4M16 2026 crib v2

pmc55

January 2026

Question 1

1(a)

The governing differential equation is:

$$\frac{dN}{dt} = \sigma_{\mathrm{a}}\phi N_{\mathrm{p}} - \lambda N$$

Its only production mechanism is through neutron absorption while it is only destroyed by decay.

This differential equation has the complementary function:

$$N = B\mathrm{e}^{-\lambda t}$$

and the particular integral:

$$N = \frac{\sigma_{\mathrm{a}}\phi N_{\mathrm{p}}}{\lambda}$$

giving the solution:

$$N = B\mathrm{e}^{-\lambda t} + \frac{\sigma_{\mathrm{a}}\phi N_{\mathrm{p}}}{\lambda}$$

Inserting the initial condition that $N(0) = 0$ gives the unknown constant as as:

$$B = -\frac{\sigma_{\mathrm{a}}\phi N_{\mathrm{p}}}{\lambda}$$

and the overall solution as:

$$N(t) = \frac{\sigma_{\mathrm{a}}\phi N_{\mathrm{p}}}{\lambda}\left(1 - \mathrm{e}^{-\lambda t}\right)$$

We can rearrange this for the flux easily:

$$\phi = \frac{\lambda N(t)}{\sigma_{\mathrm{a}} N_{\mathrm{p}}\left(1 - \mathrm{e}^{-\lambda t}\right)} \tag{1}$$

where we recognise that the activity per unit volume is given as $A = \lambda N$

Marks: 30%

1(b)

Part (i): This relies on the following formula:

$$D = \frac{1.6 \times 10^{-13}\Sigma E_{\gamma} A t}{4\pi\rho r^2}$$

A must be computed given the available data. Rearranging the previous equation (and multiplying by the 1 cm^3 volume) gives:

$$A_0 = \sigma_{\mathrm{a}} N_{\mathrm{p}} \phi (1 - \mathrm{e}^{-\lambda t})$$

Inserting the given values (including 5 hours of irradiation), we have that

$$A_0 = 2.42 \times 10^{11}\text{Bq}$$

This activity is attenuated by another hour:

$$A = A_0 \mathrm{e}^{-\lambda \times 1\text{hr}} = A_0 \mathrm{e}^{0.693} = A_0/2 = 1.21 \times 10^{11}\text{Bq}$$

Inserting all numbers into the equation (and accounting for the weighting factor of 1 for gamma rays) gives:

$$D = 1 \times \frac{1.6 \times 10^{-13} \times 1 \times 1 \times 20 \times 60 \times 1.21 \times 10^{11}}{4\pi \times 2^2 \times 1000} = 0.462 \text{ mSv}$$

Marks: 30%

Part (ii): The change here is replacing the At term in the dose equation. This is replaced by integrating the activity over time:

$$At \rightarrow \int_0^t A(t)dt = \int_0^t A\mathrm{e}^{-\lambda t}dt = \frac{A}{\lambda}(1 - \mathrm{e}^{-\lambda t})$$

This transforms the dose equation into:

$$D = \frac{1.6 \times 10^{-13} \Sigma E_\gamma}{4\pi\rho r^2} \frac{A}{\lambda} \left(1 - \mathrm{e}^{-\lambda t}\right)$$

Giving a dose of 0.413 mSv.

The assumption of a constant activity is reasonable if the exposure time is significantly lower than the half life. That isn't the case here: the exposure time is a third of the half life.

Marks: 25%

1(c)

In radiation protection, we seek to maximise the distance from sources, minimise the time exposed to sources, and apply shielding between us and the source.

We would like to shorten the processing time for the isotope, extend the distance from it (e.g., by remote handling), and apply shielding between staff and the isotope (e.g., lead).

Marks: 15%

Question 2

2(a)

For the fuel we have the following governing differential equation:

$$\frac{d^2\phi}{dx^2} + B^2\phi = 0$$

The solution of this is:

$$\phi(x) = A\cos(Bx) + C\sin(Bx)$$

Taking $x = 0$ as the left boundary and applying the boundary condition

$$\left.\frac{d\phi}{dx}\right|_{x=0} = 0$$

we have:

$$\frac{d\phi}{dx} = -AB\sin(Bx) + CB\cos(Bx)$$

This can only equal 0 if $C = 0$. Alternatively, and perhaps more simply, one could set $C = 0$ by noting the flux must be symmetric about $x = 0$. Hence we are left with:

$$\phi(x) = A\cos(Bx)$$

in the fuel region.

