
Finally, structural stability considerations of the fuel assembly may have an effect on the choice of a particular fuel element type.

Question 3

(a) 40%

The core power density distribution: $q'''(r,z) = \Phi(r,z)\ \sigma_f\ N_{U235}\ E_f$

$$N_{U235} = \frac{Number\ of\ U235\ atoms}{Core\ volume} = \frac{\frac{2000\times10^3 g}{235\ g/mole}\times 6.022\times10^{23} atoms/mole}{\pi\ 75^2\times150\ cm^3} = 1.93\times10^{21}\ \frac{\#}{cm^3}$$

$$E_f = 200\ MeV = 3.2\times10^{-11}\ J$$

$$\sigma_f\ N_{U235}\ E_f = 35\times10^{-24}\times1.93\times10^{21}\times3.2\times10^{-11} = 2.16\times10^{-12}\ \frac{J}{cm}$$

Integrate over the core volume and normalise to 100 MW to find the flux normalisation constant Φ_0.

$$Q = 200\times10^6\ W = \int_{-\frac{L}{2}}^{+\frac{L}{2}} dz \int_0^R 2\pi r dr\ \Phi(\mathrm{r,z})\ \sigma_f\ N_{U235}\ E_f =$$

$$= \Phi_0\ \sigma_f\ N_{U235}\ E_f \int_{-75}^{+75} cos\left(\frac{\pi z}{150}\right) dz \int_0^{75} \left(1-\left(\frac{r}{75}\right)^2\right) 2\pi r dr =$$

$$= \Phi_0\ \sigma_f\ N_{U235}\ E_f \left(\frac{150}{\pi} sin\left(\frac{\pi z}{150}\right)\right)\Big|_{-75}^{+75} \times \left(\pi r^2 - \frac{\pi r^4}{11250}\right)\Big|_0^{75} =$$

$$= \Phi_0\ \times 2.16\times10^{-12}\times 843750 = \Phi_0\ \times 1.8225\times10^{-6}$$

$$\Phi_0 = \frac{200\times10^6}{1.8225\times10^{-6}} = 1.0974\times10^{14}\ \frac{\#}{cm^2 s}$$

$$q'''(20,20) = 1.0974\times10^{14}\times2.16\times10^{-12}\left(1-\left(\frac{20}{75}\right)^2\right) cos\left(\frac{20\pi}{150}\right) \approx 201\ \frac{W}{cm^3}$$

(b)

(i) 15%

Let x be the fraction of fuel type a in a batch. According to Linear Reactivity Model, the core reactivity is an average of reactivities of all the assemblies comprising it. Then, neglecting leakage, we can write reactivity balance for the end of equilibrium cycle (EOC).

Fresh batch: $x(\rho_a - A_a B_c) + (1-x)(\rho_b - A_b B_c)$

Once burnt: $x(\rho_a - 2A_a B_c) + (1-x)(\rho_b - 2A_b B_c)$

Twice burnt: $x(\rho_a - 3A_a B_c) + (1-x)(\rho_b - 3A_b B_c)$

Summing up the above reactivities all weighted equally at equilibrium and setting the average to zero at EOC

$$\frac{x(3\rho_a - 6A_a B_c) + (1-x)(3\rho_b - 6A_b B_c)}{3} = 0$$

$$3x\rho_a - 6xA_aB_c + 3\rho_b - 6A_bB_c - 3x\rho_b + 6xA_bB_c = 0$$

$$x = \frac{2A_bB_c - \rho_b}{(\rho_a - 2A_aB_c - \rho_b + 2A_bB_c)} = \frac{2\times0.01\times11 - 0.24}{(0.18 - 2\times0.009\times11 - 0.24 + 2\times0.01\times11)} \approx 0.53$$

(ii) 15%

Core average reactivity at the beginning of equilibrium cycle: $\rho_{core} = \frac{1}{3}\sum_{i=1}^{3}\rho_i$

Fresh batch: $x(\rho_a - 0) + (1-x)(\rho_b - 0)$

Once burnt: $x(\rho_a - A_aB_c) + (1-x)(\rho_b - A_bB_c)$

Twice burnt: $x(\rho_a - 2A_aB_c) + (1-x)(\rho_b - 2A_bB_c)$

$$\rho_{core} = \frac{1}{3}\sum_{i=1}^{3}\rho_i = \frac{1}{3}[(3x\rho_a - 3xA_aB_c) + (1-x)(3\rho_b - 3A_bB_c)]$$

$$\rho_{core} = [(x\rho_a - xA_aB_c) + (1-x)(\rho_b - A_bB_c)] = 0.10403$$

(c) 30%

Initial reactivity:

1. Enrichment (or fissile isotopes content). This increases the probability of fission and thus reactivity. It also reduces the proportion of fertile (absorbing) fuel isotopes which also increases the reactivity.

2. Minimising the parasitic absorption through the choice and amount of coolant and structural material in the core.

3. Reduce the fuel surface area exposed to moderator. This will increase self-shielding of U238, reduce its effective absorption cross section and increase the fuel reactivity.

4. Relative volumes of fuel and moderator which affects the neutron spectrum and thus relative competitiveness of fission and absorption reactions. Thermal spectrum favours fission more (U235 fissions are predominantly thermal) than absorption (which is predominantly epithermal in U238 resonances).

5. Amount of other absorbers in the fuel e.g. burnable poisons can be varied according to operational requirements. More absorber atoms imply lower initial reactivity.

Rate of reactivity loss may depend on several factors:

1. Rate of conversion of fertile to fissile isotopes – breeding of Pu239 from captures in U238. Encouraging fertile absorptions is detrimental to initial reactivity but favourable for minimising the loss of reactivity with burnup. The reactivity vs burnup curve becomes flatter (less steep). In this case, higher fuel surface area and minimal self-shielding as well as harder neutron spectrum may be attractive.

2. The type, total amount and arrangement of burnable absorbers will affect the rate of their depletion. Gadolinium self-shielding can be varied by spreading it over more or fewer fuel pins affecting the initial reactivity suppression which is controlled by the Gd containing pins surface area and the time of Gd depletion, which is controlled by the Gd loading per pin.

3. For the same assembly fissile content, spatial distribution of fissile and fertile isotopes can promote or suppress fertile captures. Fissile-rich areas with optimised thermal spectrum can initially be supercritical supplying neutrons to fertile-rich areas with harder spectrum promoting breeding which become net producers of neutrons later in the cycle. Such arrangements will also flatten the reactivity vs burnup curve.

Question 4

(a) 30%

Recalling the solution of radial power distribution in an infinite cylindrical fuel pin (NE Databook, p.12)

$$\int_T^{T_{CL}} k(T)dT = \frac{q'''r^2}{4}$$

For temperature-independent thermal conductivity:

$$T(r) = T_{CL} - \frac{q'''r^2}{4k}$$

Volume weighted average fuel temperature:

$$T_{av} = \frac{\int_0^R T(r)\, 2\pi r dr}{\int_0^R 2\pi r dr} = \frac{\int_0^R \left(T_{CL} - \frac{q'''r^2}{4k}\right) 2\pi r dr}{\int_0^R 2\pi r dr} = \frac{\frac{2\pi R^2\, T_{CL}}{2} - \frac{2\pi\, q''' R^4}{16}}{\pi R^2} = T_{CL} - \frac{q''' R^2}{8k}$$

Comparing: $T_{CL} - T_{av} = \frac{q''' R^2}{8k}$ with $T_{CL} - T_{fo} = \frac{q''' R^2}{4k}$, one can see that the average fuel temperature is half way between the outer surface and the maximum fuel temperatures.