For the moderator, the differential equation is:

$$\frac{d^2\phi}{dx^2} - \frac{1}{L^2}\phi = 0$$

The solution of this is:

$$\phi(x) = A'\cosh(x/L) + C'\sinh(x/L) \tag{2}$$

We want to apply the corresponding boundary condition ($\frac{d\phi}{dx}\Big|_{x=a+b} = 0$), requiring the derivative:

$$\frac{d\phi}{dx} = \frac{A'}{L}\sinh(x/L) + \frac{C'}{L}\cosh(x/L) \tag{3}$$

Applying the boundary condition at $x = a + b$ gives:

$$0 = \frac{A'}{L}\sinh(\frac{a+b}{L}) + \frac{C'}{L}\cosh(\frac{a+b}{L}) \tag{4}$$

and hence:

$$C' = -A'\tanh(\frac{a+b}{L})$$

giving:

$$\phi(x) = A'\left[\cosh(x/L) - \tanh(\frac{a+b}{L})\sinh(x/L)\right] \tag{5}$$

or, by multiplying and dividing by $\cosh(\frac{a+b}{L})$ to create a new prefactor:

$$\phi(x) = A''\left[\cosh(\frac{a+b}{L})\cosh(x/L) - \sinh(\frac{a+b}{L})\sinh(x/L)\right] \tag{6}$$

By the trigonometric/hyperbolic addition rule ($\cosh(x)\cosh(y) - \sinh(x)\sinh(y) = \cosh(x-y)$, found on page 3 of the CUED Maths Data Book) this becomes:

$$\phi(x) = A''\cosh\left(\frac{x-a-b}{L}\right)$$

where $A'' \to C$ to match what the question asks.

Marks: 40%

2(b)

The system being critical ensures that there is a steady-state solution and the fluxes and currents between the fuel and moderator must be continuous.

The continuity conditions are that when $x = a$ we have the following:

$$\phi_{\text{fuel}}(a) = \phi_{\text{mod}}(a)$$

$$D_{\text{fuel}}\left.\frac{d\phi_{\text{fuel}}}{dx}\right|_{x=a} = D_{\text{mod}}\left.\frac{d\phi_{\text{mod}}}{dx}\right|_{x=a}$$

The first equation gives:

$$A\cos(Ba) = A''\cosh\left(\frac{-b}{L}\right) = A''\cosh\left(\frac{b}{L}\right)$$

The second equation gives:

$$-ABD_{\text{fuel}}\sin(Ba) = \frac{A''D_{\text{mod}}}{L}\sinh\left(\frac{-b}{L}\right) = -\frac{A''D_{\text{mod}}}{L}\sinh\left(\frac{b}{L}\right)$$

or

$$ABD_{\text{fuel}}\sin(Ba) = \frac{A''D_{\text{mod}}}{L}\sinh\left(\frac{b}{L}\right)$$

Dividing the first equation by the second eliminates the unknown constants and produces tan/tanh:

$$BD_{\text{fuel}}\tan(Ba) = \frac{D_{\text{mod}}}{L}\tanh\left(\frac{b}{L}\right)$$

as required.

Marks: 50%

2(c)

Monoenergetic neutron diffusion is a poor choice for two reasons in this scenario. First, diffusion is a poor approximation in the vicinity of interfaces of very different materials (such as between fuel and moderator). Second, the physics of (thermal) reactor lattices is strongly driven by energy variations, and much of this is lost by condensing to a single group. For example, one would expect the fast flux to be peaked in the fuel and minimum in the moderator, and vice versa for the thermal flux. The thermal flux is arguably more important for determining many quantities of interest, but its distribution is completely opposite to what has just been derived!

Marks: 10%

Question 3

3(a)

The terms in the point kinetics equations are:

1. $\frac{\mathrm{d}n}{\mathrm{d}t}$ is the rate of change of the neutron population with time.
2. $\frac{\rho-\beta}{\Lambda}n$ is the net rate at which prompt neutrons are produced from fission.
3. $\lambda_{\mathrm{d}}c$ is the rate at which precursors decay and delayed neutrons are produced.
4. $\frac{\mathrm{d}c}{\mathrm{d}t}$ is the rate of change of the delayed neutron precursor population.
5. $\frac{\beta}{\Lambda}n$ is the rate at which precursors are produced.

Marks: 20%

3(b)

The pre-precursors essentially follow the same behaviour as delayed neutrons *originally* did. They are produced in fission (at a rate of $\beta n/\Lambda$) and they are lost only by decay (at a rate of $\lambda_{\mathrm{p}}p$). This gives the requires differential equation.

The precursor equation must be changed (the neutron equation remains the same). Precursors are no longer produced directly by fission. Instead, they are produced through the decay of pre-precursors. They are lost by the same mechanism as before, i.e., their own decay to produce delayed neutrons. This gives the precursor equation as:

$$\frac{dc}{dt} = \lambda_{\mathrm{p}}p - \lambda_{\mathrm{d}}c$$

Marks: 20%

3(c)

Assuming an exponential solution is the easiest approach, although one could use Laplace transforms as well.

For the exponential ansatz, we assume the following:

$$n = n_0 \mathrm{e}^{\omega t}$$

$$c = c_0 \mathrm{e}^{\omega t}$$

$$p = p_0 \mathrm{e}^{\omega t}$$

We insert these equations into the modified point kinetics equations to obtain:

$$\omega n_0 = \frac{\rho - \beta}{\Lambda} n_0 + \lambda_\mathrm{d} c_0$$

$$\omega p_0 = \frac{\beta}{\Lambda} n_0 - \lambda_\mathrm{p} p_0$$

$$\omega c_0 = \lambda_\mathrm{p} p_0 - \lambda_\mathrm{d} c_0$$

These equations can be used to eliminate the unknown constants of n_0, c_0, and p_0. From the second equation we have:

$$p_0 = \frac{\beta}{\Lambda(\omega + \lambda_\mathrm{p})} n_0 \tag{7}$$

while from the third we have:

$$c_0 = \frac{\lambda_\mathrm{p}}{\omega + \lambda_\mathrm{d}} p_0$$

or

$$c_0 = \frac{\lambda_\mathrm{p} \beta}{\Lambda(\omega + \lambda_\mathrm{p})(\omega + \lambda_\mathrm{d})} n_0$$

Finally, inserting that into the first equation and cancelling n_0 gives:

$$\omega = \frac{\rho - \beta}{\Lambda} + \frac{\lambda_\mathrm{d} \lambda_\mathrm{p} \beta}{\Lambda(\omega + \lambda_\mathrm{p})(\omega + \lambda_\mathrm{d})}$$

Rearranging for ρ gives:

$$\rho = \Lambda\omega + \beta - \frac{\lambda_\mathrm{d} \lambda_\mathrm{p} \beta}{(\omega + \lambda_\mathrm{p})(\omega + \lambda_\mathrm{d})}$$

as required.

By Laplace transforms, the transformed equations become:

$$sN - n_0 = \frac{\rho - \beta}{\Lambda} N + \lambda_\mathrm{d} C$$

$$sP - p_0 = \frac{\beta}{\Lambda} N - \lambda_\mathrm{p} P$$

$$sC - c_0 = \lambda_\mathrm{p} P - \lambda_\mathrm{d} C$$

We don't actually need to obtain the steady-state precursor and pre-precursor populations.

Rearranging the second equation gives:

$$P = \frac{p_0}{s + \lambda_\mathrm{p}} + \frac{\beta}{\Lambda(s + \lambda_\mathrm{p})} N$$

The third equation gives:

$$C = \frac{c_0}{s + \lambda_\mathrm{d}} + \frac{\lambda_\mathrm{p}}{s + \lambda_\mathrm{d}} P$$

or, inserting the previous:

$$C = \frac{c_0}{s+\lambda_\mathrm{d}} + \frac{\beta\lambda_\mathrm{p}}{\Lambda(s+\lambda_\mathrm{d})(s+\lambda_\mathrm{p})}N + \frac{\lambda_\mathrm{p}}{(s+\lambda_\mathrm{d})(s+\lambda_\mathrm{p})}p_0$$

Inserting into the first equation gives:

$$(s - \frac{\rho-\beta}{\Lambda})N - n_0 = \frac{\lambda_\mathrm{d}c_0}{s+\lambda_\mathrm{d}} + \frac{\beta\lambda_\mathrm{p}\lambda_\mathrm{d}}{\Lambda(s+\lambda_\mathrm{d})(s+\lambda_\mathrm{p})}N + \frac{\lambda_\mathrm{d}\lambda_\mathrm{p}p_0}{(s+\lambda_\mathrm{d})(s+\lambda_\mathrm{p})}$$

or rearranging to isolate N:

$$\left[s - \frac{\rho-\beta}{\Lambda} - \frac{\beta\lambda_\mathrm{p}\lambda_\mathrm{d}}{\Lambda(s+\lambda_\mathrm{d})(s+\lambda_\mathrm{p})}\right]N = n_0 + \frac{\lambda_\mathrm{d}c_0}{s+\lambda_\mathrm{d}} + \frac{\lambda_\mathrm{d}\lambda_\mathrm{p}p_0}{(s+\lambda_\mathrm{d})(s+\lambda_\mathrm{p})}$$

We seek the roots of the term acting on N. Setting it equal to zero and rearranging directly gives the inhour equation, making the symbol change $s \to \omega$. The other terms are forcing terms due to the initial conditions. They contain the same (or, rather, fewer) roots.