(b) 15%

In this case, both the average fuel and moderator temperatures are reduced by the same $\Delta T = 5\ K$. Therefore:

$$\Delta\rho = \frac{\partial\rho}{\partial T_{fuel}} \times (-5) + \frac{\partial\rho}{\partial T_{mod}} \times (-5) = -2 \times (-5) - 20 \times (-5) = 110\ pcm$$

(c) 20%

The coolant outlet and average temperatures:

$$T_{out} = T_{in} + \frac{Q}{\dot{m}c_p}; \qquad T_{core} \approx \frac{T_{out} + T_{in}}{2} = T_{in} + \frac{Q}{2\dot{m}c_p}$$

After the change in flow rate:

$$\tilde{T}_{core} \approx \frac{T_{out} + T_{in}}{2} = T_{in} + \frac{Q}{2\tilde{m}c_p};$$

$$\Delta\tilde{T}_{core} = \tilde{T}_{core} - T_{core} = \frac{Q}{2\tilde{m}c_p} - \frac{Q}{2\dot{m}c_p} = \frac{3000\times10^6}{2\times15\times10^3\times5800} - \frac{3000\times10^6}{2\times20\times10^3\times5800} = 4.3\ K$$

Both average fuel temperature will increase by the same ΔT since the power stayed the same.

$$\Delta\rho = \frac{\partial\rho}{\partial T_{fuel}} \times 4.3 + \frac{\partial\rho}{\partial T_{mod}} \times 4.3 = -2 \times 4.3 - 20 \times 4.3 = -95\ pcm$$

(d) 35%

Increasing the power while keeping the flow rate and inlet temperature the same will result in higher core outlet and core average coolant temperature. The coolant temperature gain across the core is linear because the power distribution is uniform.

The increase in coolant outlet temperature:

$$T_{out} = T_{in} + \frac{Q}{\dot{m}c_p}$$

$$T_{core} \approx \frac{T_{out}+T_{in}}{2} = T_{in} + \frac{Q}{2\dot{m}c_p}$$

After the power is increased to $\tilde{Q}$:

$$\tilde{T}_{out} = T_{in} + \frac{\tilde{Q}}{\dot{m}c_p}$$

$$\tilde{T}_{core} \approx \frac{\tilde{T}_{out} + T_{in}}{2} = T_{in} + \frac{\tilde{Q}}{2\dot{m}c_p}$$

$$\Delta T_{core} = \tilde{T}_{core} - T_{core} = \frac{\tilde{Q}-Q}{2\dot{m}c_p} = \frac{500\times10^6}{2\times20\times10^3\times5800} \approx 2.2\ K$$

Fuel average temperature will also increase. T_{fo} can be taken to be equal to the core average coolant temperature T_{core}. The temperature rise across the pellet is proportional to power density which increases by: $\frac{3500-3000}{3000} \approx 16.7\%$, so both $(T_{CL} - T_{fo})$ and $(T_{av} - T_{fo})$ will increase proportionately.

At nominal power: $(T_{av} - T_{fo}) = \frac{q'''R^2}{8}$;

After the increase: $(\tilde{T}_{av} - \tilde{T}_{fo}) = \frac{\tilde{q}'''R^2}{8} = (T_{av} - T_{fo})\frac{\tilde{Q}}{Q}$;

This increase is only due to the power increase in the fuel. The fuel surface boundary conditions have also changed because the average coolant temperature has increased by ΔT_{core}:

$$T_{fo} \approx T_{core} = \frac{T_{out}+T_{in}}{2} = T_{in} + \frac{Q}{2\dot{m}c_p} = 570 + \frac{3000\times10^6}{2\times20\times10^3\times5800} = 583\ K$$

$\tilde{T}_{fo} \approx T_{fo} + \Delta T_{core} = 583 + 2.2 = 585.2\ K$

Putting it all together, the new average fuel temperature:

$$\tilde{T}_{av} = \tilde{T}_{fo} + \left(T_{av} - T_{fo}\right)\frac{\tilde{Q}}{Q} = T_{fo} + \Delta T_{core} + \left(T_{av} - T_{fo}\right)\frac{\tilde{Q}}{Q}$$

$$\tilde{T}_{av} = 583 + 2.2 + (900 - 583)\frac{3500}{3000} = 955\ K$$

$$\Delta\tilde{T}_{av} = \tilde{T}_{av} - T_{av} = 955 - 900 = 55\ K$$

The core reactivity change:

$$\Delta\rho = \frac{\partial\rho}{\partial T_{fuel}} \times \Delta\tilde{T}_{av} + \frac{\partial\rho}{\partial T_{mod}} \times \Delta T_{core} = -2 \times 55 - 20 \times 2.2 = -154\ pcm$$