Marks: 50%

3(d)

This question is interested in the extreme values of λ_p. As this decay constant becomes very large, the pre-precursor decays essentially instantaneously. This means that the pre-precursor dynamics can essentially be ignored and they have no relevance to reactor control/stability. This could be shown by taking the limit as $\lambda_\mathrm{p} \to \infty$. Conversely, for very small values of λ_p, it will take significantly longer to produce a delayed neutron than previously, lengthening the timescales involved in delayed neutron dynamics.

Prompt neutron dynamics are unaffected by the existence of pre-precursors; a penalty would be given if it is claimed otherwise.

Marks: 10%

Question 4

4(a)

Hold up and decay is the simplest and cheapest method of liquid and gaseous and solid waste treatment for short half life nuclides. It can also be used for some medium term half life nuclides if the quantities are low. It simply involves storing the waste for sufficient time for the activity to decay to a safe level before discharge to the environment. At least two tanks/containers are normally needed, one filling and one holding the material for the required period of time. In the case of gases charcoal absorber beds can be used as an alternative method of hold up. Care may be needed with some daughter products as they can be more radio-active than the parent. Tanks must be shielded but otherwise operator dose uptake is low and little maintenance is required.

Ion exchange using either organic or inorganic media is very useful for some longer lived nuclides in liquid waste. In this process the active ions such as cobalt 60 are exchanged for non-active ions such as sodium or hydrogen in a process very similar to that used in water treatment. Unlike conventional ion exchange, the media is not regenerated when saturated but is usually encapsulated for long term disposal. Decontamination factors of between 10 and 100 are possible with some nuclides but not all are suited to this form of treatment. It is relatively cheap and simple, the only problem being the long term disposal of the spent media. Operator dose is again low as long as the vessels are shielded though the disposal of the spent media can give rise to higher doses. It is not effective on gases.

The most intractable, usually long half-life, liquid wastes wastes may be treated by evaporation which produces a pure distillate that can be discharged to the environment and a highly active concentrate that can be immobilised by encapsulation or vitrification. It is very expensive in capital costs and also very energy intensive and can give rise to high operator dose rates due to the need for maintenance; it is normally only

used for very difficult materials such as reprocessing wastes or in locations where no radioactive discharge to the environment is permitted.

Solid wastes arise in a variety of forms and the need in all cases is to prevent the migration of the radioactive nuclides into the environment. For low level waste all that is needed is to landfill them in a sealed pit isolated from the environment and monitor the run off to ensure no contamination. This is not suitable for alpha or other long lived nuclides. Medium active waste is usually encapsulated in cement and sealed in stainless steel drums for eventual storage in an underground repository. High level wastes such as fission products are first evaporated to reduce volume then converted into glass for long term storage. Cost and operator dose uptake will depend on the activity of the wastes and the complexity of the equipment, some of which can require quite a bit of maintenance and hence operator dose.

4(b)

Part (i):

Inlet activity: 10 Bq/g

DF: 20

Outlet activity: 0.5 Bq/g

Activity captured: 9.5 Bq/g

Part (ii):

Decay constant is given by: $\lambda_{\text{Ag-110m}} = \ln 2/(252 \times 24) = 1.146 \times 10^{-4}$ h^{-1}

The Ag-110m will decay whilst captured and the number of atoms of Ag-110m arising is given by:

$P = Q \times A_{\text{Ag-110m}} \times \rho \times 1000 \times 3600/\lambda_{\text{Ag-110m}}$

where Q is the flow rate (m^3/h), A the activity captured (Bq/g), ρ is the effluent density (kg/m^3), $\lambda_{\text{Ag-110m}}$ is the decay constant in h^{-1}, 1000 is the conversion from kg to g, and 3600 is the conversion from hours to seconds.

$P = 50 \times 9.5 \times 1000 \times 1000 \times 3600/(1.146 \times 10^{-4}) = 1.49 \times 10^{16}$ atoms/h

Decay of Ag-110m over 2 years is given by:

$N_{\text{Ag-110m}} = (P/\lambda_{\text{Ag-110m}})(1 - \exp(-\lambda_{\text{Ag-110m}}T))$

$= (1.49 \times 10^{16}/1.146 \times 10^{-4})(1 - \exp(-1.146 \times 10^{-4} \times 2 \times 8760))$

$= 1.13 \times 10^{20}$ atoms Ag-110m.

Activity on the medium:

Mass of medium (M) is given by $500 \times 1200 = 6 \times 10^5$ kg

Total activity is given by $N_{\text{Ag-110m}} \times \lambda_{\text{Ag-110m}}/3600$ Bq

$= 1.13 \times 10^{20} \times 1.146 \times 10^{-4}/3600 = 3.59 \times 10^{12}$ Bq

Specific activity is given by $3.59 \times 10^{12}/(6 \times 10^5 \times 1000) = 5980$ Bq/g